

Determination of Cutting Orientation in Zigzag Milling Operations: A Geometrical Approach

Byeong Keuk Kim*, Joon Young Park* and Nam-Sook Wee**

ABSTRACT

This paper describes new methods to minimize the cutting time in zigzag milling operation of two dimensional polygonal surfaces. Previous works have been focused on mainly experimental approaches by considering some machining parameters such as, spindle speed, depth of cut, cutter traverse rate, cutter diameter, number of teeth, tool wear, life of tool, and so on. However, in this study, we considered two geometrical factors one by one in order to see the effect separately, which are the length of cut and the number of cutter traverse. In an N -sided concave or convex polygon, an algorithm has been developed which minimize the total length of cut. Also, a heuristic approach was used to minimize the number of cutter traverse.

Key words : Zigzag milling, Length of cut, Number of cutter traverse

1. Introduction

Modern machine tools are made to have high accuracy and various uses. However, most process planning processes need experienced workers. It means that effective methods to minimize machining time depend on an experienced operator. Although many manufacturing systems are used in process plans or analytical designs, determination of the optimal cutting orientation remains as an open problem.

Among the cutting operations, milling is the most widely used to fabricate complex geometric surfaces. Milling operations can be classified into two types; window frame milling and stair case milling which is also called a zigzag milling. It has been known that the zigzag milling takes less time than the window frame milling when an object is machined^[10]. In the zigzag milling operation, minimum cutting length for a polygonal object was found when the tool path was generated paral-

lel to the longest edge^[6,9,11]. However, in these works, since the shapes of the polygon was limited to triangles, rectangles, and pentagons which are convex, optimal cutting orientation for more complex objects could not be found.

In this paper, we present the optimal cutting orientation for minimizing the machining time when an N -sided concave or convex polygonal object is machined using a zigzag milling operation. Cutting conditions^[1], such as feed rate, depth of cut, spindle speed, cutter diameter, and the number of teeth were assumed to be constant. Determination of the optimal cutting orientation was performed in two ways where the total cutting length and the number of cutter traverse are minimized respectively. Fig. 1 shows the concave polygon of 28 edges. We cautiously considers to select either two ways in the polygon.

When the total cutting length was minimized, an optimal cutting orientation was found for N -sided convex or concave polygons. When the number of cutter traverse was minimized, an optimal cutting orientation was found for triangles and rectangles while a heuristic approach was used for N -sided concave or convex polygons.

*학생회원, Dept. of Industrial Engineering, Dongguk Univ.

**종신회원, Dept. of Industrial Engineering, Dongguk Univ.

***정회원, Dept. of Industrial Engineering, Hansung Univ.

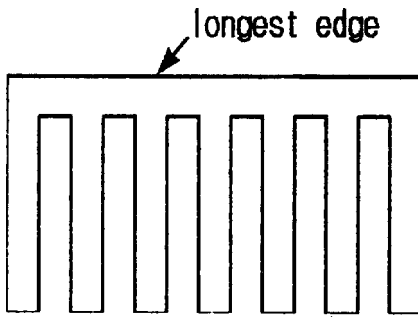


Fig. 1. The concave polygon of 28 edges.

2. Mathematical Model

In order to set up a mathematical model of optimal cutting orientation, following notations are used. The object to be machined is assumed to be a polygonal shape.

- T_L : Total length of cut
- M_A : Area of polygon
- L_A : Length of direct cut
- L_E : Length of indirect cut
- D : Cutter diameter
- l_i : Length of i_{th} edge
- l'_i : Length of i_{th} sweep in a zigzag milling operation
- θ : Cutting orientation with respect to the base edge ($0^\circ \leq \theta \leq 180^\circ$)
- θ_i : Angle between i_{th} edge and the tool path orientation
- θ_{ij} : Angle between i_{th} edge and j_{th} edge
- θ_{ic} : Angle between i_{th} edge and convex hull
- h : Pseudo-height of polygon measured perpendicularly to the cutting orientation

Here, L_A means the length of tool path inside the polygon including the initial length of cut which is parallel to an edge, and L_E means the length of tool path along the edges of the polygon excluding the initial length of cut which is parallel to an edge. Fig. 2 shows the length of L_A and L_E in the triangular polygon.

2.1 Length of cut

Following assumptions are made when the

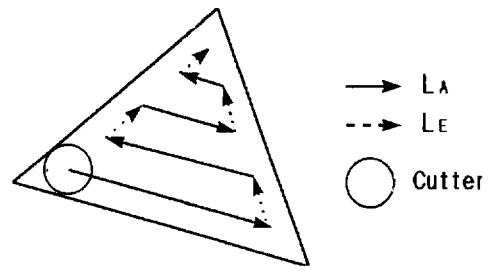


Fig. 2. The definition of L_A and L_E .

length of cut is considered as a factor in minimizing the cutting time.

Assumptions:

- (1) Cutting starts from a vertex of the polygon.
- (2) Center of the tool travels every vertex of the polygon.
- (3) Cutting ends at a vertex of the polygon.
- (4) Direction of the cutting inside the polygon is parallel to one of the edges.

2.1.1 Mathematical model of a triangle

Since a triangle is the simplest polygon, it is considered first. Fig. 3 shows a triangular object. In this case, the total length of cut T_L can be formulated as $L_A + L_E$. Fig. 4 shows a tool path parallel to the edge l_1 in the polygon. Here, we can formulate the relationship between the length of cut and the area of polygon, M_A .

$$\begin{aligned}
 M_A = & \left\{ (l'_1 + l_1) \times \frac{1}{2} \times \frac{1}{2} \right\} + \left\{ (l'_1 + l'_2) \times D \times \frac{1}{2} \right\} \\
 & + \left\{ (l'_2 + l_2) \times D \times \frac{1}{2} \right\} + \dots \dots \dots \quad (1) \\
 & + \left\{ (l'_{n-1} + l_n) \times D \times \frac{1}{2} \right\} + \left\{ (l_n) \times \frac{D}{2} \times \frac{1}{2} \right\}
 \end{aligned}$$

Here, we can notice that the initial cutting starts from the position apart from an edge by the distance of tool radius. Equation (1) can be simplified as follows.

$$\begin{aligned}
 M_A = & \left\{ (l'_1 + l'_2 + \dots + l'_{n-1} + l_n) \times D \right\} \\
 & + \left\{ \frac{D}{4} \times l_1 \right\} - \left\{ \frac{D}{4} \times (l'_1 + l_n) \right\} \quad (2)
 \end{aligned}$$

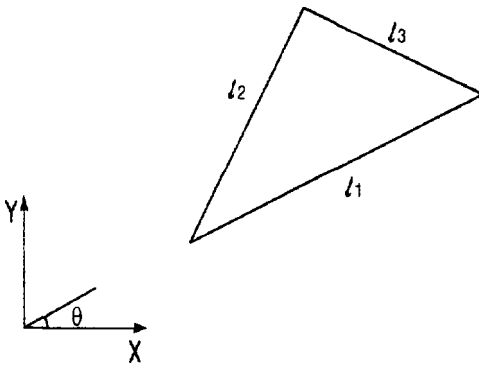


Fig. 3. Triangular object.

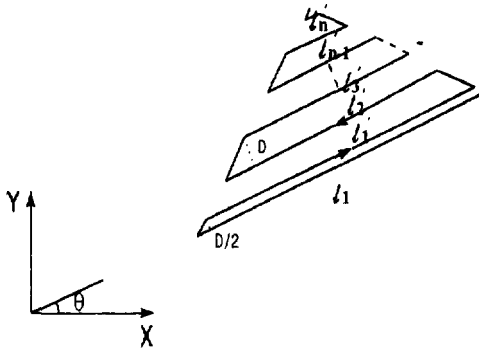


Fig. 4. Tool path in a triangular object.

l'_1 and l'_n can be represented using the relationship with θ_2 .

$$l'_1 : l_2 - \frac{D}{(2 \times \sin \theta_2)} = l_1 : l_2$$

$$l'_n = \frac{D}{(2 \times \sin \theta_2)} = l_1 : l_2$$

Thus, l'_1 and l'_n are simplified as follows.

$$l'_1 = \frac{l_1}{l_2} \times (l_2 - \frac{D}{2 \times \sin \theta_2}) \tag{3}$$

$$l'_n = \frac{l_1}{l_2} \times (\frac{D}{2 \times \sin \theta_2}) \tag{4}$$

From equations (3) and (4), $l'_1 + l'_n$ equals to l_1 .

Hence, we have

$$M_A = \{(l'_1 + l'_2 + \dots + l'_{n-1} + l'_n) \times D\} \text{ and}$$

$$l'_1 + l'_2 + \dots + l'_{n-1} + l'_n \text{ can be denoted as } L_A.$$

Therefore, the length of direct cut in the triangular polygon, L_A , can be formulated as follows.

$$L_A = \frac{M_A}{D} \tag{5}$$

The length of indirect cut, L_E , can be represented as follows using the relationship with l_2 and l_3 .

$$L_E = \frac{l_2 + l_3}{2} \tag{6}$$

From (5) and (6), total length of cut, T_L , is computed as follows.

$$T_L = \left\{ \frac{M_A}{D} \right\} + \left\{ \frac{l_2 + l_3}{2} \right\} \tag{7}$$

2.1.2 Mathematical model of a quadrilateral

To extend the above idea for a more generalized case, a concave quadrilateral is considered. Analysis for a concave polygon is a new attempt which has not been performed in the previous work^[6,9-11]. Fig. 5 shows a quadrilateral object.

Similar to the case of a triangular object, the total length of cut for a quadrilateral can be denoted as follows.

$$T_L = L_A + L_E$$

Fig. 6 shows the tool path parallel to the edge l_1 in the quadrilateral polygon. The indirect cut is computed as follows.

$$L_E \approx \left\{ \frac{1}{2} \times \left(l_2 \pm \frac{D}{2 \times \sin \theta_2} \right) \right\} + \left\{ \frac{1}{2} \times l_3 \right\} \tag{11}$$

$$+ \left\{ \frac{1}{2} \times \left(l_4 \mp \frac{D}{2 \times \sin \theta_4} \right) \right\}$$

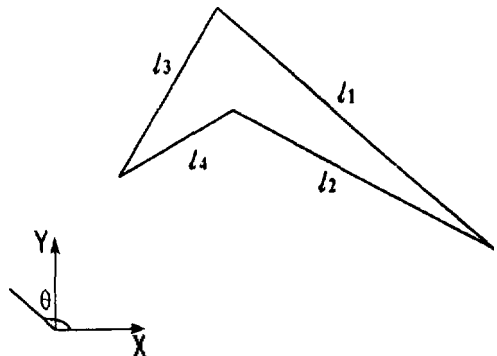


Fig. 5. Quadrilateral object.

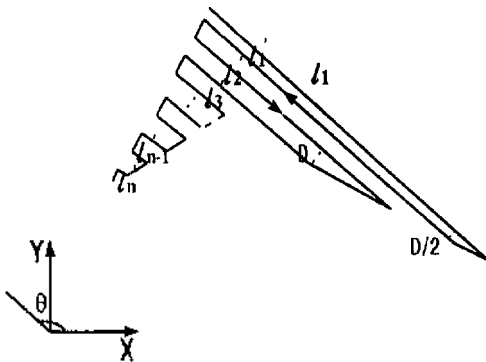


Fig. 6. Tool path in a quadrilateral object.

In the equation (11), we can see that the length of indirect cut along the l_2 and l_4 is not the half of the edge length. This is because the center of tool travels the vertex where l_2 and l_4 intersect. Therefore, if the number of edges is bigger than or equal to 4, the length of indirect cut should be modified by the radius of tool. This modification is necessary for every pair of edges which share a vertex other than starting and ending vertices. In the modification, if there is a need to add or subtract a certain distance for an edge, the subtraction or the addition for the corresponding edge should be done and vice versa. From equations (10) and (11), the total length of cut in a quadrilateral polygon can be represented as follows.

$$T_L = \left\{ \frac{M_A}{D} \right\} + \left\{ \frac{1}{2} \times (l_2 \pm \frac{D}{2 \times \sin \theta_2}) \right\} + \left\{ \frac{1}{2} \times l_3 \right\} + \left\{ \frac{1}{2} \times (l_4 \mp \frac{D}{2 \times \sin \theta_4}) \right\} \quad (12)$$

As it was shown in the mathematical model of the triangular and quadrilateral object, the length of direct cut was found to be equal to M_A/D , and the length of indirect cut roughly corresponds to the half of total edge lengths excluding the edge which is parallel to the cutting orientation. Particularly, a modification should be made for every pair of edges which share a vertex other than starting and ending vertices. From these observations, we can find the total length of cut for an N -sided convex or concave polygon as illustrated in the Fig. 7.

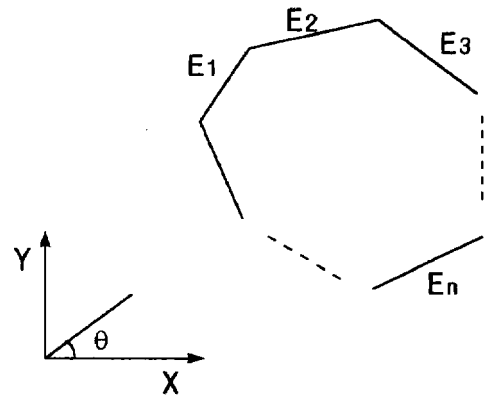


Fig. 7. N -sided polygon.

$$\begin{aligned} \text{total length of cut} = & \left\{ \frac{\text{area of polygon}}{\text{diameter of tool}} \right\} \\ & + \left\{ \frac{\text{sum of each edge length except edge parallel to cutting direction}}{2} \right\} \\ & \pm \left\{ \frac{\text{diameter of tool}}{2 \times \sin(\text{angle between the cutting orientation and edge})} \right\} \end{aligned}$$

The proposed model can be applied to an arbitrarily shaped N -sided convex or concave polygon. It can be also noticed that the length of direct cut is always constant as the division of the polygon area by the tool diameter regardless of the cutting orientation. Thus the total length of cut is influenced by the length of indirect cut which is the function of sum of all edges excluding the edge parallel to the cutting direction. Therefore, the total length of cut is the shortest when the tool path is generated parallel to the longest edge in the polygon.

2.2 Number of cutter traverse

In this section, number of cutter traverse is minimized to see the influence to the total cutting time. The number of cutter traverse corresponds to the division of the pseudo-height of the polygon by the tool diameter. Here, the pseudo-height of the polygon is defined as the minimum distance between two lines which are parallel to the cutting direction and contain the polygon.

2.2.1 Mathematical model of a triangle

In order to express the relationship between the pseudo-height of polygon and θ , it is necessary to divide the range of θ as follows. Fig. 8 shows a triangular object, and Fig. 9 shows the pseudo-height of polygon which is perpendicular to the cutting orientation.

Case 1: θ lies within the range of θ_{12}

$$h = \{l_2 \times \sin(\theta_{12} - \theta)\} + \{l_1 \times \sin(\theta)\} \tag{13}$$

Case 2: θ lies within the range of θ_{23} .

$$h = \{l_3 \times \sin(\theta_{23} - \theta)\} + \{l_2 \times \sin(\theta)\} \tag{14}$$

Case 3: θ lies within the range of θ_{31} .

$$h = \{l_1 \times \sin(\theta_{31} - \theta)\} + \{l_3 \times \sin(\theta)\} \tag{15}$$

As the direction of the cutting varies, the pseudo-height of triangular polygon changes according to the equations (13)-(15). Fig. 10 shows the range of the cutting orientation in triangular polygon.

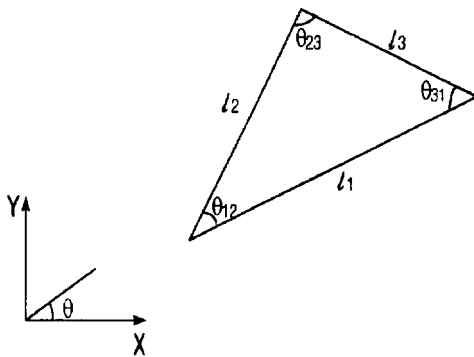


Fig. 8. Triangular object.

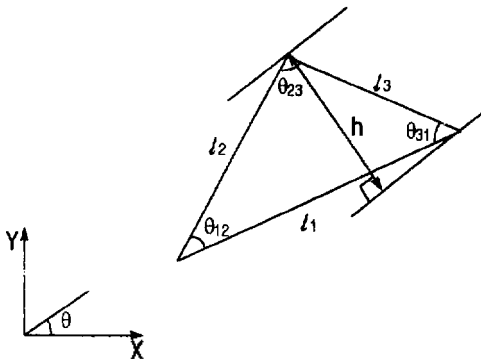


Fig. 8. The pseudo-height of triangular object.

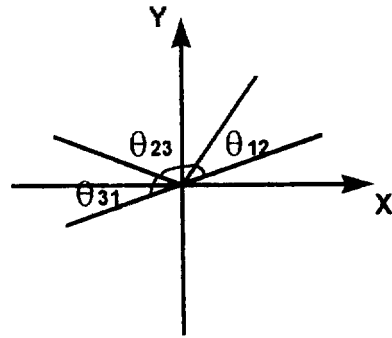


Fig. 10. The range of the cutting orientation in triangular polygon.

2.2.2 Mathematical model of a quadrilateral

In a concave quadrilateral, it should be noted that the number of cutter traverse can be bigger than one in a single scan of the polygon. This case happens within the so-called reflex profile region. Fig. 11 shows a reflex profile region.

Reflex profile region is the area where the tool movement without cutting is not necessary^[4]. Fig. 12 shows a quadrilateral polygon and the Fig. 13 shows the pseudo-height of polygon which is perpendicular to the cutting orientation. In a quadrilateral, it is necessary to divide the range of θ as follows.

Case 1: θ lies within the range of θ_{4c} .

$$h = \{l_1 \times \sin(\theta)\} + \{l_4 \times \sin(\theta_{12} + \theta_{2c} - \theta)\} \tag{16}$$

Case 2: θ lies within the range of θ_{34} .

$$h = \{l_3 \times \sin(\theta_{34} + \theta_{12} - \theta)\} + \{l_2 \times \sin(\theta_{2c} + \theta_{4c} + \theta)\} + \{l_4 \times \sin(\theta - \theta_{4c})\} \tag{17}$$

Case 3: θ lies within the range of θ_{13} .

$$h = \{l_3 \times \sin(\theta)\} + \{l_1 \times \sin(\theta_{13} - \theta)\} \tag{18}$$

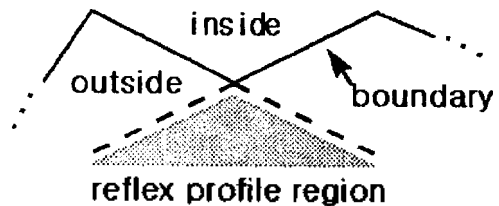


Fig. 11. Reflex profile region.

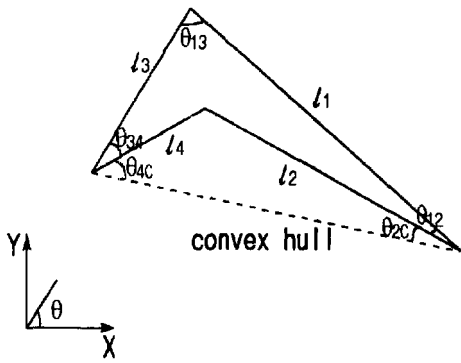


Fig. 12. Quadrilateral object.

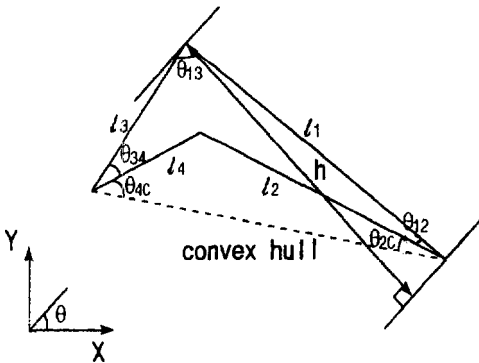


Fig. 13. The pseudo-height of quadrilateral object.

Case 4: θ lies within the range of θ_{12} .

$$h = \{l_1 \times \sin(\theta)\} + \{l_2 \times \sin(\theta_{12} - \theta)\} + \{l_4 \times \sin(\theta_{12} + \theta_{2c} + \theta_{4c} - \theta)\} \quad (19)$$

Case 5: θ lies within the range of θ_{2c}

$$h = \{l_1 \times \sin(\theta)\} + \{l_4 \times \sin(\theta_{12} + \theta_{2c} - \theta)\} \quad (20)$$

As the direction of the cutting varies, the pseudo-height of quadrilateral polygon changes according to the equations (16)–(20). Fig. 14 shows the range of the cutting orientation in quadrilateral polygon.

2.2.3 Mathematical model of an N-sided polygon

In the previous sections, mathematical models to minimize the number of cutter traverse for a triangular and a quadrilateral object were proposed. Since too many special cases should be considered as the number of edges increases, we investigated how to determine the range of optimal cutting

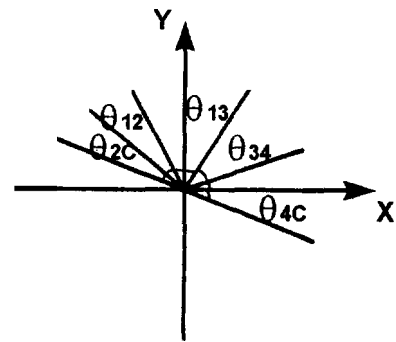


Fig. 14. The range of the cutting orientation in quadrilateral polygon.

orientation instead of the exact orientation in that case. In order to facilitate the process of finding the range of the optimal cutting orientation, decomposition of an object into trapezoidal regions was applied. Fig. 15 shows the trapezoidal decomposition in a concave octagon. In the trapezoidal decomposition, the polygon is decomposed into several trapezoids by drawing lines from the each vertex in an arbitrary direction until it meets the boundary of the polygon. Although triangles, *abd*, *def*, *ijl*, *mnp*, and trapezoids, *bfgh*, *ghik*, *jkmn*, can be generated through the process, triangles are considered as the special case of trapezoids^[7]. Fig. 16 shows a trapezoidal decomposition in a quadrilateral region. In that figure, A and A' denote the two possible scanning directions which divide the quadrilateral region into several trapezoids. With the direction A and A', three and two triangles are generated. If the pseudo-height of

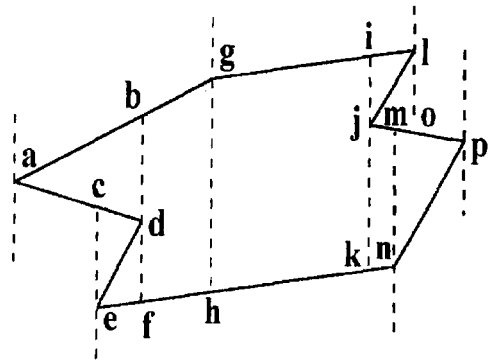


Fig. 15. Trapezoidal decomposition of an octagon.

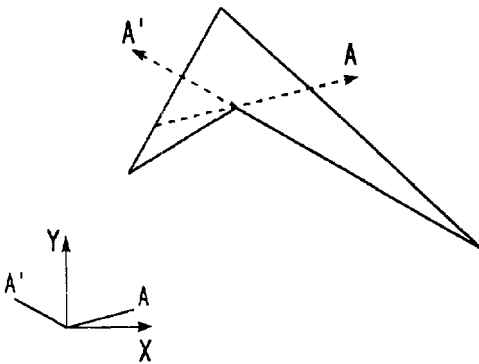


Fig. 16. Trapezoidal decomposition in a quadrilateral region.

decomposed trapezoid equals to the pseudo-height of the tool, the direction of minimizing the number of trapezoids becomes the optimal orientation.

We propose a heuristic approach to find the optimal orientation minimizing the number of cutter traverse in an object. To facilitate the computation, we consider the case where the cutting orientation is parallel to an edge. After the polygon is decomposed into trapezoids, pseudo-height of the each trapezoid can be added to represent the number of cutter traverse. If we apply this operation for every edge, we can find an edge which minimizes the number of cutter traverse. Now, the optimal orientation is conjectured to be within the θ around the edge. The θ here is the angle between the edge which minimizes the number of cutter traverse and another edge which has the smallest angle with the edge. Thus the proposed heuristic approach can be presented as follows;

Step 1: Select an edge.

Decompose the polygonized object into several trapezoids by drawing lines from each vertex to the direction parallel to the edge.

Step 2: Apply the Step 1 for every edge.

Step 3: Select the edge which minimizes the sum of the pseudo-height of trapezoids.

Step 4: Calculate the sum of the pseudo-height of the trapezoids generated by using an orientation which is inclined by θ from the edge.

Step 5: By trial and error, find the θ which min

imizes the number of cutter traverse.

In summary, we can determine the range where the optimal orientation lies from the steps 1 through 3, and the optimal orientation can be found by the step 4 and step 5.

3. Simulation

In this research, we investigated the two elements, the length of cut and the number of cutter traverse to determine the orientation of minimizing the total cutting time. Through the mathematical analysis for the length of cut, we concluded that the optimal orientation should be selected as parallel to the longest edge in the polygon.

In a computer simulation, we investigated how the the length of cut changes as the angle varies. The length of cut was computed for each orientation from 0 to 180 degrees by changing the angle by one degree. Through the mathematical analysis for the number of traverse, only the range, where the orientation of minimum number of cutter traverse lies, was computed. Thus more precise value for the number of cutter traverse was found by changing the angle from 0 to 180 degrees.

We can summarize the result of the simulation as follows. Given a quadrilateral object as shown in Fig. 17, direction of the minimum length of tool path was found to be parallel to the edge a which is the longest edge in the polygon. See Fig. 18. Direction of having the minimum number of cutter traverse was also found to be near to the direction of the edge a as shown in Fig. 19. It was within the range of 3 degrees. This result is the same as

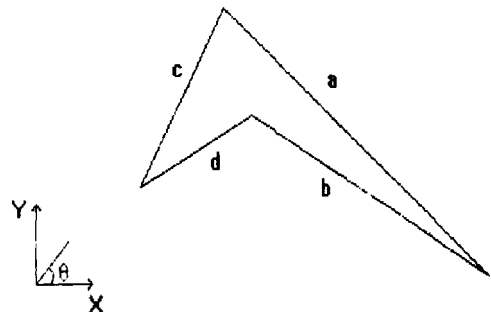


Fig. 17. Quadrilateral polygon.

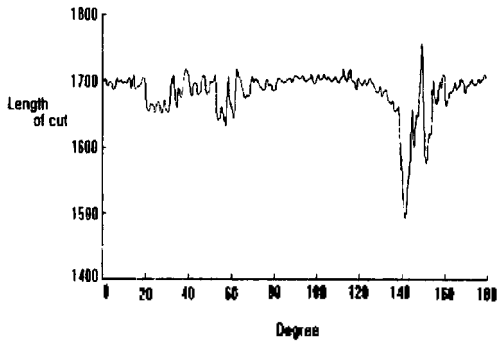


Fig. 18. Length of cut variation in quadrilateral.

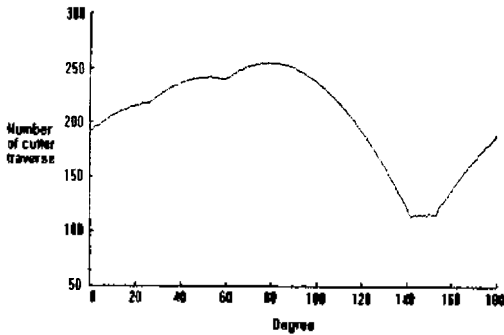


Fig. 19. Number of cutter traverse variation in concave quadrilateral.

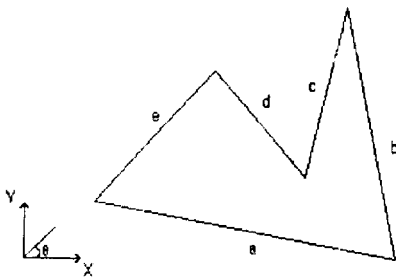


Fig. 20. Pentagonal object.

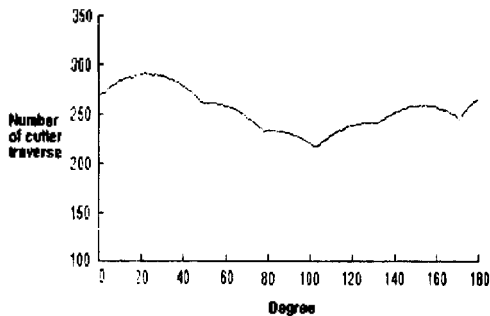


Fig. 21. Number of cutter traverse variation in concave pentagonal.

the one we could get from the heuristic approach.

In order to convince the above result, one more example was tested as shown in Fig. 20. In this case, if we apply the heuristic approach, we could see that the minimum number of trapezoids occurs in the direction which is parallel to edge *b*. The result of the computer simulation as shown in Fig. 21 also found the direction of edge *b* as the optimal orientation which minimizes the number of cutter traverse.

4. Conclusion

In this research, we investigated how to minimize the total cutting time in milling operation by considering the two factors, total length of tool path and the number of cutter traverse. We found that the orientation for the minimum total length of tool path is the direction which is parallel to the longest edge of the object.

For the orientation minimizing the number of cutter traverse, a heuristic approach was developed which minimize the number of trapezoids. In the computer simulation for a quadrilateral and a pentagon, the heuristic approach was shown to be correct. The research of considering the number of cutter traverse and the total length of cut at the same time remains as a future research. And there will be studied the optimal cutting orientation of free surface in 3D space.

References

1. Chang, T.C., Wysk, R.A., Davis, R.P. and Choi, B., "Milling parameter optimization through a discrete variable transformation", *International Journal of Production Research*, Vol. 20, No. 4, pp. 507-516, 1982.
2. Chang, T.C., Wysk, R.A. and Wang, H.P., *Computer-Aided manufacturing*, Prentice-Hall, 1991.
3. Cowton, C.J. and Wirth, A., "On the economics of cutting tools", *International Journal of Production Research*, Vol. 31, No. 10, pp. 2441-2446, 1993.
4. Held, M., *On the Computational Geometry of pocket Machining*, Springer-Verlag, 1991.
5. Kim, B.K., "Determination of Cutting Orientation in Zigzag Milling Operation (-S: A Geometrical Ap-

proach) M.S. thesis of Dongguk univ., 1996.

6. Lakkaraju, R.K., Raman, S. and Irani, S.A., "An analytical model for optimization of NC tool cutting path", *International Journal of Production Research*, Vol. 30, No. 1, pp. 109-127, 1992.
7. Mulmuley, K., *Computational Geometry*, Prentice-Hall, 1994.
8. Marshall, S. and Griffiths, J.S., "A survey of cutter path construction techniques for milling machines", *International Journal of Production Research*, Vol. 32, No. 12, pp. 2861-2877, 1994.
9. Prabhu, P.V., Gramopadhye, A.K. and Wang, H.P., "A general mathematical model for optimizing NC tool path for face milling of flat convex polygonal surfaces", *International Journal of Production Research*, Vol. 28, No. 1, pp. 101-130, 1990.
10. Wang, H., Chang, H., Wysk, R.A. and Chandawarkar, A., "On the Efficiency of NC Tool Path Planning for Face Milling Operations", *Transactions of the ASME*, Vol. 109, No. 4, pp. 370-376, 1987.
11. Wang, H.P., Chang, H. and Wysk, R.A., "An analytical Approach to Optimize NC Tool Path Planning for Face Milling Flat Convex Polygonal Surfaces", *IIE Transactions*, Vol. 20, No. 3, 1988.



김 병 극

1995년 경일대학교 산업공학과 학사
 1997년 동국대학교 산업공학과 석사
 1997년~현재 동국대학교 산업공학과
 박사과정 2학기 재학중
 관심분야: NC 가공, CAPP, Geometric
 Modeling



박 준 영

1982년 한양대학교 기계공학과 학사
 1985년 University of Minnesota 산업공
 학과 석사
 1991년 University of Michigan 산업공학
 과 박사
 1992년~1995년 한성대학교 산업공학
 과 조교수
 1995년~현재 동국대학교 산업공학과
 부교수
 관심분야: Geometric Modeling, Compu-
 ter Graphics, Mesh Genera-
 tion, STEP



위 남 숙

1979년 서울대학교 수학과 학사
 1983년 univ. of California, Berkeley 산
 업공학과 석사
 1988년 univ. of California, Berkeley 산
 업공학과 박사
 1996년~현재 한성대학교 산업공학과 교수
 관심분야: Computational Geometry, CAD,
 Graphics