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Objective Estimation of Velocity Streamfunction Field with Discretely Sampled Oceanic Data II: with Application of Least-square Regression Analysis

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A least-square regression analysis is applied for the estimation of velocity streamfunction field based on discretely sampled current meter data. The coefficients of a streamfunction that is expanded in terms of trigonometric basis function are obtained by enforcing the horizontal non-divergence of two-dimensional flow field. This method avoids interpolation and gives a root-mean-square (rms) residual of fit which includes the divergent part and noisiness of oceanic data. The implementation of the method is done by employing a boundary-fitted, curvilinear orthogonal coordinate which facilitates the specification of boundary conditions. An application is successfully made to the Texas-Louisiana shelf using the 32 months current meter data (31 moorings) observed as a part of the Texas-Louisiana Shelf and Transport Processes Study (LATEX). The rms residual of the fitting is relatively small for the shelf, which indicates the field is well represented by the streamfunction.

Key words : objective analysis, streamfunction, coastal circulation, ocean current

1. Introduction

In recent years, the demand for the understanding of synoptic structure of ocean current has been increased in the field of environment, fishery as well as oceanography. In oceanography, the synoptic map is required for the understanding of ocean currents and the initialization of the computer model to forecast the oceanic condition. For the demand, comprehensive observations have been conducted in variety ways of traditional current mooring such as the Mid Ocean Dynamics Experiment (MODE) (Bretherton *et al.*, 1976) and Texas-Louisiana Shelf Circulation and Transport Processes Study (LATEX) (Nowlin *et al.*, 1991), and remote sensing such as AVHRR (for example, NOAA-6) and altimeter (Seasat, Geosat and Topex/Poseidon) (Wunsch and Gasposchkin, 1980). However, the direct sampling had to be limited in space and time due to different reasons. Due to the sparse sampling, we confront with a necessity of the objective analysis (interpolation) for synoptic mapping from discretely sampled oceanic data. The main question we asked in this and last (Cho, 1997) papers is that how the low-frequency current is objectively mapped from discretely measured current meter data observed over the Texas-Louisiana continental shelf as a part of LATEX.

In the last paper (Cho, 1997), I introduced an objective method for the generation of velocity streamfunction dealing with the Poisson equation and implemented the methodology with the LATEX current meter data. The primary disadvantages of the method are a necessity of prior interpolation of the observed velocity field to estimate the vorticity field and the problem of specifying boundary condition, which are inherent in the solution of Poisson equation. In this paper we will employ a simpler method in order to avoid the problems of Poisson equation. The method in the present study is based on the least-square regression analysis, first applied in oceanography context by Vastano and Reid (1985). They estimated sea surface topography in a portion of the Oyashio Frontal Zone in the northwestern Pacific that includes the First and Second Oyashio Intrusions and an anticyclonic eddy, using the infrared satellite imaginary (NOAA-6 AVHRR). The method was also applied in the California Current off Point Sur (Njoku *et al.*, 1985). The method is now an important tool in oceanography for both the analysis of observation (Njoku *et al.*, 1985) and the initialization of numerical model (Arango, 1990).

The method is basically spectral one and assumes that the streamfunction field can be represented by a series of orthogonal basis function. The coefficients of the series are obtained by a least-square fit to the observed velocity data, which is a process to impose the non-divergence constraint for the streamfunction. This method also produces the root-mean-square (rms) residual of the fitting which includes the noisiness and divergent part of measured data. The residual explains whether the oceanic field is well represented by the streamfunction. In addition to the theory of the method, we will also discuss the implementation of the method in terms of the LATEX current meter measurements. This is the first implementation of the method using the moored current meter data. For the implementation of the method we employed boundary-fitted curvilinear orthogonal coordinates in which boundary condition is easily specified. We expect the method can be extended for the construction of sea surface topography from altimeter data.

In section 2 we describe the methodology for the generation streamfunction. Section 3 discusses the implementation of the method which includes the current meter data, the generation of boundary-fitted curvilinear orthogonal grid, test for the number of modes, and resulting velocity streamfunction. Section 4 is a concluding discussion.

2. Methodology

The method adopted in this study is a least-square fitting of observed velocities using a series of trigonometric basis functions for representation of the streamfunction, whose gradient determines the flow field in a manner similar to geopotential anomaly (Vastano and Reid, 1985). We use the boundary-fitted curvilinear orthogonal coordinates. The streamfunction is taken as

$$\psi(\xi,\eta) = \sum_{n}\sum_{m}(A_{n,m}\cos m\alpha\xi + B_{n,m}\sin m\alpha\xi)\sin n\beta\eta \qquad (1)$$

where $\alpha=\pi/L_\xi$, $\beta=\pi/2L_\eta$, ξ and η represent the alongshelf and cross-shelf axes, L_ξ, L_η represent the alongshelf and cross-shelf ranges of the domain, and $A_{n,m}$ and $B_{n,m}$ are coefficients to be determined. The streamfunction, ψ, become zero at the coastal boundary by the last sine term of the equation (1), thus specifying the condition of no flow across the coastal boundary. The major advantage of the present method is to avoid the need for interpolation. The coefficients $A_{n,m}$ and $B_{n,m}$ are obtained by minimizing the following error measure:

$$\sigma^2 = \frac{1}{j_{\max}} \sum_j [\,(u-u_j)^2 + (v-v_j)^2\,], \tag{2}$$

where

$$u = -\frac{\partial\psi}{\partial\eta} = -\sum_n\sum_m (A_{n,m} \cos m\alpha\xi + B_{n,m} \sin m\alpha\xi)\, n\beta \cos n\beta\eta \tag{3}$$

and

$$v = \frac{\partial\psi}{\partial\xi} = \sum_n\sum_m (-A_{n,m} \sin m\alpha\xi + B_{n,m} \cos m\alpha\xi)\, m\alpha \sin n\beta\eta \tag{4}$$

The solution for the coefficients involves the inversion of the matrix obtained from a linear set of equations. The standard lower/upper triangular (LU) decomposition technique (Press *et al.*, 1986) is used for the matrix inversion. Using the coefficients, we can obtain the streamfunction field with (1). The rms residual of the fitting is also computed using the equation (2).

3. Application

3.1 Data

For an application of the methodology discussed in the last section, we used the LATEX current meter data measured over the entire Texas-Louisiana continental shelf at 31 moorings with 75 current meters (Fig. 1) from April 1992 and November 1994. The basic data processing is to remove the high frequency motions such as tide and inertial motion. The 40-hr low-pass Lanczos filter was used for the purpose. For detailed descriptions of the data see Cho (1997) and Nowlin *et al.* (1991).

3.2 Boundary-fitted orthogonal curvilinear grid

The realistic representation of coastal line and subsequent implementation of boundary condition are especially difficult in the mapping of shelf region. Therefore, a boundary-fitted, orthogonal curvilinear coordinate system is employed here for the implementation of the methodology of streamfunction generation. The advantage of boundary-fitted coordinates is that complex configurations of the lateral boundaries can be mapped simple geometries. Boundary conditions are simpler and specified on straight lines such that the no-flow condition across coastal boundary is satisfied as precisely as the mapping allows. In addition, the bathymetry is represented

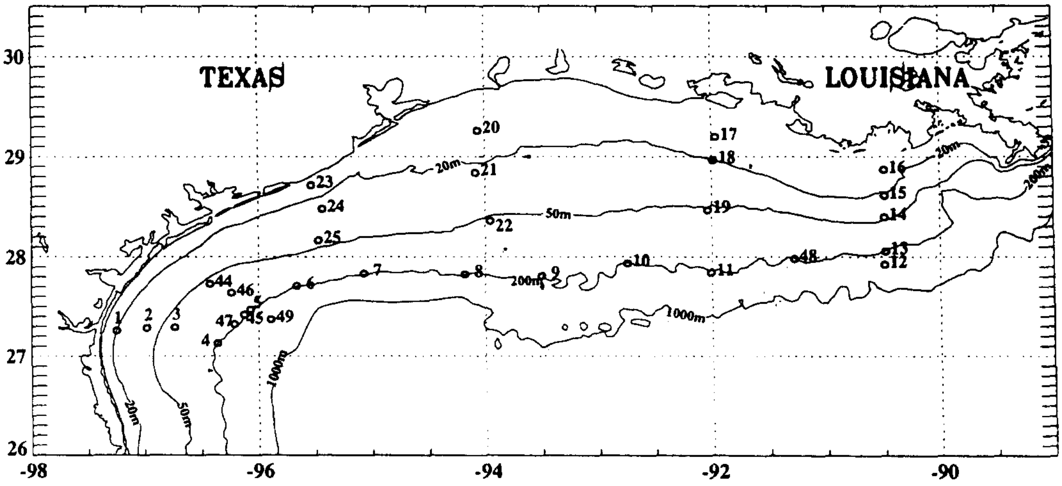

Fig. 1. Map of the Texas-Louisiana continental shelf showing the locations of current meter moorings by the Texas-Louisiana Shelf Circulation and Transport Processes Study (LATEX).

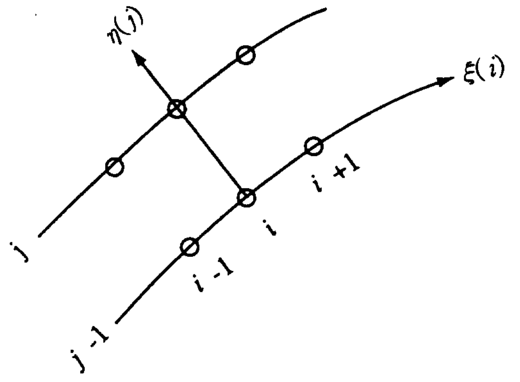

Fig. 2. The orthogonal curvilinear grid system (Mellor, 1993).

in a more natural manner on the coastal region in which the low-frequency circulations are strongly constrained by bathymetry (Csanady, 1982).

Many methodologies for grid generation have been developed using conformal, curvilinear, elliptic, or algebraic mapping techniques. Reviews of these method can be found in a collection of papers from symposium on numerical grid generation compiled in the book edited by Thompson (1982). The mapping technique employed here is a curvilinear transformation described in detail by Mellor (1993). Fig. 2 shows the orthogonal curvilinear coordinate system. The method uses the orthogonality conditions;

$$(\frac{\partial \xi}{\partial s})_j = -(\frac{\partial \eta}{\partial s})_i, \quad (\frac{\partial \eta}{\partial s})_j = (\frac{\partial \xi}{\partial s})_i \tag{5}$$

Equation (5) can be discretized by finite difference as following and can be solved numerically such that

$$\xi_{i,j} - \xi_{i,j-1} = \frac{\delta_j s}{\delta_i s}[\, \eta_{i+1,j} - \eta_{i-1,j} + \eta_{i+1,j-1} - \eta_{1,j-1}\,] \tag{6}$$

and

$$\eta_{i,j} - \eta_{i,j-1} = \frac{\delta_j s}{\delta_i s}[\, \xi_{i+1,j} - \xi_{i-1,j} + \xi_{i+1,j-1} - \xi_{1,j-1}\,], \tag{7}$$

where

$$\delta_i s = \frac{1}{4}[\, (\xi_{i+1,j} - \xi_{i,j})^2 + (\eta_{i+1,j} - \eta_{i+1,j})^2\,]^{1/2} + \frac{1}{4}[\, (\xi_{i+1,j-1} - \xi_{i-1,j-1})^2 + (\eta_{i+1,j-1} - \eta_{i-1,j-1})^2\,]^{1/2},$$

$$\delta_j s = [\, (\xi_{i,j} - \xi_{i,j-1})^2 + (\eta_{i,j} - \eta_{i,j-1})^2\,]^{1/2}.$$

Fig. 3 shows the full grid generated by this method. The inshore and offshore boundaries were fitted to smoothed versions of the coast and the 1000-m isobath. The land-sea interface is mapped to points sampled from the Central Intelligence Agen-

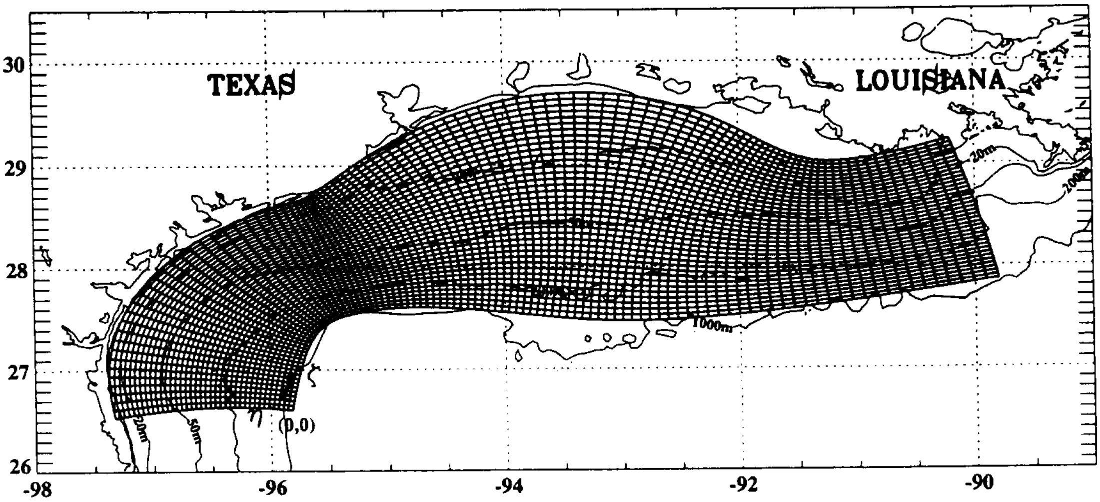

Fig. 3. The boundary fitted orthogonal curvilinear coordinate system used for the computation of velocity streamfunction using a least square analysis.

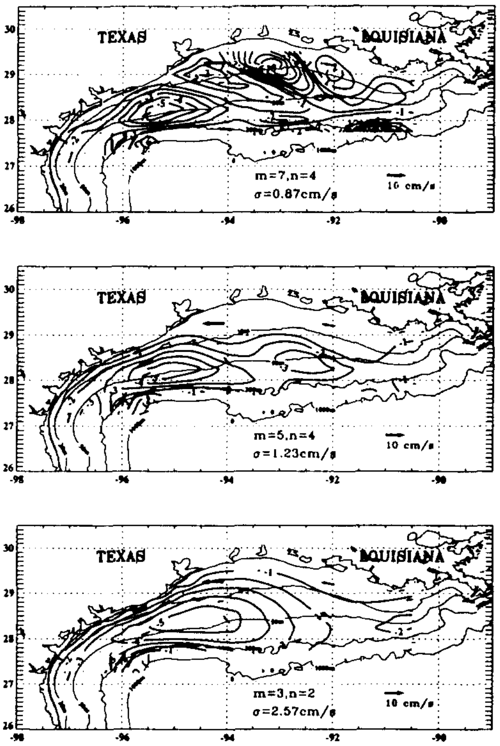

Fig. 4. The velocity streamfunction fields using different numbers of harmonics in alongshelf (m) and cross-shelf axes (n). The unit of streamfunction is $10^7 cm/sec^2$. Arrows are 32-month average currents at the upper meters (10m) for each LATEX mooring.

cy of Geographic and Cartographic Research World Data Bank II. Bathymetry is interpolated from a database provided by the LATEX and compiled by James Herring of Dynalysis (Princeton) using Naval Oceanographic Survey, Digital Bathymetric Data Base 5-minute, and gridded Texas A&M University data sets. The observed velocity is transformed into the orthogonal system and then the streamfunction equation (1) is obtained by minimizing equation (2).

3.3 Test for the number of harmonics

In order to compute the streamfunction, we first tested the appropriate number of modes for the LATEX data. Fig. 4 shows the contours of near surface (10-m) streamfunction generated with different number of harmonics based on the top meter velocity data averaged over the entire observation period. The top figure is the streamfunction pattern using seven harmonics in the alongshelf direction (m=7) and four harmonics in the cross-shelf direction (n=4). The middle one is m=5, n=4 and the bottom is m=3, n=2. Increasing the number of harmonics results in the reduced rms residual of the fitting such that σ=0.87 cm/sec for m=7 and n=4, σ=1.23 cm/sec for m=5 and n=4, and σ=2.37 cm/sec for m=3 and n=2. However, increasing the number of harmonics results in noisy patterns of streamfunction (i. e., patterns inconsistent with the resolution of the data). A reasonably smooth streamfunction pattern demands the use of lower harmonics at the expense of higher rms residual. In this application, the number of degrees of freedom of the LATEX observation field is at most 31. For the m=7 and n=4 case, the system is close to being undetermined and the noisiness of the streamfunction field should be expected. We think that the number of harmonics that produces a reasonable smooth streamfunction pattern is not same for different area and data. However, for simplicity, m=3 and n=2 are used for the application of LATEX data.

The streamfunction pattern (m=3 and n=2) is fairly consistent with one determined using Poisson equation (Cho, 1997) and with observations. Indeed, the duality of the streamfunction pattern obtained from two different methods here provides a degree of confidence in the results that would otherwise not exist. The shelf-wide overall mean velocity streamfunction field from April 1992 to November 1994 forms an elongated cyclonic gyre which was also deduced from hydrographic observations by Cochrane and Kelly (1986). In the southwest of the shelf, the flow is downcoast at the inshore region and upcoast at the outer shelf. The streamfunction pattern obtained using least-square analysis resolves this structure better than that obtained by the method of Poisson equation (Cho, 1997). The range of the streamfunction values from the coast to the center of the gyre is a little smaller in the streamfunction by the method of Poisson equation, which suggests the interpolation also plays a role of the smoothing of the field.

The rms residual is 2.37 cm/sec for the streamfunction field when m=3 and n=2. The rms residual can be compared with the rms speed of 6.32 cm/sec for the observation field. The ratio of the two rms values is 0.38 (or 0.14 in relative error variance). The rms residual accounts for discrepancies in both speed and direction. The relatively small residual value indicates that the field is represented adequately by

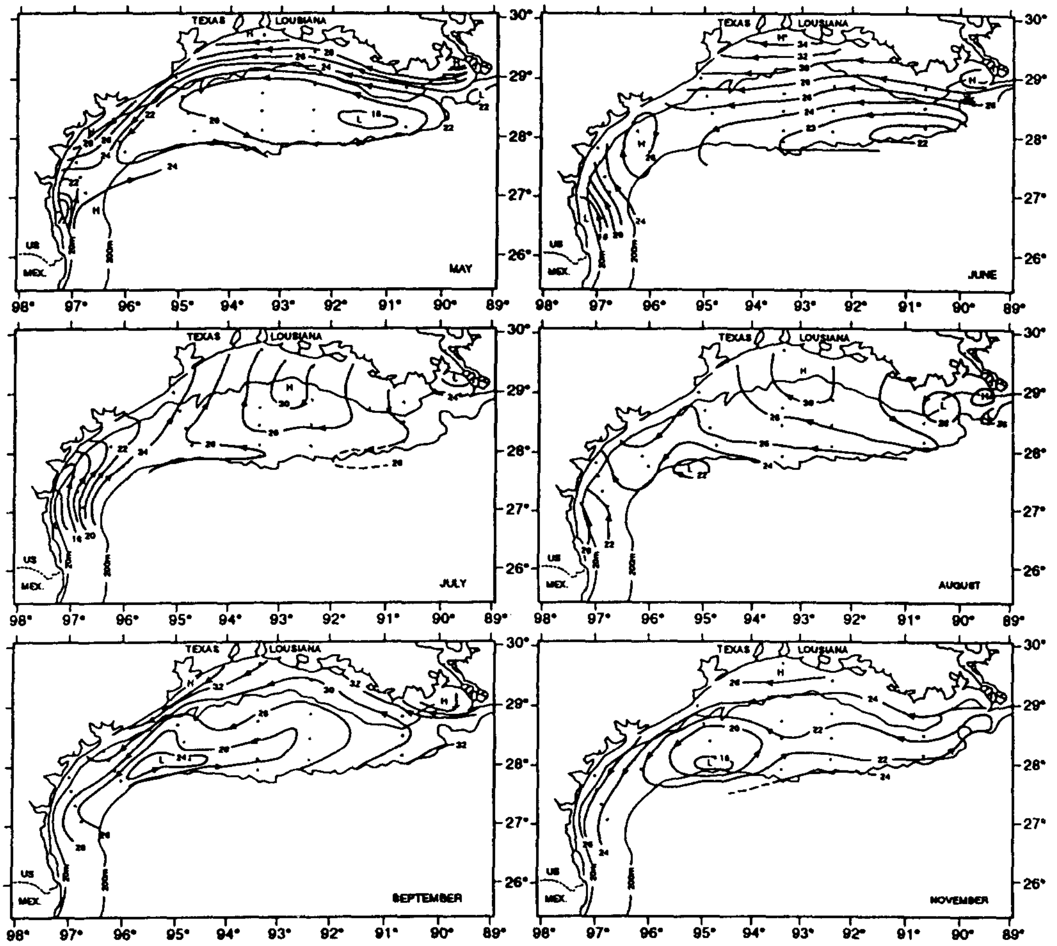

Fig. 5. Monthly mean geopotential anomaly (dyn cm) of the sea surface relative to 70 decibars based on GUS Ⅲ cruises in 1963, 1964 and 1965. (Adapted from Cochrane and Kelly, 1986).

the streamfunction equation (1). The rms residual is possibly due to a horizontally divergent component of flow that results from averaging and observation. Barron (1994) used the same methods to generate a streamfunction field over the shelf based on AVHRR data (October 24-25, 1992). In his study, the rms residual was 9.6 cm/sec and rms speed was 27.1 cm/sec. In spite of using a higher number of harmonics (m=4 and n=4) in his study, the ratio (0.35) is comparable to the ratio for the present study (m=3 and n=2). Using the foregoing method, the overall summer mean and nonsummer mean velocity streamfunction based on the near top-meter LATEX data are generated to characterize the shelf-wide flow patterns in two seasons and are discussed in the next chapter.

3.4 Velocity streamfunction

The distinct characteristics of the low-frequency circulation over the Texas-Louisiana shelf is a contrast between the nonsummer and summer, which is based on the Cochrane and Kelly (1986) seasonal circulation study (Fig. 5). They used monthly mean geopotential anomaly maps based on the GUS III data to infer a general seasonal circulation pattern. This pattern projects an elongated cyclonic gyre extending from the Rio Grande to the Mississippi River Delta. This gyre persists from September to May, with the center of the low migrating upcoast during this period. During the summer (June through August), when winds are dominantly out of the southeast to southwest (Fig. 6) and thus have upcoast component of wind stress (Fig. 7), the coastal current reverses, initially in the coastal bend region and then progressing upcoast, resulting in an anticyclonic cell centered off the Texas-Louisiana border in July and August.

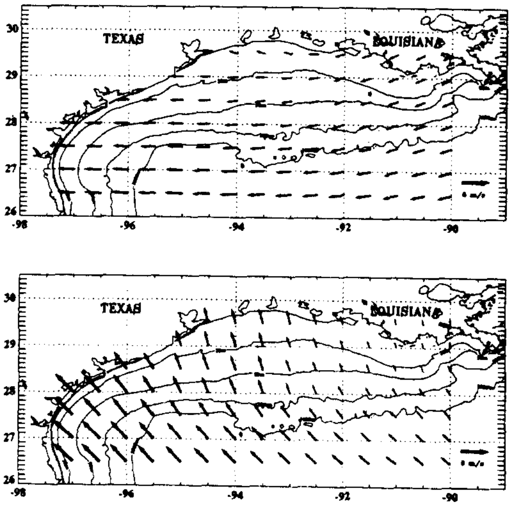

Fig. 6. The shelf-wide mean 10-m wind fields during LATEX observation period (April 1992 to November 1994): nonsummer (top) and summer (bottom).

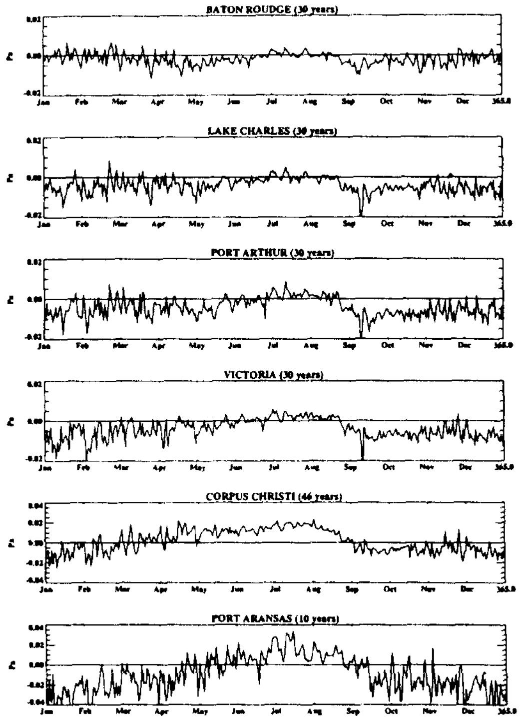

Fig. 7. Alongshelf component of long term daily mean wind stress at airport stations near the Texas-Louisiana coast.

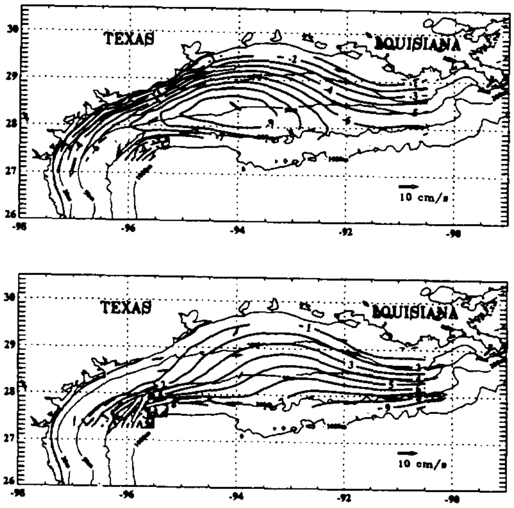

Fig. 8. Shelf-wide mean 10-m velocity streamfunction fields using record-length nonsummer mean (top) and summer mean (bottom) of LATEX observation. Arrows represent corresponding mean current vectors from upper meters (10m). The unit of streamfunction is 10^7cm/sec^2.

In order to test the Cochrane and Kelly circulation schema, we generated the shelf-wide near-surface (10m) velocity streamfunction for the nonsummer and summer based on the record-length (32 months) average of LATEX mooring observation as Fig. 8. The rms speeds are 8.25 cm/sec for the nonsummer mean and 8.86 cm/sec for the summer mean. The corresponding rms residuals of the fitting are 3.61 cm/sec and 4.50 cm/sec. The nonsummer streamfunction pattern generally supports the Cochrane and Kelly' scheme (1986). The nonsummer average shows an elongated cyclonic gyre over the shelf. The gyre is possibly forced by wind stress (Figs. 6, 7), buoyancy effect by river discharge, Loop Current Eddies (LCEs) (Cochrane and Kelly, 1986; Sturges, 1993; Oey, 1995, Cho, 1997). Mean summer flow directed upcoast over all the shelf. However, the Cochrane and Kelly schema (Fig. 5) in summer showed that the contours of geopotential anomaly was directed to the normal to the coastal line. This fact suggests that the coastal current is not in the geostrophic balance, which could not be explained by the geostrophic method of Cochrane and Kelly (1986). Thus the results of the present study induces the further study of the momentum balance of the coastal jet during summer season.

4. Summary and Discussion

A least-square regression analysis is presented for the generation of velocity streamfunction from discretely sampled current meter data and successfully applied in the Texas-Louisiana continental shelf based on the LATEX data. The streamfunction in this method is expanded by a series of trigonometric basis function and the coefficients of the series are obtained by maximizing the energy from non-divergent component of flow. The primary advantage of this method is that it avoids the interpolation of the observed velocity field, which is required in the solution of Poisson's equation (Cho, 1997). Adopting a boundary-fitted orthogonal, curvilinear coordinate system, the method also facilitates the problem of specifying boundary conditions. In addition, rms residual of the streamfunction fit is calculated and explains whether the field is well represented by streamfunction.

In the application for the LATEX data, we had to use a few harmonics (3 in alongshelf direction and 2 in cross-shelf direction) in order to reduce the instability of the method. Using small number of modes requires the expense of higher rms residual of the streamfunction fit which cannot be evaded because of the limitation of the number of mooring. The problem increasing the resolution of the streamfunction is one of future task. In spite of using small modes, the resulting rms residual is relatively small compared to other study, which suggests that the observed velocity field over the Texas-Louisiana shelf is well represented by the streamfunction field. This is possibly due to the low-pass filtering of the current meter data. Thus generated nonsummer streamfunction pattern supports the Cochrane and Kelly's circulation schema over the shelf that forms a cyclonic gyre. However, the summer mean pattern is a little different with the map of geopotential anomaly by Cochrane and Kelly (1986).

In a series of paper, we tried to construct the shelf-wide velocity streamfunction with two different approaches. Both methods (least-square analysis and Poisson equation) result in consistent pattern with observations and with each other. This fact increases the confidence of the methodologies adopted in our study. We hope our study will be used for the mapping and understanding of synoptic structure of low-frequency current, for the initialization of the computer modeling, and for the assimilation of observation data into ocean models.

The primary limitation of the current meter data used in the present study is that it cannot resolve the small scale structures of flow field due to the spacing of moorings. The increase of the sampling interval in order to improve the resolution will not come true in the near future due to the high cost of mooring. It have to be considered that how might the objective methods using hydrographic data be improved, or perhaps, used to augment the analysis using current meter data. Of course this is beyond the present study. One of other possible approach to improve streamfunction analysis is to make use of satellite altimeter data. Satellite altimetry has proven an effective tool to measure surface geostrophic current (Wunsch and Gaposchkin, 1980). Even though the interval of satellite track is quite wide, the alongtrack sampling resolution is very high. A time sampling rate is 10-20 days. Applying the altimetry data in the coastal ocean is quite challenging due to the accuracy of tide and geoid. However, we expect the application would be possible in the coastal ocean by providing dynamic constraint such as non-divergence employed in the present study. This is something that might be explored in the future.

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객관적 분석을 통한 속도 유선함수(streamfunction) 산출 II: 최소자승 회귀분석법의 응용

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(1997년 5월 17일 접수)

해양의 몇몇 정점에서 관측된 유속자료로부터 속도 유선함수를 최소자승 회귀분석법을 응용하여 산출하였다. 유선함수는 삼각함수의 합으로 표현되며 이들 함수의 상수들은 2차원 유속장의 비발산 부분을 최소화하여 구하였다. 위의 방법은 기존의 방법들이 요구하는 관측치의 보강이 필요치 않으며 관측 및 흐름장의 발산에 기인하는 오차를 계산할 수 있는 장점이 있다. 연안역의 복잡한 해안경계 문제를 쉽게 표현하기 위하여 연안을 경계로 하는 곡선직교좌표를 도입하여 유선함수를 구하였다. 텍사스-루이지아나 대륙붕 순환 및 수송 연구(LATEX)를 위하여 31개 정점에서 관측된 해류계자료를 이용 텍사스-루이지아나 대륙붕상의 속도유선함수를 성공적으로 산출하였다. 위의 해역에서 유선함수 산출오차는 비교적 적게 나타나 유선함수가 표층 유속장을 잘 나타냄을 알 수 있다.