An Analytical Approach to the Spark Resistance Formula Caused by Electrostatic Discharge.

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Abstract

A modern electric system located at a certain distance from the discharge may respond with unexpected sensitivity, when an eletrostatic discharge (ESD) phenomenon occurs heteronomously between metallic objects. For analyzing the transient electromagnetic fields caused by ESD, two resistance formulas - Toepler and Rompe-Weizel - are introduced. The experimental results given by Wilson-Ma are used to compare which of these resistance formulas is proper.

1. Introduction

High-performance information equipments employing highly-integrated digital-IC with the low dissipation power are more fragile to electrostatic discharges (ESD). This is mainly because the immunity of high-tech equipments to the transient eletromagnetic noises caused by the ESD event has been degraded. It is well recognized that the eletromagnetic interference level of this kind is not always proportional to the occurrence voltage of the ESD and also that the lower voltage ESD sometimes gives the more serious damage to the equipments. Although the existence of the above phenomenon had already been pointed out by Honda[1], recent analytical studies about ESD event have been reported from the standpoint of the occurrence

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eletromagnetic fields[2],[3]. These reports, however, analyze the eletromagnetic fields on the basis of the measured waveform of the discharge current. Therefore, the distinctive phenomenon in the ESD event has not theoretically been explained, though the existence was confirmed experimentally. We believe that since this phenomenon is quite similar to the properties of the electromagnetic ignition noises caused by the gap breakdown of plug[4],[5], the occurrence mechanism will also be the same. In this paper, two resistence formulas(Rompe-Weizel's and Toepler's) will be applied to calculate the ESD current. By using the experimental data given by Wilson and Ma, verification of the proper resistence fomula will be made.

2. Theory

The portion immediately before ESD occurs can be in the condition of an electrical dipole moment. For simplicity, the charged object causing the ESD is assumed here to be a point charge. It is also assumed that the electric dipole moment having the charge quantites of $\pm q$ discharges across a spark gap ℓ. Fig. 1 (a) shows this situation . The portion right after the ESD has occurred can be modeled as a current dipole with the length of ℓ through which the ESD current i(t) flows, as shown in Fig.1 (b).

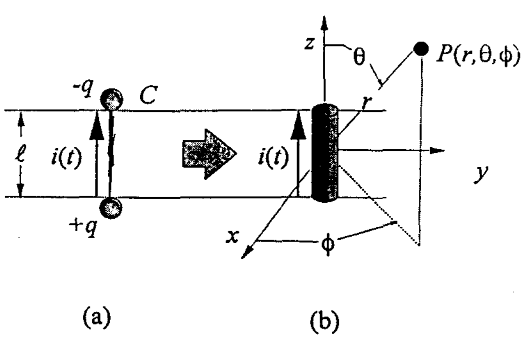

Fig.1 (a) Electric dipole moment and (b) dipole model

In this case the occurrence electromagnetic fields can be theoretically derived in terms of a function of the ESD current. Consider an ESD current as a single shot impulsive current. Denoting by I_m the current peak value and by τ the nominal duration period, it follows that $q = I_m \times \tau$. Then the ESD current i(t) can be expressed as

$$i(t) = I_m \times F(t/\tau)$$

where, $F(\cdot)$ is a dimensionless funtion representing the ESD current waveform and the following relation holds

$$\int_{\infty}^{0} F(x)dx = 1 \quad \cdots\cdots (1)$$

Using Eq. (1), we have the electromagnetic fields at a location $P(r, \theta, \Phi)$ in Fig.1 (b) given by

$$E_r(t) = \frac{1}{2\pi} \left(\frac{\ell}{c\tau}\right)^2 \frac{Z_o I_m}{l} \cos\theta \left\{ \frac{1}{(r/c\tau)^3} \left[1 - \int_0^{t/\tau} F(x' - \frac{r}{c\tau}) dx \right] + \frac{1}{(r/c\tau)^2} F(\frac{t}{\tau} - \frac{r}{c\tau}) \right\} \quad \cdots\cdots (2)$$

$$E_\theta(t) = \frac{1}{4\pi} \left(\frac{\ell}{c\tau}\right)^2 \frac{Z_o I_m}{l} \sin\theta \left\{ \frac{1}{(r/c\tau)^3} \left[1 - \int_0^{t/\tau} F(x' - \frac{r}{c\tau}) dx \right] + \frac{1}{(r/c\tau)^2} F(\frac{t}{\tau} - \frac{r}{c\tau}) + \frac{1}{(r/c\tau)} \frac{d}{d(t/c\tau)} F(\frac{t}{\tau} - \frac{r}{c\tau}) \right\} \quad \cdots (3)$$

$$H_\theta(t) = \frac{1}{4\pi} \left(\frac{\ell}{c\tau}\right)^2 \frac{I_m}{l} \sin\theta \left\{ \frac{1}{(r/c\tau)^2} F(\frac{t}{\tau} - \frac{r}{c\tau}) + \frac{1}{(r/c\tau)} \frac{d}{d(t/c\tau)} F(\frac{t}{\tau} - \frac{r}{c\tau}) \right\} \quad \cdots (4)$$

where c is the speed of light and $Z_o = \sqrt{\mu_o / \varepsilon_o}$. For the function F(·), considering the dipole as a Spark channel, we can derive the F(·) from solving the equation governing the electrical behavior of a capacitance around the discharge. For Spark resistence, both Toepler's formulas and Rompe Weisel's are commonly used to examine the development process of the spark. According to the kind of formula, the function F (·) is derived differently from these resistence formula. The derivation of F(·) is as follows.

2.1 Rompe-Weizel formula

Rompe-Weizel formula for spark resistence is expressed as follows[5]:

$$r(t) = \frac{\ell}{\sqrt{(2\alpha/p)\int_0^t i(t')dt'}} \qquad (5)$$

where r(t) is spark resistence at time t, ℓ is spark length, α is spark coefficient determined by gas pressure, p is pressure and i(t) is the spark current flowing through the gap. The stray capacitance across the gap before the ESD occurs is denoted by C_o, the spark voltage is V_s. The capacitance discharge circuit is as follows:

$$\begin{aligned} v(t) &= V_s - \frac{1}{C_o}\int_0^t i(t')\,dt' \\ v(t) &= r(t)\,i(t) \\ &= i(t) \times \frac{\ell}{\sqrt{(2\alpha/p)\int_0^t i(t')^2 dt'}} \end{aligned} \qquad (6)$$

Setting

$$\begin{aligned} z &= (2\alpha/p)(V_s/\ell)^2 t, \\ V &= v(t)/V_s, \\ I &= \frac{i(t)}{C_o V_s (2\alpha/p)(V_s/\ell)^2} \end{aligned} \qquad (7)$$

We can change Eq.(6) into the following form.

$$\begin{aligned} V(x) &= 1 - \int_0^x I(x')dx' \\ V(x) &= \frac{I(x)}{\sqrt{\int_0^x I(x')^2 dx'}} \end{aligned} \qquad (8)$$

Solving Eq. (8) under the conditions of V(0)=1 and I(0)=0, we can obtain

$$\begin{aligned} V(x) &= 1 - \int_0^x I(x')dx' \\ V(x) &= \frac{I(x)}{\sqrt{\int_0^x I(x')^2 dx'}} \end{aligned} \qquad (8)$$

$$\begin{aligned} I(x) &= \frac{1}{4}\exp\left(\frac{x - x_o}{2}\right) \\ &\cdot \left\{1 + \exp\left(\frac{x - x_o}{2}\right)\right\}^{-1.5} \end{aligned} \qquad (9)$$

where x_o is an integral constant. From $\frac{\partial I}{\partial x} = 0$ and (7), we can obtain as follows

$$I_m = \frac{C_o V_s}{\tau} = \frac{C_o V_s (\alpha/p)(V_s/l)^2}{3\sqrt{3}} \qquad (10)$$

$$\begin{aligned} F(t/\tau) &= \frac{3\sqrt{3}}{2}\exp\left\{3\sqrt{3}\left(\frac{t}{\tau} - x_o\right)\right. \\ &\cdot \left[1 + \exp\left\{3\sqrt{3}\left(\frac{t}{\tau} - x_o\right)\right\}\right]\Bigg\}^{-1.5} \end{aligned} \qquad (11)$$

$$\begin{aligned} \frac{\partial F(t/\tau)}{\partial(t/\tau)} &= \frac{27}{4}\exp\left\{3\sqrt{3}\left(\frac{t}{\tau} - x_o\right)\right\} \\ &\cdot \left[1 + \exp\left\{3\sqrt{3}\left(\frac{t}{\tau} - x_o\right)\right\}\right]^{-2.5} \\ &\cdot \left[2 - \exp\left\{3\sqrt{3}\left(\frac{t}{\tau} - x_o\right)\right\}\right] \end{aligned} \qquad (12)$$

The maximum slope of the ESD current is given by

$$\begin{aligned} &\left.\frac{di}{dt}\right|_{max} \\ &= \frac{2\sqrt{21} - 3}{75\sqrt{6 - \sqrt{21}}} C_o V_s (\alpha/p)^2 (V_s/l)^4 \end{aligned} \qquad (13)$$

Equation (10) and (11) show that the ESD current can be calculated from the discharge length ℓ, discharge voltage V_s, and capacitance of the ESD occurrence portion. Furthermore, as seen in Eq(10) and Eq(13) the peak value and maximum slope of the ESD current can be determined by the discharge voltage V_s and the electric field strength(V_s / ℓ), which are largely dependent on the latter. Fig. 2 shows the waveform of the discharge current and its time derivative that determines the radiated field.

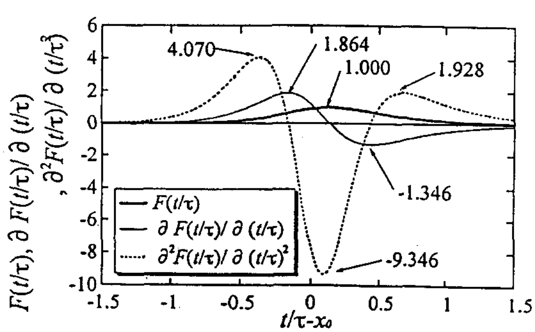

Fig. 2 Waveforms of dimensionless F(t/τ) and its derivative dF(t/τ)/d(t/τ)

2. 2 Toepler formula

When ESD phenomenon occured, Toepler proposed the following formula for spark resistance.

$$r(t)=\frac{K_T\cdot\ell}{\int_0^t i(t)dt} \quad \cdots\cdots (14)$$

where K_T is a Toepler constant, ℓ is the spark length and i(t) is the spark current. Denoting by v(t) the voltage across the gap, we have for the capacitance discharge circuit.

$$v(t)=V_s-\frac{1}{C_o}\int_0^t i(t')dt' \quad \cdots\cdots (15)$$

$$v(t)=r(t)\,i(t)=i(t)\times K_T\cdot\frac{\ell}{\int_0^t i(t)dt} \quad \cdot (16)$$

Here two equation become a nonlinear equation. By solving these nonlinear equation, the current and the derivative current are expressed as follows.

$$i(t)=C_oV_s\alpha\beta\frac{\exp\{\beta(t-t_o)\}}{(1+\alpha\exp\{\beta(t-t_o)\})^2} \quad \cdots (17)$$

$$\frac{\partial i(t)}{\partial t}=C_oV_s\alpha\beta^2\frac{1-\alpha\exp\{\beta(t-t_o)\}}{(1+\alpha\exp\{\beta(t-t_o)\})^3}\exp\{\beta(t-t_o)\} \quad \cdots\cdots (18)$$

$$\alpha=\frac{Q_{inj}}{C_oV_s},\quad \beta=\frac{V_s}{K_T\ell}$$

where $Q_{inj}(=10^{-10}[C])$ is the charge which is injected into the discharge part, $K_T=3.7\times10^{-7}[Vs/m]$ is a Toepler constant. These datas are extracted from Daout and Ryser[6]. If spark voltage V_S, spark capacitance C_o and spark length ℓ is fixed, the spark current waveform can be calculated from the Eq.(17) and (18). From these equations, we obtain

$$I_m=\frac{\beta C_oV_s}{4}=\frac{C_o V_s^2}{4K_T\ell} \quad \cdots\cdots (19)$$

$$\frac{\partial i(t)}{\partial t}\Big|_{max}=\frac{\sqrt{3}}{18}\frac{C_o V_2^3}{(K_T\ell)^2} \quad \cdots\cdots (20)$$

From (17) and (19), the ESD current can be calculated from the discharge length ℓ, discharge voltage V_S and capacitance of the ESD occurrence portion.

3. Numerical calculation

In the section, applying the theories mentioned above to the experimental data given by Wilson and Ma, we verify which of resistence formulas is valid for the ESD resistence model. Fig.3 shows the ESD dipole that Wilson and Ma studied. For this configuration, the electic field is found to be

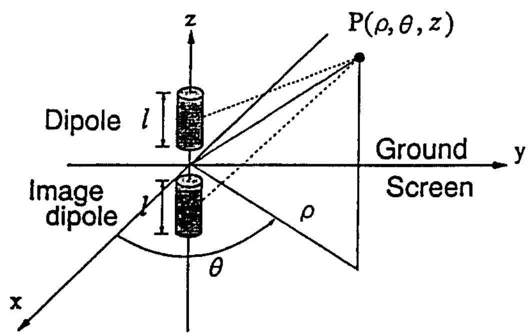

Fig. 3 Dipole model of ESD in close proximity to perfect ground screen that Wilson and Ma studied.

$$\overline{E}(\overline{r},t) \approx a_\rho dl \frac{\rho z}{R^2}\frac{\eta_o}{2\pi}\left\{\frac{3i(u)}{R^2}+\frac{1}{cR}\frac{\partial i(u)}{\partial u}\right\} + a_z dl\frac{\eta_o}{2\pi}\left\{\left[\frac{3z^2}{R^2}-1\right]\frac{i(u)}{R^2} + \left[\frac{z^2}{R^2}-1\right]\frac{1}{cR}\frac{\partial i(u)}{\partial u}\right\} \quad (21)$$

where R is the distance from the discharge point to the observation point (ρ, ϕ, z), η_o is the free space wave impedance, c is the propagation speed, and i(u) is the time-dependent ESD current wave form evaluated at time u=t-R/C. In order to validate Eq(21), Wilson and Ma conducted the following experiment. The small brass ball was placed in close proximity to the ground screen. A commercially available ESD simulator was used to produce sparks between the ball and ground. The spark voltage, the ESD current and the electric field was measured simultaneously. Substituting the ESD current waveforms measured for serveral sparks, they calculated the electric fields and confirmed that the calculated waveforms agree with the measured results. Here we use the theoretical current derived in former section and first of all, apply Rompe-Wiezel's resistence formula to the ESD current. In this place, in order to calculate the discharge current waveform, the gap length ℓ, and stray capacitance C_o across the spark gap are needed. By using Eq.(10), (13), ℓ and C_o are obtained as follows.

$$C_o = 27 \cdot \frac{2\sqrt{21}-3}{75\sqrt{6-\sqrt{21}}} \cdot \frac{1}{V_s} \cdot I_m^{\,2} / \frac{di}{dt}\Big|_{\max} \quad (22)$$

$$\ell = \sqrt{3\sqrt{3} \cdot \frac{2\sqrt{21}-3}{75\sqrt{6-\sqrt{21}}} \cdot a \cdot V_s^2 \cdot I_m / \frac{di}{dt}\Big|_{\max}} \quad (23)$$

Where, I_m and $\frac{di}{dt}\big|_{\max}$ is obtained from the measured current given by Wilson and Ma. The current can be derived from this formula and measured discharge voltage V_s. Next, by using Toepler's resistance formula, the other current is calculated. In this case, ℓ and C_o are obtained from Eq.(19), (20)

$$\ell = \frac{\sqrt{3}}{18} \cdot \frac{4V_s I_m}{K_T \cdot \frac{\partial i(t)}{\partial t}\Big|_{\max}} \quad (24)$$

$$C_o = \frac{4K_T \ell I_m}{V_s^2} \quad (25)$$

Table.1 shows the gap length ℓ and stray capacitance C_o according to spark voltage 2kV, 4kV and 6 kV. The gap length ℓ increases with spark voltage. The stray capacitance also grows with spark voltage. By making use of the calculated value ℓ, C_o, we calculate the ESD current and electric field waveform. In Fig. 4, 5, 6 the waveforms of the spark currents and the electric fields are showed according to spark voltage 2kV, 4kV, 6kV. In the course of calculating the spark current, the inductive component and the radiated component was considered. For the dipole length, we did not use the gap the length but the effective dipole length which was estimated so that the peak value of the radiated electric field coincides with the result measured for the 2kV spark. In the figures, the "o" lines are the experimental results given by Wilson and Ma. The solid lines are the calculated results given by Rompe-Weizel's resistance formula. The dotted lines are given by Toepler's Formula. From the figures, the calculated results approximately agree with the measured waveforms of the ESD currents and electric fields. The differences of the results given by the two resistance formulas do not almost exist. As seen in the figures, the wave tails of the measured ESD currents are more gentle than the calculated ones. This may be because the discharge current due to the charge stored in the circuit capacitance of the ESD simulator flows through the gap late after the spark.

Table 1 Gap length capacitance estimated from the measured waveform by Wilson and Ma

Spark voltage Vs[kV]	Gap length ℓ[mm]		Gap capacitance Co[pF]	
	Rompe Weizel	Toepler	Rompe Weizel	Toepler
2.0	0.438	0.253	13.03	10.76
4.0	0.827	0.451	13.13	10.84
6.0	1.892	1.573	16.44	13.58

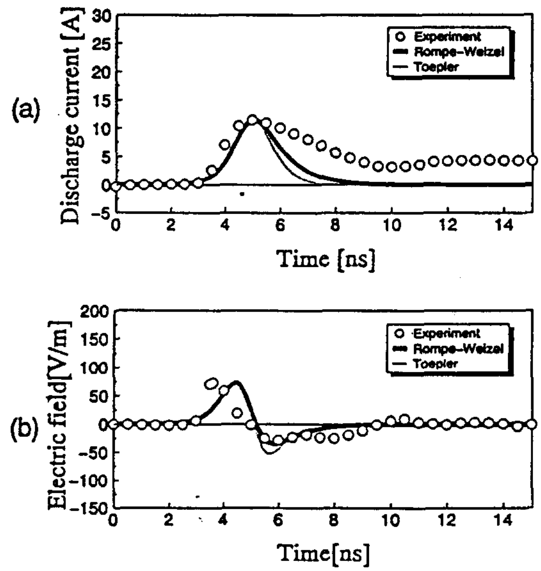

Fig. 4 Waveforms (a) spark current and (b) electric field for a 2kV

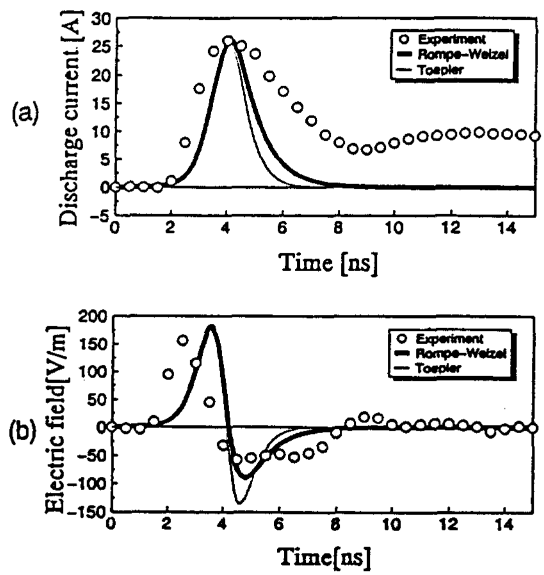

Fig. 5 Waveforms (a) spark current and (b) electric field for a 4kV

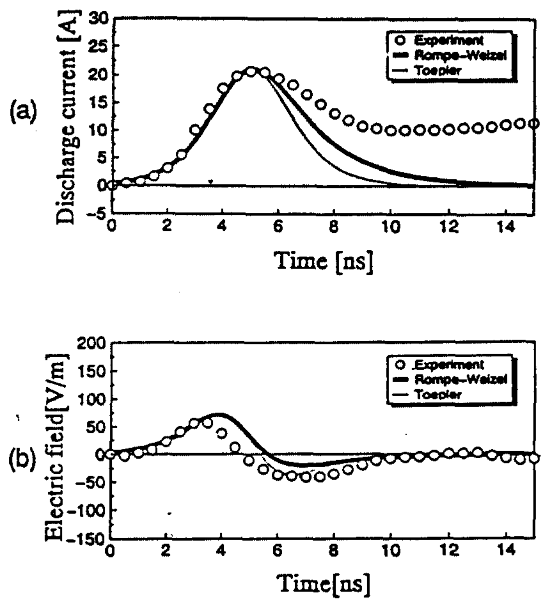

Fig. 6 Waveforms (a) spark current and (b) electric field for a 6kV

4. Conclusion

For analyzing the transient electromagnetic fields radiated by ESD, the Rompe-Weizel's resistance formula and Toepler's one are introduced. In order to compare the properties of two resistance, the experimental results given by Wilson ans Ma are used. The calculated results given by two resistance formulas approximately agree with the measured waveforms of the ESD currents and electric fields. We can see that the theoretical results obtained by Rompe-Weizel's resistance formular do not differ greatly from the results provided given by Toepler.

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