Responses of the Plasmasphere to Impulsive Disturbances in the Magnetotail

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(Manuscript received 15 October, 1997)

Abstract

We present a theoretical study on how the plasmasphere responses to the sudden impulses in the magnetosphere. A mechanism on how Pi 2 pulsations are excited in the magnetosphere is also proposed. When impulsive disturbances associated with the substorm onset are assumed in the magnetotail, their propagation toward the sunward direction is investigated with a wave equation. The propagation speed undergoes serious variations owing to the existence of the plasmasphere, which results in various reflection and tunneling of traveling disturbances at the plasmapause. In order to examine the effect of the plasmapause on initial impulsive disturbances, we analytically solve the wave equation based on the model of reasonable Alfven speed profile. The exact solution shows that virtual resonant states exist inside the plasmaspheric cavity. We obtain the result that these unique modes strongly persist for arbitrary incoming impulses from the source in the magnetotail, which quantitatively corresponds to the signature of Pi 2 pulsations.

1. Introduction

In the process of substorm, it is widely accepted that the phase of substorm onset is often accompanied with the signature of Pi 2 pulsations. The Pi 2 pulsations are well-known ULF(ultra low frequency) wave phenomena in the nightside magnetosphere. The important characteristics of Pi 2 (e.g., *Yumoto et al.*, 1990; *Sutcliffe and Yumoto*, 1991; *Takahashi et al.*, 1995) can be summarized as (1) mostly nightside phenomena which are ocassionally observed in the dayside region, (2) almost compressional wave events, and (3) low-latitude phenomena(L < 5) on both ground-based and satellite measurements. Currently, there have been several scenarios on the generation mechanisms of Pi 2 pulsations which are well represented by *Yeoman and Orr* (1989). One mechanism is the surface wave at the plasmapause (*Sutcliffe*, 1975) excited by the incoming compressional waves from the tailward source. The other is the plasmaspheric cavity modes which have been supported by many numerical studies(*Allan et al.*, 1986; *Zhu and Kivelson*, 1989; *Fujita and Glassmeier*, 1995; *Lee*, 1996). However, the simple cavity modes are not well accepted near the plasmapause which is neither a good reflector nor a good transmitter. In fact, there has been no comprehensive theoretical study on the mechanism of impulsive disturbances in the magnetotail and Pi 2 pulsations so far. No quantitative investigation has been made on how the plasmasphere and magnetosphere respond to the sudden impulses produced by the substorm onset in the tailward region. It is also uncertain how arbitrary impulsive disturbances associated with the substorm onset in the magnetotail can consistently produce relatively stable signature of Pi 2 pulsations. At present, the questions associated with this subject, which have to be self-consistenly answered, may be listed as:

1) How Pi 2 pulsations can consistently appear as a precursor of the substorm onset for various types of substorms?
2) Why Pi 2 pulsations are not observed by the satellite measurements in the nightside outer magnetosphere, while they are often observed in the plasmasphere?
3) Is the current theory of cavity modes able to explain the Pi 2 pulsations in the plasmasphere as a cavity resonance?
4) Why the amplitude of Pi 2 pulsations observed in space is always larger than the peaks from the simultaneous observations of any disturbances/noises in the outer magnetosphere and even in the magnetotail?

Recently, an analytic analogy between the inhomogeneous wave equation in magnetohydrodynamics and the Schrödinger's equation in quantum mechanics is first introduced and adopted to obtain an exact solution for the propagation of compressional waves in the nightside region (*Lee*, 1996; 1997). In this work, the plasmapause is considered as an obstacle for waves since it is not an exact boundary like in the cavity/waveguide modes owing to the finite Alfven speed gradients and the limited radial size. *Lee* (1997) showed that virtual resonant states exist in the plasmasphere and are differentiated from the previous cavity modes by proving that such resonant states firmly remain in the presence of quite arbitrary sources whether they are shortly impulsive or continuously driven for a relatively long period.

In this paper, we will attempt to solve the puzzles listed above by adopting the analytical approach. First, we present a brief introduction to the theoretical model (*Lee*, 1997). Then, each answer to the questions above will be given by investigating the corresponding cases in the analytical model. Finally, our results will be discussed and compared with the recent statistical observational feature (*Takahashi et al.*, 1995).

2. Model and Equations

When disturbances propagate radially inward, its speed is approximated by the Alfven speed V_A From now on, we consider a compressional wave as the disturbances that can propagate across magnetic field lines from the source region to the low-latitude equatorial region. If we adopt an MHD wave equation for compressional electric field components,

$$\frac{d^2E}{dx^2} + \left(\frac{\omega^2}{V_A^2} - k_y^2 - k_z^2\right)E = 0, \tag{1}$$

is obtained for arbitrary Alfven speeds. The wave speed has rapid variations near the plasmapause owing to the sudden increase of density in the plasmasphere. Fig. 1 shows the Alfven speed profile based on the observations (*e.g., Chappell,* 1988; *Takahashi and Anderson,* 1992). In (1), we approximate the speed profile with the step-like function as shown in Fig. 1 and waves are assumed by $E(x)e^{i(k_y y + k_z z - \omega t)}$ with the simplified geometry where x, y and z represent the radial, azimuthal and north-south direction, respectively.

To maintain the continuity of the wave functions E_1 , E_2 and E_3 in Fig. 1, the boundary conditions are given by

$$d(\ln E_1(x_0 + a)) = d(\ln E_2(x_0 + a)), \tag{2}$$

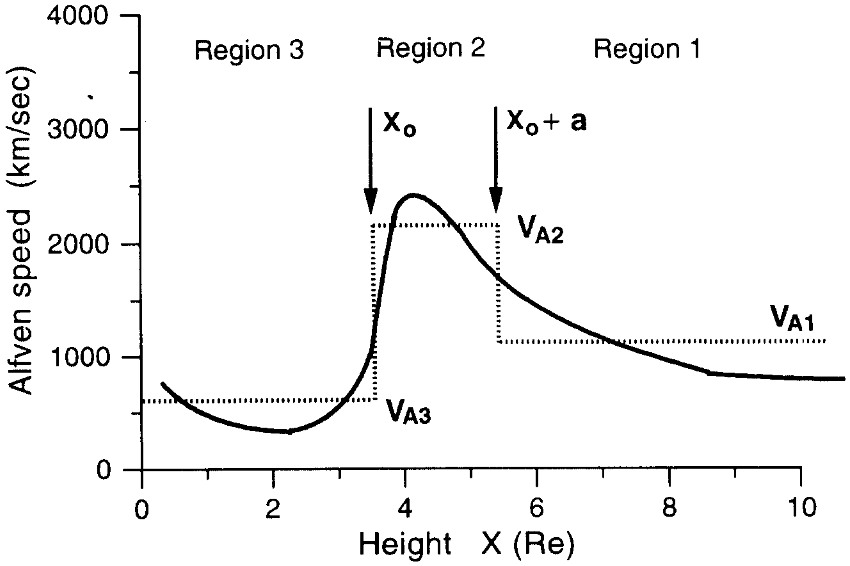

Fig. 1. The Alfven speed assumed in this model(after *Lee*[1997]): the realistic profile (solid curve) and the modeled profile (dotted curve). The position $x=0$ corresponds to the Earth's surface.

$$d(\ln E_2(x_0) = d(\ln E_3(x_0)), \tag{3}$$

$$E_3(0) = 0, \tag{4}$$

and each solution from (1) can be written as

$$E_1(x) = e^{-ik_1x} + Ae^{ik_1x}, \tag{5}$$

$$E_2(x) = Be^{\kappa x} + Ce^{-\kappa x}, \tag{6}$$

$$E_3(x) = T\sin k_3x, \tag{7}$$

where

$$\kappa^2 = k_y^2 + k_z^2 - \frac{\omega^2}{V_{A2}^2}, \tag{8}$$

and

$$k_i^2 = \frac{\omega^2}{V_{Ai}^2} - k_y^2 - k_z^2 \quad (i = 1, 3). \tag{9}$$

From the conditions (2)-(4), after the straightforward calculations, the full solutions are obtained as follows:

$$A = \tau\{(1 + i\delta)\xi_1e^{-\kappa a} - (1 - i\delta)\xi_2e^{\kappa a}\}e^{-2ik_1(x_0 + a)}, \tag{10}$$

$$B = -2\tau\xi_2e^{-ik_1(x_0 + a)}e^{-\kappa a}, \tag{11}$$

$$C = \tau\xi_1e^{-ik_1(x_0 + a)}e^{\kappa a}, \tag{12}$$

$$T = -4\tau\delta e^{-ik_1(x_0 + a)}, \tag{13}$$

where

$$\delta = \frac{\kappa}{k_1}, \quad \eta = \frac{k_3}{k_1}, \tag{14}$$

$$\begin{aligned}\frac{1}{\tau} = {} & \xi_0[\sin(\phi - k_3x_0)e^{-\kappa a} - \sin(\phi + k_3x_0)e^{+\kappa a} \\ & - i\delta\{\sin(\phi - k_3x_0)e^{-\kappa a} + \sin(\phi + k_3x_0)e^{+\kappa a}\}],\end{aligned} \tag{15}$$

$$\xi_1 = \xi_0\sin(\phi - k_3x_0), \tag{16}$$

$$\xi_2 = \xi_0\sin(\phi + k_3x_0), \tag{17}$$

$$\xi_0 = \sqrt{\eta^2 + \delta^2} = \frac{\sqrt{k_3^2 + \kappa^2}}{k_1}, \tag{18}$$

and

$$\phi = \tan^{-1}\frac{\eta}{\delta} = \tan^{-1}\frac{k_3}{\kappa}. \tag{19}$$

Now let us look at the solution in Region 3. With (13) and (15), the coefficient T can be written as $T(\omega) \equiv |T|e^{i\theta}$where the amplitude $|T|$ and phase θ become

$$\begin{aligned}\frac{4\delta/\xi_0}{|T|} = {} & [\{\sin(\phi - k_3x_0)e^{-\kappa a} - \sin(\phi + k_3x_0)e^{+\kappa a}\}^2 \\ & + \delta^2\{\sin(\phi - k_3x_0)e^{-\kappa a} + \sin(\phi + k_3x_0)e^{+\kappa a}\}^2]^{1/2},\end{aligned} \tag{20}$$

$$\theta = \alpha - k_1(x_0 + a), \tag{21}$$

where

$$\alpha = \tan^{-1}\delta\frac{\sin(\phi - k_3x_0)e^{-\chi a} + \sin(\phi + k_3x_0)e^{+\chi a}}{\sin(\phi - k_3x_0)e^{-\chi a} - \sin(\phi + k_3x_0)e^{+\chi a}}. \tag{22}$$

3. Results

We investigate the above equations based on the model with appropriate parameters. The location of the boundaries at the plasmapause is at x_o= 3.5 RE and x_o + a = 5.5 RE, respectively. The Alfven speeds are assumed as V_{A1} = 1000 km/sec, V_{A2} = 2200 km/sec,

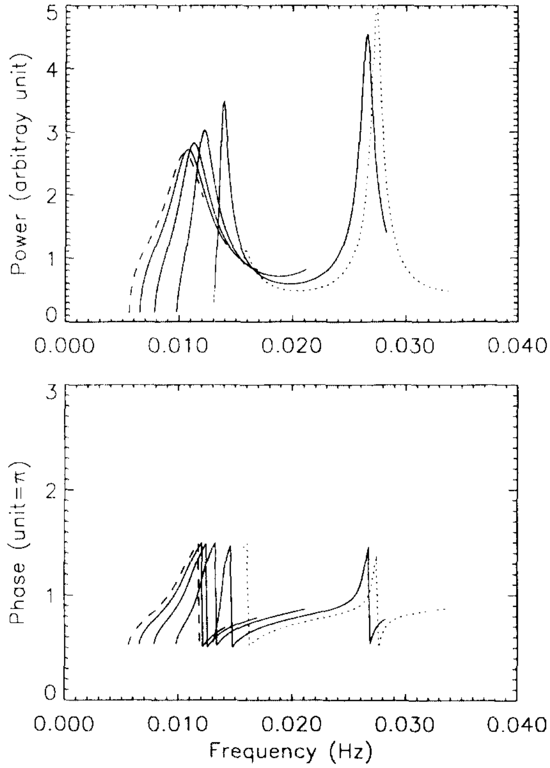

Fig. 2. The amplitude |T(ω)| (top) and phase angle $\theta(\omega)$ (bottom) for $(k_y^2+k_z^2)^{1/2}V_{A1} < \omega < (k_y^2+k_z^2)^{1/2}V_{A2}$ To satisfy the range of Pi 2 period(40-150 seconds), the wavenumber $(k_y^2+k_z^2)^{1/2}$ varies from 2π/10RE (dotted curve) to 2π/30RE (dashed curve). The other solid curves are drawn with intermediate wavenumbers between the two values. The power unit is normalized by s_0 which is the initial amplitude of disturbance assumed at the source.

and V_{A3} = 700 km/sec. Fig. 2 shows the wave amplitude and phase of the plasmaspheric response to the impulse in the magnetotail. Each curve is drawn for different k_y and k_z values, where $(k_y^2+k_z^2)^{1/2}$ varies from $2\pi/10\mathrm{R_E}$ to $2\pi/30\mathrm{R_E}$. These values are determined by the condition that the trapped wave frequencies should lie in the appropriate range of Pi 2 oscillations, for instance, 40-150 seconds. It is evident that relatively strong spectral peaks are found near $f =$ 0.010 – 0.015 Hz and $f =$ 0.026 – 0.028 Hz, which are the fundamental trapped resonance and the second trapped resonance. These two peaks are the virtual resonances of the plasmasphere for the impulsive incoming disturbances, which are very consistent with the recent Pi 2 observations (*Lin et al.*, 1991; *Takahashi et al.*, 1995). The phase relation also shows a very interesting feature that the angle is shifted by about π for each resonant mode. This suggests that Pi 2 pulsations may show such phase shifts if

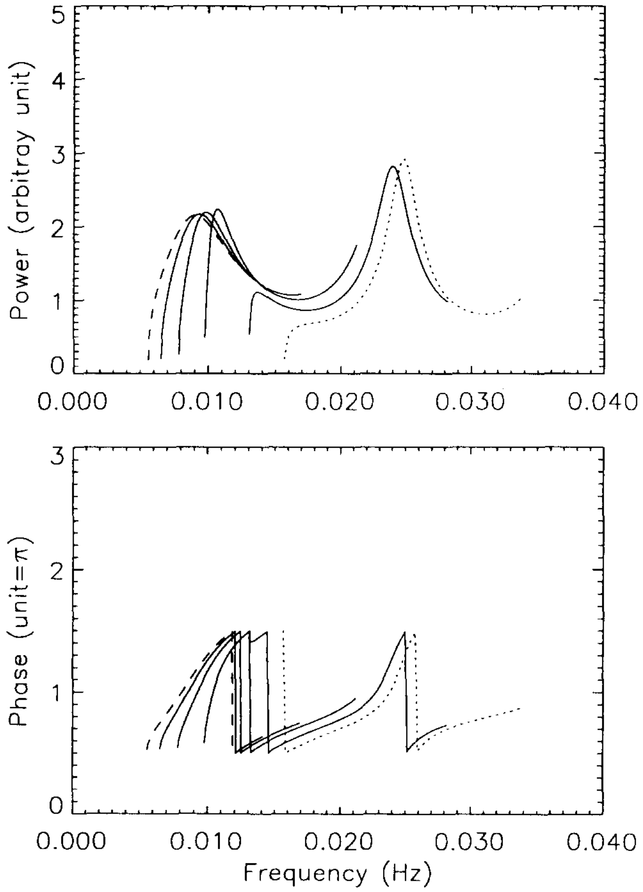

Fig. 3. The $|T(\omega)|$(top) and $\theta(\omega)$ (bottom) for $x_0=4.0R_E$ and $x_0+a=5.0R_E$, respectively, with the same Alfven speed as shown in Fig. 1. V_{A1}

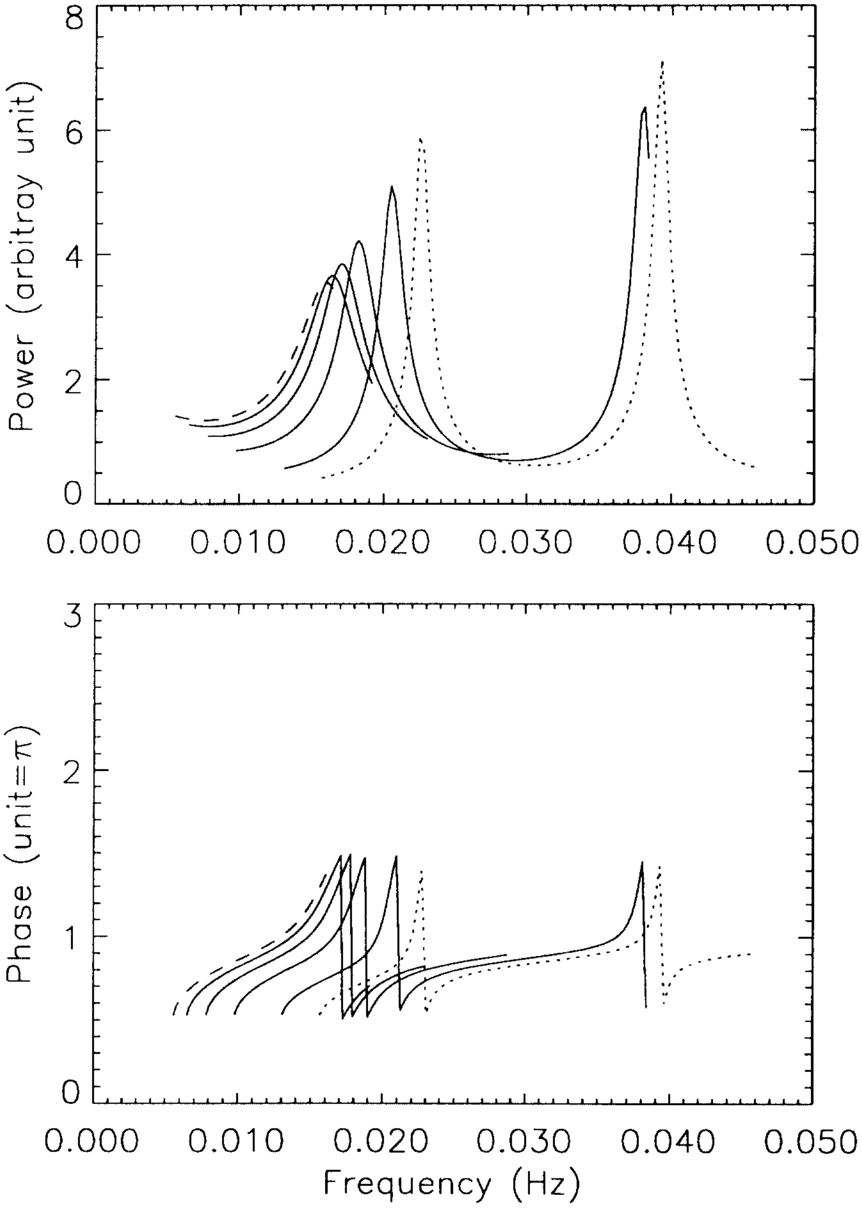

Fig. 4. The $|T(\omega)|$ (top) and $\theta(\omega)$ (bottom) for V_{A1} =1000 km/sec, V_{A2} =3000 km/sec and V_{A3} =1000 km/sec. The boundaries are assumed to be the same as shown in Fig. 1.

they arise owing to the virtual resonances of the plasmasphere. Fig. 3 and Fig. 4 represent the plasmaspheric responses for different boundaries and Alfven speed profiles. In Fig. 3, the location of the boundaries is assumed to be at x_0 = 4.0R$_E$ and x_0+a = 5.0R$_E$, respectively, with the same Alfven speed from those as shown in Fig. 1. It is evident that the peaks become relatively wide-banded since the plasmapause is now assumed to be thinner and less opaque compared to the previous case. In Fig. 4, we assume different Alfven speeds from those in Fig. 1 with the same boundary condition as in Fig. 1. The Alfven speeds are assumed as V_{A1} = 1000 km/sec, V_{A2} = 3000 km/sec and V_{A3} = 1000 km/sec . It is still the case that the two-peaked resonant states appear similarly as in Fig. 2 and Fig. 3. In fact, the spectral feature shown in Fig. 2 - Fig. 4 is found to be highly persistent for the change of parameters assumed above such as the Alfven speed and the plasmapause thickness, which simply moves the peak frequencies and the bandwidths of resonant modes. We also neglected the damping rate here, which changes only the bandwidth of each resonant peak in figures.

4. Discussion and Summary

Our analytic results are based on a simple model, but still provide a clear physics in dynamical wave properties near the plasmapause. Our model is simplified in two respects: One is the simplicity on the Alfven speed profile adopted. The other is the simplicity on the geometry we assumed. It is basically one-dimensional, so that the localization of sources in the east-west and north-south direction is neglected. These aspects have been well discussed by *Lee* (1997) where this box-like model is first introduced. Our main results derived from this analytic approach are summarized with the questions listed in Introduction. Each answer to the corresponding question is presented below, which is summarized as follows:

1) From (20), the plasmasphere is expected to always produce significant double peaked power in the Pi 2 frequency range. Whether the source is long-lived or short-lived, the effects of the source characteristics are imposed on this spectral feature as a secondary factor and such peaks would consistently remain the same. Thus, Pi 2 pulsations reflect the nature of plasmaspheric spatial structure independent of any changes in the outer space, which may explain the stable occurrences of Pi 2 pulsations.
2) The resonant modes exist inside the plasmasphere and extends a bit to the plasmapause region. Thus, the outer space has less possibilities to see these internally excited oscillations.
3) The cavity modes require relatively firm surrounding boundaries to be estabilished. They also need a relatively short impulse from the source to be defined: otherwise, it becomes a driven case, which no longer allows any cavity/waveguide modes. Our results above assumed arbitrary (soft) boundary conditions at the plasmapause, which persistently show the existence of virtual resonant modes without any necessary conditions of the cavity mode formation.

4) This is another important aspect in the Pi 2 phenomena. The cavity modes only select the corresponding wave power at each resonant frequency among the energy from the source. The theory of cavity modes indicates that the cavity makes frequencies discrete in spectrum, but it cannot significantly amplify the certain wave modes compared to the initial disturbances. In Fig. 2 - Fig. 4, each spectral peak is significantly larger than the initial wave amplitude given by unit in (5). This enhancement occurred since the source energy is continuously delivered into the plasmasphere and piled up in each resonant mode. This fact explains how we see amplified signals from less strong input in space.

Acknowledgments

This work was supported by the Korean Science and Engineering Foundation (96-0702-03-01-3) and the Basic Science Research Institute Program, Ministry of Education (BSRI 96-5408).

References

Allan, W., E. M. Poulter, and S. P. White, Hydromagnetic wave coupling in the magnetosphere-plasmapause effects on impulse-excited resonances, *Planet. Space Sci.*, *34*, 1139, 1986.

Chappell, C. R., The terrestrial plasma source: a new perspective in solar-terrestrial processes from Dynamics Explorer, *Rev. Geophys.*, *26*, 229, 1988.

Fugita, S., and K. H. Glassmeier, Magnetospheric cavity resonance oscillations with energy flow across the magnetopause, *J. Geomagn. Geoelectr.*, *47*, 1277, 1995.

Lee, D. H., Dynamics of MHD wave propagation in the low-latitude region, *J. Geophys. Res.*, *101*, 15371, 1996.

Lee, D. H., On the generation mechanism of Pi 2 pulsations in the magnetosphere, submitted to *Geophys. Res. Lett*, 1997.

Lin, C. A., L. C. Lee, and Y. J. Sun, Observations of Pi 2 pulsations at a very low-latitude (L=1.06) station and magnetospheric cavity resonances, *J. Geophys. Res.*, *96*, 21105, 1991.

Sutcliffe, P. R., The association of harmonics in Pi 2 power spectra with the plasmapause, *Planet. Space Sci.*, *23*, 1581, 1975.

Sutcliffe, P. R., and K. Yumoto, On the cavity mode nature of low-latitude Pi 2 pulsations, *J. Geophys. Res.*, *96*, 1543, 1991.

Takahashi, K., and B. J. Anderson, Distribution of ULF energy ($f < 80$ mHz) in the inner magnetosphere: A statistical analysis of AMPTE CCE magnetic field data, *J. Geophys. Res.*, *97*, 10751, 1992.

Takahashi, K., S.-I. Ohtani, and B. J. Anderson, Statistical analysis of Pi 2 pulsations observed by the AMPTE CCE spacecraft in the inner magnetosphere, *J. Geophys. Res.*, *100*, 21929, 1995.

Yeoman, T. K., and D. Orr, Phase and spectral power of mid-latitude Pi 2 pulsations: Evidence for a plasmaspheric cavity resonance, *Planet. Space Sci.*, *37*, 1367, 1989.

Yumoto, K., K. Takahashi, T. Sakurai, P. R. Sutcliffe, S. Kokubun, H. Lühr, T. Saito, M. Kuwashima, and N. Sato, Multiple ground-based and satellite observations of global Pi 2 magnetic pulsations, *J. Geophys. Res.*, *95*, 15175, 1990.

Zhu, X. M., and M. G. Kivelson, Global mode ULF pulsations in a magnetosphere with a nonmonotonic Alfven velocity profile, *J. Geophys. Res.*, 94, 1479, 1989.