

Objective Estimation of Velocity Streamfunction Field with Discretely Sampled Oceanic Data I: with Application of Helmholtz Theorem

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An objective method for the generation of velocity streamfunction is presented for dealing with discretely sampled oceanic data. The method treats a Poisson equation (forced by vorticity) derived from Helmholtz theorem in which streamfunction is obtained by isolating the non-divergent part of the two-dimensional flow field. With a mixed boundary condition and vorticity field estimated from observed field, the method is implemented over the Texas-Louisiana shelf based on the current meter data of the Texas-Louisiana Shelf Circulation and Transport Processes Study (LATEX) measured at 31 moorings for 32 months (April 1992 - November 1994). The resulting streamfunction pattern is quite consistent with observations. The streamfunction field by this method presents an opportunity to initialize and to verify computer models for local forecasts of environmental flow conditions for oil spill, nutrient and plankton transports as well as opportunity to understand shelf-wide low-frequency currents.

Key words : Objective analysis, streamfunction, coastal circulation, ocean current

1. Introduction

This is the first of two papers in which an objective method for the estimation of velocity streamfunction based on discretely sampled oceanic data is introduced and applied in the context of the Texas-Louisiana Shelf Circulation and Transport Processes Study (LATEX). In Part I the Helmholtz theorem is used and a Poisson equation forced by vorticity is solved to estimate velocity streamfunction field. In Part II a least-square regression analysis is used to find the coefficients of streamfunction expanded in terms of a series of trigonometric basis function (Cho, 1997). The objective of two papers lies on the

synoptic mapping of the low-frequency flow field from the discrete sampled oceanic data.

In recent years there has been an increasing demand for the time dependent spatial structure of flow field over the ocean due to the interests in the environment and fishery such as oil spill, nutrient and plankton transports. In oceanography, the flow field is essential for the understanding of oceanic circulation and can be also used to initialize numerical modeling. However, the high cost of oceanographic observation, especially direct one by mooring, has been an obstacle for the demand. Even though ob-

served by direct mooring, the number of mooring had to be limited due to the cost. The shortage of the observation data invokes an interpolation task in order to ascertain the overall pattern of the flow over a wide area such as Texas-Louisiana continental shelf in the present study.

The LATEX program, one of the largest shelf circulation program in oceanographic history, was initiated to study the seasonal and interannual variability of low-frequency currents and associated dynamical processes over the entire Texas-Louisiana shelf (Nowlin *et al.*, 1991). The LATEX observations were comprehensive, including current meter moorings, bottom wave recorders, meteorological buoys, drifting buoys, hydrographic stations and ADCP for the period from April 1992 to November 1994. The present study deals with the LATEX current meter observations. The LATEX moorings involved the deployment of 75 current meters on 31 moorings over the entire shelf (Fig. 1) for a period of about 32 months.

The problem addressed here is how to produce an objective shelf-wide synoptic

map with the discretely sampled LATEX current meter data spaced $O(10\text{km})$ to $O(100\text{km})$. Because of the sampling interval, we cannot resolve the small scale processes having scales smaller than mooring spacing. The primary interest of the present study is in the low-frequency currents over the Texas-Louisiana continental shelf. The high frequency motions such as inertial motion, internal wave and tides are filtered out from the moored current data. In momentum balance, a low-frequency circulation of ocean is in approximately geostrophic (quasigeostrophic) and horizontally non-divergent to an accuracy of a few per cent (Gill, 1982). The streamfunction can be introduced for the horizontally non-divergent field. Then question narrows down to estimate scalar streamfunction field from the observed vector field. It would be desirable to calculate the streamfunction field rather than velocity field itself because (1) it imposes the non-divergence constraint for the low-frequency current and thus generates dynamically meaning field by reducing divergent part and observational noise, and (2) it increases the degree of freedom by cal-

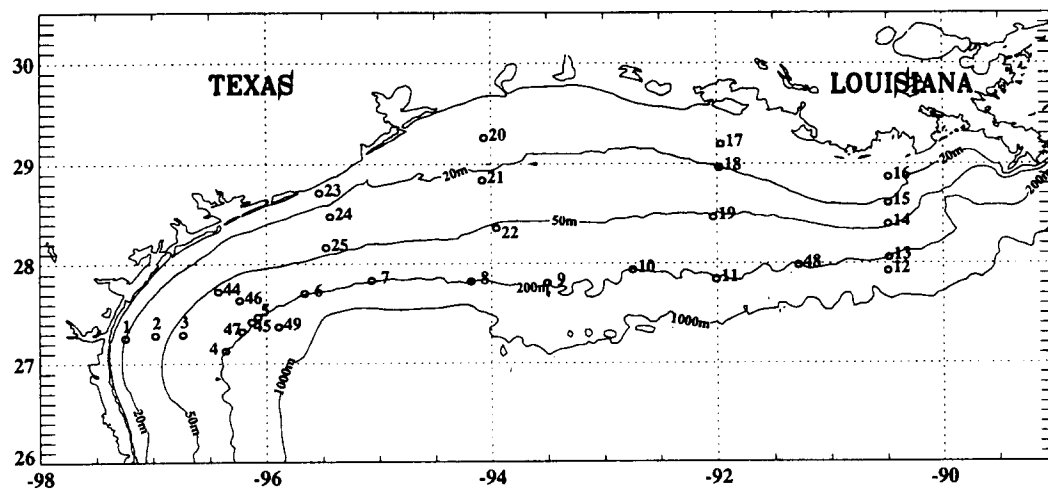


Fig. 1. Map of the Texas-Louisiana continental shelf showing the locations of current meter moorings by the LATEX.

culating one scalar field from two components of velocity vector. Thus generated streamfunction also provides information on the transport of upper layer (von Schwind, 1980).

The streamfunction can be produced in several ways (Pritchard, 1948; Hawkins and Rosenthal, 1965; Liebelt, 1967; Hubertz *et al.*, 1972; Vastano and Reid, 1985; Carter and Robinson, 1987). Pritchard (1948) evaluated the streamfunction by a line integration involving the velocity components. Using this method he produced streamline chart for a portion of the tropical Atlantic and Pacific Oceans. However, we did not consider this method because it does not impose the non-divergence constraint. Another way is to use statistical estimation theory (Liebelt, 1967). The approach of this method is based on the Gauss-Markov theorem. This methodology was applied for the sampling strategy and objective analysis of synoptic mapping of the Mid Ocean Dynamics Experiment (MODE) observation (Bretherton *et al.*, 1976). However, it is difficult to use this method in the shelf region such as Texas-Louisiana shelf in which the field is inhomogeneous and anisotropic. In addition, the decorrelation scale (ranging from 28km to 32km) over the shelf (Li *et al.*, 1996) is small compared to the spacings of the LATEX current mooring.

We imposed a criteria in the selection of methodology for streamfunction analysis such that it can enforce horizontal non-divergence. Through the constraint, we hope to reduce the sampling noise of the observation as well as to produce dynamically meaningful flow field. In this paper, we choose and test a method using Helmholtz theorem in which two dimensional non-divergent

part of flow field is isolated by solving Poisson's equation. This method has been used successfully in the atmospheric field (Hawkins and Rosenthal, 1965) and in the oceanic field (Hubertz *et al.*, 1972; Carter and Robinson, 1987).

The objective of this paper is to test the above streamfunction analysis procedure over the Texas-Louisiana shelf based on the LATEX current mooring data in order to produce the shelf-wide streamfunction field. We begin with a discussion in section 2 of a methodology for generating streamfunction. The implementation of the method is introduced in terms of an appropriate boundary condition and the interpolation of observed field for the vorticity field in section 3. The observation data and its processing is also described in section 3. Finally, a summary and discussion are given in section 4.

2. Methodology

Examined here is a method to produce the streamfunction field from the discretely sampled oceanic data. The method involves solving Poisson's equation using an estimated field of vorticity based on the observed flow. Any two-dimensional velocity field, $\bar{V}(x, y)$, can be represented by a divergent and non-rotational component and a non-divergent and rotational component, by Helmholtz's theorem, as follows;

$$\bar{V} = -\nabla\phi + k \times \nabla\psi \quad (1)$$

where ϕ is the velocity potential, ψ is the streamfunction, and k is a locally vertical unit vector. Because of the non-divergent characteristics of the low-frequency currents, we are interested in the second part of the equation (1). The relation of the streamfunction to sea surface topography (dynamic height) is made by invoking a geostrophic approximation for the flow such that

$$\bar{v} = \frac{\partial \psi}{\partial s} = \frac{g}{f} \frac{\partial \eta}{\partial s} \quad (2)$$

where f is Coriolis parameter, g is the acceleration of gravity, and the coordinate s is taken normal to streamlines and contours of η , the sea surface topography. Using the relation (2), the streamfunction can be compared to the dynamic height field estimated from hydrographic observation (Cochrane and Kelly, 1986) and to the sea surface topography measured from satellite altimeter (Wunsch and Gasposchkin, 1980).

For the velocity potential, we can obtain a relation by taking the divergence of the equation (1) such that

$$\nabla^2 \phi = -\nabla \cdot \bar{v} \quad (3)$$

Equation (3) explains that the horizontal divergence determines the velocity potential. Taking the z -component curl in equation (1), we get a Poisson equation forced by the horizontal vorticity field of flow such that

$$\nabla^2 \psi = \mathbf{k} \cdot \nabla \times \bar{v} \quad (4)$$

In order to solve this Poisson equation for velocity streamfunction, we need information on the curl field of the flow and an appropriate boundary condition at each point of boundary of the domain for which streamfunction is to be evaluated. To calculate the vorticity field from the velocity field, the discretely observed velocity has to be interpolated at regular grid points. Thus two tasks are needed to find the streamfunction by the method adopted in this study; one is to interpolate the observations onto a regular grid and the other is to find appropriate boundary conditions. The implementation is explained in next section.

3. Application

3.1 Data

The data used for the present study are the moored current meter meas-

urements observed on the entire Texas-Louisiana continental shelf observed from April 1992 to November 1994 as a part of LATEX. The array consisted of 75 current meters measuring current speed and direction, temperature, and conductivity on 31 moorings (Fig. 1). Five types of current meters (Endeco 174 SSM and DMT, Aanderaa RCM 4/5 and 7/8, and InterOcean S4) were deployed in the moorings. The mooring positions and current meter depths are listed in Table 1. In water shallower than 50 m, the mooring has two meters at depths near 10-12 m and near bottom. In water depth equal to or deeper than 50 m, meters were placed near 10-12m, at mid-depth, and near the bottom, except for moorings 12, 44, and 45, which had only two meters (Jochens and Nowlin, 1994). The periods during which data were observed at each current meter are shown in Fig. 2.

The mooring array was designed to identify a variety of physical processes on the Texas-Louisiana shelf and consisted of several sub-arrays. For a detailed description of the mooring strategy see Nowlin *et al.* (1991).

The current meters recorded the speed and direction of flow at 5-min to 2-hr interval (mostly at 30-min). Raw data first were filtered by a 3-hour low-pass Lanczos filter to reduce sampling noise, and then the filtered time series data were resampled at a 1-hr interval. The hourly data set was then filtered by a 40-hr low-pass Lanczos filter (Harris, 1978) having a width of 193 points of symmetric weight with a half-amplitude point of 40 hours to eliminate the tidal and inertial motions that are energetic on the shelf (Chen *et al.*, 1995). Fig. 3 shows the shape of the Lanczos filter and its frequency response function. The 40-hr low-pass data were sub-sampled at 6-hr intervals. These final 40-

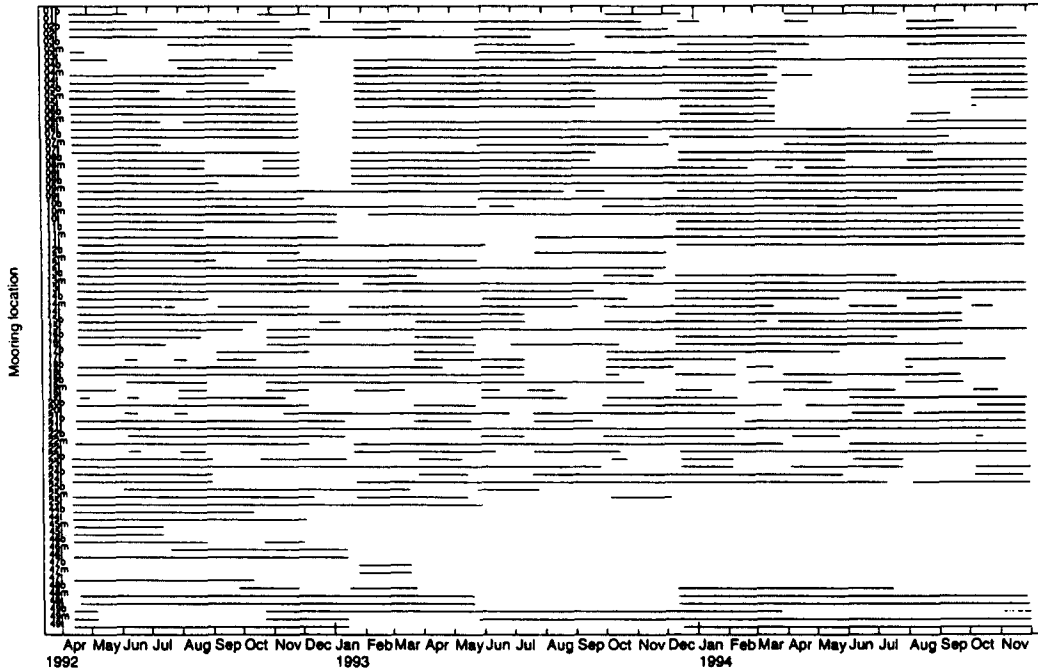


Fig. 2. Lines denote periods for which LATEX current meter data are available during the observation period (April 1992 to November 1994).

hr low-pass and 6-hr interval data were used in the present study.

3.2 Interpolation of observed field

Table 1. The mooring number, water depth, position, and vertical depths of instruments of LATEX current meter moorings

Mooring Number	Water Depth	Latitude (°N)	Longitude (°W)	Top CM Depth	Middle CM Depth	Bottom CM Depth
1	21m	27°15.38'	97°14.74'	10m		19m
2	37m	27°17.03'	96°58.81'	10m		30m
3	66m	27°17.38'	96°44.17'	10m	30m	61m
4	201m	27°07.57'	96°21.51'	12m	100m	190m
5	199m	27°28.10'	96°04.40'	12m	100m	190m
6	201m	27°42.51'	95°39.84'	12m	100m	190m
7	199m	27°50.04'	95°04.17'	12m	100m	190m
8	201m	27°49.47'	94°10.77'	14m	100m	190m
9	200m	27°48.50'	93°30.18'	12m	100m	190m
10	200m	27°56.13'	92°44.70'	12m	100m	190m
11	200m	27°50.52'	92°00.25'	12m	100m	190m
12	500m	27°55.43'	90°29.68'	17m	100m	190m
13	200m	28°03.44'	90°29.15'	12m	100m	190m
14	47m	28°23.67'	90°29.57'	11m		42m
15	27m	28°36.50'	90°29.49'	10m		24m
16	19m	28°52.02'	90°29.45'	10m		17m
17	7m	29°11.76'	91°37.89'	3m		5m
18	22m	28°57.76'	91°58.96'	10m		21m
19	51m	28°27.91'	92°02.09'	3m	21m	44m
20	15m	29°15.65'	94°03.82'	3m		13m
21	24m	28°50.24'	94°04.77'	10m		21m
22	55m	28°21.29'	93°57.35'	3m	20m	48m
23	15m	28°42.77'	95°32.15'	9m		13m
24	30m	28°28.43'	95°26.23'	11m		27m
25	45m	28°09.72'	95°28.54'	13m	23m	38m
44	56m	27°43.53'	96°25.44'	9m		49m
45	200m	27°25.09'	96°07.59'	10m		190m
46	91m	27°38.28'	96°14.02'	10m	50m	84m
47	200m	27°19.30'	96°12.77'	10m	100m	190m
48	200m	27°58.98'	91°17.00'	10m	100m	190m
49	503m	27°22.15'	95°53.64'	10m	100m	495m

We need the curl field of flow to solve the equation (4) and thus require the interpolation of the observed field into grids for the estimation of vorticity field. The

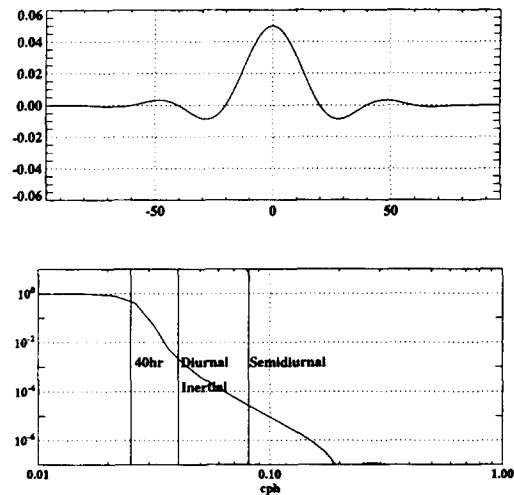


Fig. 3. The shape of the Lanczos filter in the time domain (top) and its frequency response function (bottom).

choice of an interpolation scheme is difficult task in place such that the sampling space is larger than the decorrelation scale and the field is inhomogeneous and anisotropic such as Texas-Louisiana shelf (Li *et al.*, 1996). These characteristics excludes the statistical interpolation scheme (Bretherton *et al.*, 1976). We tested different schemes for the interpolation including Akima's and settled down a spline method developed by Franke (1982) for the interpolation.

The algorithm of the interpolation is based on a weighted sum of locally defined thin plate splines, and yields an interpolation function that is differentiable. The basis function of the interpolation is

$$Q(x,y) = \sum_k A_k d_k^2 \log d_k + a + bx + cy \quad (5)$$

where $d_k^2 = (x-x_k)^2 + (y-y_k)^2$, and $Q(x,y)$ is interpolated velocity component (u,v) at the position (x,y) . The coefficients; A_k, a, b, c are determined by solving the linear system of equations such that

$$\sum_k A_k d_k^2 \log d_k + a + bx + cy \Big|_{(x,y)=(x_i,y_i)} = f_i \quad (6)$$

$$\sum_k A_k = 0 \quad (7)$$

$$\sum_k A_k x_k = 0 \quad (8)$$

$$\sum_k A_k y_k = 0 \quad (9)$$

where f_i represents observations at point (x_i, y_i) .

Fig. 4 shows the interpolated velocity field (open arrows) on a uniform grid based on current meter observations (shown in dark arrows). The data used for this interpolation are mean values averaged over the entire observation period (32 months). The interpolated velocity field has a structure very consistent with observations, with very smooth transition in the gaps between moorings. The horizontal curl field of flow is estimated from the interpolated flow field to solve equation (4).

3.3 Boundary Condition

Several boundary conditions have been used to solve the Poisson equation (Hawkins and Rosenthal, 1962; Hubertz *et al.*, 1972; Carter and Robinson, 1987). First one is to apply the normal derivative boundary condition, which comes from the definition of the streamfunction:

$$\frac{\partial \psi}{\partial x} = v, \quad \frac{\partial \psi}{\partial y} = -u \quad (10)$$

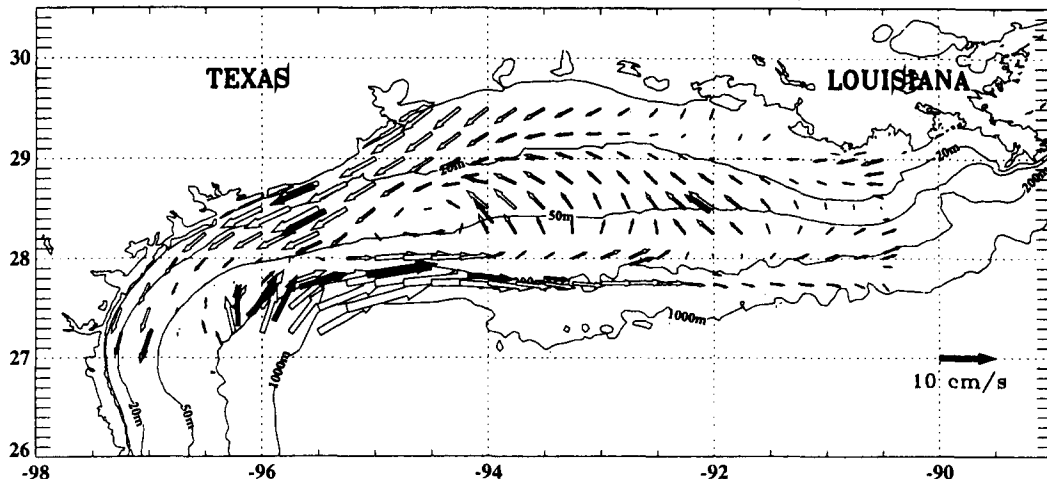


Fig. 4. The interpolated shelf-wide flow field (open arrows) from the observation (dark arrows) using a spline method (Franke, 1982). The observations used are averages of LATEX current at 10-m depth over the entire record-length of 32 months.

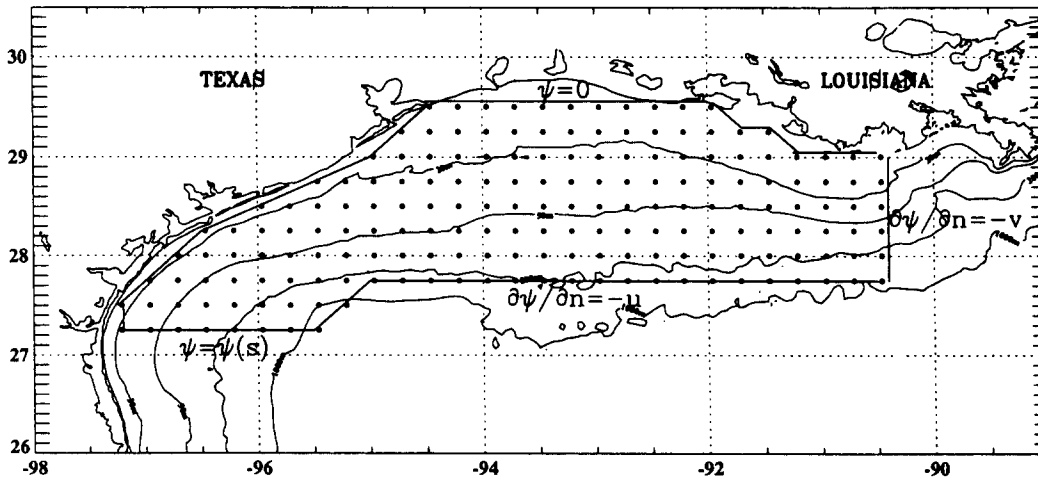


Fig. 5. The grid and boundary conditions for solving a Poisson equation.

This method has an advantage that boundary data are determined directly from the interpolated field (Fig. 4). This boundary condition works well in the field in which the observational noise and the divergent part of flow are small (Hawkins and Rosenthal, 1962). Carter and Robinson (1997) used other boundary condition to reduce the noisiness of observation data. In the method, we have following relation by taking scalar product of (1) with the unit outward normal vector, n ,

$$\frac{\partial \psi}{\partial s} = -\bar{V}_n + \frac{\partial \phi}{\partial n} \quad (11)$$

where s is the distance along the boundary and \bar{V}_n is the velocity normal to the boundary. By integration (11) around the boundary, we have

$$\int \bar{V}_n ds = \int \frac{\partial \phi}{\partial n} \equiv \frac{\partial \phi}{\partial n} S, \quad (12)$$

where S is the total length of the boundary. Using this relation, we have following approximation:

$$\frac{\partial \psi}{\partial s} = -\bar{V}_n + \frac{\partial \phi}{\partial n} \quad (13)$$

By fixing the value of ψ at the starting point, and then integrate (13) around the boundary to get the boundary values for

use in the solution of (4). Another approach is to integrate (11) directly using the information of velocity potential which is obtained from the equation (3). This final approach is ideal because it maximizing the energy from ψ field while minimizing the energy from the ϕ field. Hawkins and Rosenthal (1965) reviewed all three methods described here. Their conclusion was that the first method is best for their purpose. We also follow their results on the basis that the observation data are already filtered out the divergent high frequency motions. Also our primary test shows that the resulting streamfunction field is not sensitive on the choice of above boundary conditions because of the characteristics of the data. In the present study, a mixed boundary condition finally chosen for the solution of equation (4). Fig. 5 shows the grid and boundary condition. The grid spacing is 1/4 degree in the x (eastward) and y (northward) directions. The solution was not sensitive to the grid spacing. The Neumann boundary condition is used at the seaward and eastern boundaries. This boundary condition specifies the normal gradient of ψ in terms of the tangential

component of velocity along the boundary. Along the coast and at the southwestern boundaries, the Dirichlet boundary condition is used.

3.4 Velocity streamfunction

Using the curl field estimated from the interpolated field (Fig. 4) and the mixed boundary condition (Fig. 5), the Poisson equation (4) is solved for the shelf-wide streamfunction field by the method of simultaneous over-relaxation (SOR) (Press *et al.*, 1986). The contoured depiction of this field is shown in Fig. 6. The dynamic height (sea surface topography) can be evaluated by (2) from the streamfunction field. The streamlines represent 32-month averaged near surface (10 m) flow field over the Texas-Louisiana continental shelf. The resulting streamfunction pattern is quite consistent with observations and thus provides a degree of confidence in the result. The near surface mean streamfunction shows an elongated cyclonic gyre pattern over the shelf. The gyre pattern supports the Cochrane and Kelly's scheme (1986) of the low-fre-

quency circulation on the shelf deduced from the geopotential anomaly field and also supports the Oey's numerical experiment on the shelf (1995). The inshore limb, directing downcoast direction, is a typical coastal jet driven by the downcoast alongshelf wind explained by the coastal Ekman pumping mechanism. (Cochrane and Kelly, 1986). The origin of the eastward flow in the outer shelf is understood to be formed by the activity of Loop Current Eddies (LCEs) detached from the Loop Current (LC) in the Gulf of Mexico (Cochrane and Kelly, 1986; Oey, 1995) and by the continuation of the western boundary current driven by negative wind stress curl in the western Gulf of Mexico (Sturges, 1993). The eastern convergence is made by the intrusion of the Mississippi Canyon and the western convergence is done by the convergence of coastal current by the morphology of coastal boundary (Cochrane and Kelly, 1986) or the collision and stalling processes of LCEs over the shelf (Oey, 1995).

4. Discussion and summary

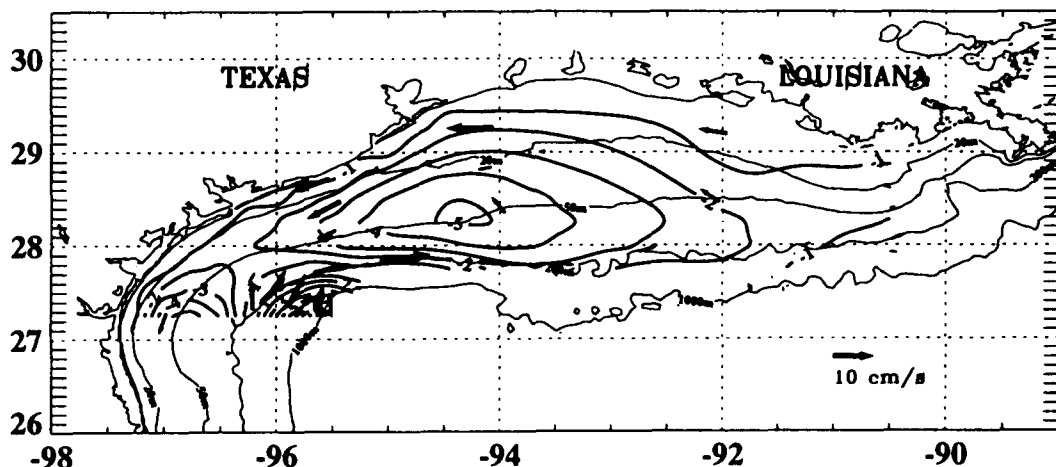


Fig. 6. Velocity streamfunction field on the Texas-Louisiana shelf generated by a Poisson equation. The observation used to compute the velocity streamfunction are the record-length averages of LATEX current meter observations at 10-m depth. The unit of streamfunction is $10^7 \text{ cm}^2/\text{sec}$.

An objective method is adopted to construct a shelf-wide streamfunction field and successfully implemented over the Texas-Louisiana continental shelf based on the LATEX current meter data. The method isolated the non-divergent part of the two-dimensional flow field by solving a Poisson equation forced by vorticity estimated from the measured currents, using mixed boundary conditions. The interpolated field of velocity by the scheme employed in this study provides the smooth transition between moorings and shows consistency with observation. It is believed that the smoothing also filters out the noisiness of the observation data. In the coastal region, the interpolation of oceanic data has been a big problem because of the anisotropy and inhomogeneity of the field even though the data are sampled within the decorrelation scale such as hydrographic and altimeter data. From our result, the spline method can be used in the shelf region such as Texas-Louisiana continental shelf. However, the necessity for interpolation in the streamfunction generation is obviously a primary disadvantage. This problem will be evaded using different approach in the next paper (Cho, 1997). For the boundary condition to solve the Poisson equation, we applied the streamfunction definition, which is to integrate the interpolated velocity along the boundary except the coastal boundary. The data used in this study are not sensitive to the choice of the types of boundary condition because the data already remove the divergent high frequency motions by filtering and the noisiness of the data by averaging. However, the selection of boundary condition have to be very cautious according to the data type. The methodology adopted in this study generated stable streamfunction field for the entire

shelf circulation from the discrete current meter data. The streamfunction field by this method presents an opportunity to initialize and verify computer models for local forecasts of environmental flow conditions as well as the understanding of the shelf-wide low-frequency currents.

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객관적 해석을 통한 속도 유선함수(streamfunction) 산출 I: 헬름홀츠(Helmholtz) 정리의 응용

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(1997년 5월 15일 접수)

본 연구는 해양에서 비규칙적으로 관측된 유속자료를 이용 속도유선함수를 객관적해석을 통하여 산출하였다. 이를 위하여 헬름홀츠(Helmholtz) 정리를 응용 2차원 유속장의 비발산 부분만 나타내는 유선함수와 와도와의 관계를 규정하는 포이송(Poisson)방정식을 도출하고, 혼합경계조건과 관측치로부터 산출된 와도장을 이용 유선함수를 구하였다. 위의 방법을 실현하기 위하여 텍사스-루이지아나 대륙붕 순환 및 수송 연구(LATEX)의 일환으로 텍사스-루이지아나 대륙붕의 31개 정점에서 32개월(1992년 4월 SIM 1994년 11월)간 관측된 해류계자료를 이용하였다. 본 방법으로 산출된 텍사스-루이지아나 대륙붕의 속도유선함수는 관측치와 잘 일치하였다. 본 연구에서 산출된 유선함수는 특정 해역의 저주파 운동의 이해뿐만 아니라 기름유출, 영양염 및 플랑크톤 수송과 관련한 환경유동모델의 초기화 및 검증에 응용될 것으로 기대된다.