▩응용논문

# A Comparison on Confidence Intervals for P(X>Y) with Explanatory Variables *

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## Abstract

In this paper, we obtain some approximate confidence intervals for the reliability of the stress-strength model when the stress and strength each depend on some explanatory variables, respectively. Also we compare the confidence intervals via Monte Carlo simulation.

## 1. Introduction

An important extension of the stress-strength model allows the strength $X$ and the stress $Y$ to depend on some explanatory variables. In many cases, an experimenter has access to the measurements of some explanatory variables that affect the strength or influence the stress. The additional information can play an important role in the analysis by extending the classical stress-strength model to include explanatory variables.

Duncan(1986) gave some specific examples of the strength-stress model with explanatory variables. Guttman, Johnson, Bhattacharyya and Reisser(1988) obtained an approximate confidence interval for reliability, $R=P(X>Y\mid \boldsymbol{z}, \boldsymbol{t})$, where $X$ and $Y$ are independent normal variables with explanatory variables $\boldsymbol{t}$ and $\boldsymbol{z}$, respectively. Since the true distribution of the estimator for $R$ is often kewed and

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biased for a small sample and/or large value of $R$, the interval based on uttman et. al. may deteriorate the accuracy. So we will use the bootstrap method to rectify these problems. Efron(1979) initially introduced the bootstrap method to assign the accuracy for an estimator. To construct approximate confidence interval for a parameter, Efron(1981, 1982, 1987) and Hall(1988) proposed the percentile method, the bias correct method(BC method), the bias correct acceleration method (BCa method), and the bootsrap percentile-t method, etc.

In this paper, we obtain some approximated bootstrap confidence intervals for the reliability of the stress-strength model when they are linearly related to explanatory variables. Also we compare approximate confidence intervals via Monte Carlo simulation.

## 2. Preliminaries

Suppose that $X$ is related to $p$ explanatory variables $\boldsymbol{t}$ and $Y$ is related to $q$ explanatory variables $\boldsymbol{z}$ according to the linear relationships,

$$X=\beta_0+\boldsymbol{\beta}'(\boldsymbol{t}-\bar{\boldsymbol{t}})+\varepsilon$$

and

$$Y=\alpha_0+\boldsymbol{\alpha}'(\boldsymbol{z}-\bar{\boldsymbol{z}})+\delta, \tag{2.1}$$

where $\boldsymbol{\beta}=(\beta_1,\beta_2,\cdots,\beta_p)'$ and $\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\cdots,\alpha_q)'$ are regression coefficients and the errors $\varepsilon$ and $\delta$ are independent random variables with $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, respectively.

Let $(X_i, \boldsymbol{t}_i)$ and $(Y_j, \boldsymbol{z}_j)$, $i=1,\cdots,m$, $j=1,\cdots,n$ be samples from the models in (2.1), and let $\bar{\boldsymbol{t}}=m^{-1}\sum_{i=1}^{m}\boldsymbol{t}_i$, $\bar{\boldsymbol{z}}=n^{-1}\sum_{j=1}^{n}\boldsymbol{z}_j$, $\boldsymbol{T}=(\boldsymbol{t}_1, \boldsymbol{t}_2,\cdots, \boldsymbol{t}_m)'$ and $\boldsymbol{Z}=(\boldsymbol{z}_1, \boldsymbol{z}_2,\cdots, \boldsymbol{z}_n)'$, where $\boldsymbol{T}$ is $m\times p$ matrix and $\boldsymbol{Z}$ is $n\times q$ matrix. And let $R(\boldsymbol{t}, \boldsymbol{z})$ be reliability for a given $\boldsymbol{t}$ and $\boldsymbol{z}$, that is, $R(\boldsymbol{t}, \boldsymbol{z})=P(X>Y\mid \boldsymbol{t}, \boldsymbol{z})$. Then the reliability for the strength-stress model becomes

$$R(\boldsymbol{t}, \boldsymbol{z}) = \Phi(\rho), \tag{2.2}$$

where $\rho = \left(\beta_0 - \alpha_0 + \boldsymbol{\beta}'(\boldsymbol{t} - \bar{\boldsymbol{t}}) - \boldsymbol{\alpha}'(\boldsymbol{z} - \bar{\boldsymbol{z}})\right)/\sqrt{\sigma_1^2 + \sigma_2^2}$.

Guttman et. al.(1988) show that the inferences of $\rho$ are based on the statistic given as

$$\hat{\rho} = \left(\widehat{\beta_0} - \widehat{\alpha_0} + \widehat{\boldsymbol{\beta}}'(\boldsymbol{t} - \bar{\boldsymbol{t}}) - \widehat{\boldsymbol{\alpha}}'(\boldsymbol{z} - \bar{\boldsymbol{z}})\right)/\sqrt{\widehat{\sigma_1^2} + \widehat{\sigma_2^2}}, \tag{2.3}$$

where $(\widehat{\beta_0}, \widehat{\boldsymbol{\beta}}, \widehat{\alpha_0}, \widehat{\boldsymbol{\alpha}})$ denotes the least squares estimators of $(\beta_0, \boldsymbol{\beta}, \alpha_0, \boldsymbol{\alpha})$ and $(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})$ denotes the mean squares due to residual, that is,

$$\widehat{\sigma_1^2} = (m-p)^{-1}\sum_{i=1}^{m}(X_i - \widehat{\beta_0} - \widehat{\boldsymbol{\beta}}'(\boldsymbol{t} - \bar{\boldsymbol{t}}))^2$$

and

$$\widehat{\sigma_2^2} = (n-q)^{-1}\sum_{j=1}^{n}(Y_j - \widehat{\alpha_0} - \widehat{\boldsymbol{\alpha}}'(\boldsymbol{z} - \bar{\boldsymbol{z}}))^2. \tag{2.4}$$

They suggested that the estimator of $R(\boldsymbol{t}, \boldsymbol{z})$ is given by

$$\hat{R}(\boldsymbol{t}, \boldsymbol{z}) = \Phi(\hat{\rho}). \tag{2.5}$$

Also they proved that the distribution of $\hat{\rho}$ is asymptotically normal with mean $\rho$ and variance $\sigma_\rho{}^2 = (1/N) + (\rho^2/2e)$, where $N = (\sigma_1^2 + \sigma_2^2)/(A_1(m)\sigma_1^2 + A_2(n)\sigma_2^2)$, $e = (\sigma_1^2 + \sigma_2^2)^2/(\sigma_1^4/(m-p) + \sigma_2^4/(n-q))$, $A_1(m) = \boldsymbol{t}'(\boldsymbol{T}'\boldsymbol{T})^{-1}\boldsymbol{t}$ and $A_2^2(n) = \boldsymbol{z}'(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{z}$.

Therefore, they obtained an approximated $100(1-2\alpha)\%$ normal confidence interval for $R(\boldsymbol{t}, \boldsymbol{z})$ is given by

$$\left(\Phi(\hat{\rho} + \hat{\sigma}_\rho \cdot z^{(\alpha)}) \; , \; \Phi(\hat{\rho} + \hat{\sigma}_\rho \cdot z^{(1-\alpha)})\right), \tag{2.6}$$

where $\hat{\sigma}_\rho$ is computed by using $\widehat{\sigma_1^2}$ and $\widehat{\sigma_2^2}$ in the formulars for $N$ and $\epsilon$, $z^{(\alpha)}$ is the $100\alpha$th percentile of the standard normal distribution and $\Phi$ denotes the cumulative distribution function of a standard normal random variable.

# 3. The Bootstrap Procedure

In this section, we describe the bootstrap procedure to obtain confidence intervals for $R(\boldsymbol{t}, \boldsymbol{z})$.

Given $\underline{\boldsymbol{X}}=(x_1, x_2, \cdots, x_m)$ and $\underline{\boldsymbol{Y}}=(y_1, y_2, \cdots, y_n)$ from (2.1), the bootstrap procedure may be described as following steps:

(1) Compute the plug-in estimators for $(\beta_0, \boldsymbol{\beta}, \alpha_0, \boldsymbol{\alpha})$ and $(\sigma_1^2, \sigma_2^2)$, say $(\widehat{\beta}_0, \widehat{\boldsymbol{\beta}}, \widehat{\alpha}_0, \widehat{\boldsymbol{\alpha}})$ and $(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})$, respectively.

(2) Construct the sample distribution functions for the strength, say $F_m$(from $\underline{\boldsymbol{X}}$) and for the stress, say $G_n$(from $\underline{\boldsymbol{Y}}$). That is,

$$F_m \sim N(\widehat{\beta}_0 + \widehat{\boldsymbol{\beta}}'(\boldsymbol{t} - \bar{\boldsymbol{t}}),\ \widehat{\sigma_1^2})$$

and

$$G_n \sim N(\widehat{\alpha}_0 + \widehat{\boldsymbol{\alpha}}'(\boldsymbol{z} - \bar{\boldsymbol{z}}),\ \widehat{\sigma_2^2}). \tag{3.1}$$

(3) Generate random samples of size $m$ and $n$ from fixed $F_m$ and $G_n$, respectively. The corresponding samples $\underline{\boldsymbol{X}}^* = (x_1^*, x_2^*, \cdots, x_{n_2}{}^*)$ and $\underline{\boldsymbol{Y}}^* = (y_1^*, y_2^*, \cdots, y_{m_2}{}^*)$ are called the bootstrap samples.

(4) Compute the bootstrap estimators for $\rho$, that is,

$$\widehat{\rho}^* = \left(\widehat{\beta}_0{}^* - \widehat{\alpha}_0{}^* + \widehat{\boldsymbol{\beta}}^{*'}(\boldsymbol{t} - \bar{\boldsymbol{t}}) - \widehat{\boldsymbol{\alpha}}^{*'}(\boldsymbol{z} - \bar{\boldsymbol{z}})\right) / \sqrt{\widehat{\sigma}_1{}^{*^2} + \widehat{\sigma}_2{}^{*^2}}, \tag{3.2}$$

where $\widehat{\beta}_0{}^*$, $\widehat{\alpha}_0{}^*$, $\widehat{\boldsymbol{\beta}}^{*'}$, $\widehat{\boldsymbol{\alpha}}^{*'}$, $\widehat{\sigma}_1{}^{*^2}$ $\widehat{\sigma}_2{}^{*^2}$ are computed based on bootstrap samples.

(5) Evaluate bootstrap estimators $\widehat{R}^*(\boldsymbol{t}, \boldsymbol{z}) = \Phi(\widehat{\rho}^*)$ for $R(\boldsymbol{t}, \boldsymbol{z})$.

(6) Repeat B times step (3)-(5), we obtain $\widehat{R}^{*b}(\boldsymbol{t}, \boldsymbol{z}) = \Phi(\widehat{\rho}^{*b})$, $b = 1, 2, \cdots, B$.

Let $\widehat{F}(s) = P(\widehat{R}^*(\boldsymbol{t}, \boldsymbol{z}) \le s)$ be the cumulative distribution function of the bootstrap distribution of $\widehat{R}^*(\boldsymbol{t}, \boldsymbol{z})$. Then we can obtain an approximation of $\widehat{F}(s)$ by $\widehat{F}^*$ as follows:

$$\widehat{F}^{*}(s) = \frac{1}{B}\sum_{b=1}^{B} I(\,\hat{R}^{*b}(\,t,\,z) \le s), \tag{3.3}$$

where $I(\,\cdot\,)$ is an indicator function.

The bootstrap confidence limits that depend on the extreme tails of $\widehat{F}^{*}$ will require a large number of bootstrap samples to achieve acceptable accuracy. Efron(1993) recommended that B should be $\ge 500$ or 1000 in order to make the variability of the bootstrap confidence limits acceptably low.

## 4. The Bootstrap Confidence Intervals for $R(\,t,\,z)$

Now, we will construct some bootstrap confidence intervals for $R(\,t,\,z)$ in this section.

Let $\widehat{F}^{*-1}(\alpha)$ be the $100\alpha$ empirical percentile of the $\hat{R}^{*b}(\,t,\,z)$. That is,

$$\widehat{F}^{*-1}(\alpha) = \inf\{s :\ \widehat{F}^{*}(s) \ge \alpha\}. \tag{4.1}$$

Then first, an approximated $100(1-2\alpha)\%$ interval by bootstrap percentile method (Efron(1981)) for $R(\,t,\,z)$ is given as

$$(\,\widehat{F}^{*-1}(\alpha),\ \widehat{F}^{*-1}(1-\alpha)). \tag{4.2}$$

Second, an approximated $100(1-2\alpha)\%$ interval by bias correct(BC) method (Efron(1982)) for $R(\,t,\,z)$ is given as

$$(\,\widehat{F}^{*-1}(\alpha_1),\ \widehat{F}^{*-1}(\alpha_2)), \tag{4.3}$$

where $\alpha_1 = \Phi(2\,\hat{z}_0 + z^{(\alpha)})$, $\alpha_2 = \Phi(2\,\hat{z}_0 + z^{(1-\alpha)})$ and

$$\hat{z}_0 = \Phi^{-1}(\,\widehat{F}^{*}(\,\hat{R}(\,t,\,z))) = \Phi^{-1}\left[\frac{1}{B}\sum_{b=1}^{B} I(\,\hat{R}^{*b}(\,t,\,z) \le \hat{R}(\,t,\,z)\right].$$

Third, an approximated $100(1-2\alpha)\%$ interval by the bias correct acceleration (BCa) method(Efron(1987)) requires computations of bias correction and acceleration constants. In fact, the bias correct constant is same as that of bias correct method. To obtain the acceleration constant $a$, for multiparameter family case we replace the multiparameter family $\Sigma=\{f_{\boldsymbol{\eta}}(\Lambda)\}$ by the least favorable one parameter family $\hat{\Sigma}=\{f_{\widehat{\boldsymbol{\eta}}+\lambda\widehat{\boldsymbol{\mu}}}(\Lambda)\}$(See Efron(1987)), where $\boldsymbol{\eta}$ denotes parameter vector, $\widehat{\boldsymbol{\eta}}$ denotes sufficient statistics vector for $\boldsymbol{\eta}$, $\widehat{\boldsymbol{\mu}}$ denotes the least favorable direction at $\boldsymbol{\eta}=\hat{\boldsymbol{\eta}}$, and $\Lambda=(X, Y)$. He used the least favorable one parameter family to calculate an approximate value for the acceleration constant $a$, $\hat{a}=\frac{1}{6\sqrt{n}}\frac{\hat{\Psi}^{(3)}(0)}{(\hat{\Psi}^{(2)}(0))^{3/2}}$, where $\Psi(\cdot)$ is a function such that the densities $f_{\boldsymbol{\eta}}(x,y)$ are of the form $f_{\boldsymbol{\eta}}(x,y)=f_0(x,y)\cdot\exp(\boldsymbol{\eta}'(x,y)-\Psi(\boldsymbol{\eta}))$.

In our cases, we can obtain $\hat{a}$ from many algebraic calculation, that is,

$$\hat{a} = \frac{1}{6}\frac{(a_3+a_4)}{(a_1+a_2)^{3/2}}, \tag{4.4}$$

where

$$a_1 = \frac{m}{2}\left(-\frac{\psi_3^2}{d_3^2}+\frac{2\psi_1^2d_3^2-4\psi_1\psi_3d_1d_3+2\psi_3^2d_1^2}{d_3^3}\right),$$

$$a_2 = \frac{n}{2}\left(-\frac{\psi_4^2}{d_4^2}+\frac{2\psi_2^2d_4^2-4\psi_2\psi_4d_2d_4+2\psi_4^2d_2^2}{d_4^3}\right),$$

$$a_3 = \frac{m}{2}\left(-\frac{2\psi_3^3}{d_3^3}+\frac{-6\psi_1^2\psi_3d_3^2+12\psi_1\psi_3^2d_1d_3-6\psi_3^3d_1^2}{d_3^4}\right),$$

$$a_4 = \frac{n}{2}\left(\frac{2\psi_4^3}{d_4^3}+\frac{-6\psi_2^2\psi_4d_4^2+12\psi_2\psi_4^2d_2d_4-6\psi_4^3d_2^2}{d_4^4}\right),$$

$$\psi_1 = \phi\left(\frac{d_1-d_2}{\sqrt{d_3+d_4}}\right)\frac{d_3}{m\sqrt{d_3+d_4}},\quad \psi_2 = -\phi\left(\frac{d_1-d_2}{\sqrt{d_3+d_4}}\right)\frac{d_4}{n\sqrt{d_3+d_4}}$$

$$\psi_3 = -\phi\left(\frac{d_1-d_3}{\sqrt{d_3+d_4}}\right)\frac{d_3(d_1-d_2)}{m(d_3+d_4)^{3/2}},$$

$$\psi_4 = -\phi\left(\frac{d_1-d_2}{\sqrt{d_3+d_4}}\right)\frac{d_4(d_1-d_2)}{n(d_3+d_4)^{3/2}}$$

and $d_1=\widehat{\beta}_0+\widehat{\boldsymbol{\beta}}'(\boldsymbol{t}-\bar{\boldsymbol{t}})$, $d_2=\widehat{\alpha}_0+\widehat{\boldsymbol{\alpha}}'(\boldsymbol{z}-\bar{\boldsymbol{z}})$, $d_3=\widehat{\sigma_1^2}$ and $d_4=\widehat{\sigma_2^2}$, where $\phi(\cdot)$ denotes standard normal probability density function.
Therefore an approximated $100(1-2\alpha)\%$ BCa interval for $R(\boldsymbol{t},\boldsymbol{z})$ is given as

$$\left(\widehat{F}^{*-1}(\alpha_3)\ ,\ \widehat{F}^{*-1}(\alpha_4)\right), \tag{4.5}$$

where

$$\alpha_3 = \Phi[\widehat{z}_0 + (\widehat{z}_0+z^{(\alpha)})/(1-\widehat{a}(\widehat{z}_0+z^{(\alpha)}))]$$

and

$$\alpha_4 = \Phi[\widehat{z}_0 + (\widehat{z}_0+z^{(1-\alpha)})/(1-\widehat{a}(\widehat{z}_0+z^{(1-\alpha)}))].$$

Finally, to obtain an approximated interval by bootstrap percentile-t interval (Hall(1988)) for $R(\boldsymbol{t},\boldsymbol{z})$, we let $\widehat{\rho}^*{}_T$ be an approximate bootstrap pivotal quantity of $\widehat{\rho}$, that is,

$$\widehat{\rho}^*{}_T=(\widehat{\rho}^*-\widehat{\rho})/\ \widehat{\sigma}_\rho^*, \tag{4.6}$$

where $\widehat{\sigma}_\rho^*$ is computed from $\widehat{\sigma_\rho}$ based on bootstrap samples.
Let $\widehat{F}^*{}_T$ denote the empirical distribution function of $\widehat{\rho}^*{}_T$ given as

$$\widehat{F}^*{}_T(s) = \frac{1}{B}\sum_{b=1}^{B} I(\ \widehat{\rho}_T{}^*\le s), \tag{4.7}$$

for all s. And let $\widehat{F}_T{}^{*-1}(\alpha)$ denotes the $100\alpha$ th empirical percentile of the $\widehat{\rho}^*{}_T$ given as

$$\widehat{F}_T{}^{*-1}(\alpha)=\inf\{s:\ \widehat{F}^*{}_T(s)\ \ge\ \alpha\}. \tag{4.8}$$

Then, an approximated $100(1-2\alpha)\%$ bootstrap percentile-$t$ interval for $R(t, z)$ is given by

$$\left( \Phi(\hat{\rho}+\hat{\sigma}_{\rho}\cdot \quad \hat{F}^{*-1}{}_{T}(\alpha))\ ,\ \Phi(\hat{\rho}+\hat{\sigma}_{\rho}\cdot \quad \hat{F}^{*-1}{}_{T}(1-\alpha))\ \right). \qquad (4.9)$$

## 5. Comparisons

In this Section, we evaluate the approximated confidence intervals presented in Section 4 through simulation. Monte Carlo studies are performed for investigating the adequacy of some methods given in Section 4.

Regression parameters $n, m, \beta_0, \alpha_0, \boldsymbol{\beta}, \boldsymbol{\alpha}$ are chosen so that $R(t, z)$ takes the values of 0.1, 0.3, 0.5, 0.7, 0.9. For the sake of convenience, we only consider the simple linear regression models for $X$ and $Y$. We set both $t_i$ and $z_i$ as $\pm(i-1)/n,\ \ i=1,2\cdots,n/2$, symmetrically around the point zero. For each case, we try simulation when $t=\bar{t}=0$ and $z=\bar{z}=0$. The equally chosen sample sizes $n$ and $m$ are 10, 20, 30, 50. The number of pairs of samples generated for each combination of $R(t, z)$ and $n(=m)$ is 500. For each independent random samples, the approximated bootstrap confidence intervals were constructed by each method with bootstrap replications $B=1000$ times. Also the used confidence level $(1-2\alpha)$ is 0.90. <Table 4.1> and <Table 4.2> give the actual coverage probabilities and interval lengths of the approximated confidence intervals, respectively. The graphs for some cases of table 4.1-4.2 are given Figures 1-4.

<Figure 1> represents the plot of coverage probabilities against reliability when $n=m=10$. <Figure 1> illustrates that the approximated bootstrap confidence intervals is nearly always better than Guttman et.al.'s interval regardless of the reliabilities. In particular, Guttman et.al.'s interval is worse than the bootstrap intervals for extreme values of reliabilities, say 0.1 or 0.9.

<Figure 2> represents the plot of coverage probabilities against sample size when $R(\bar{t}, \bar{z})=0.9$. <Figure 2> show that the approximation to the nominal confidence level 90% of bootstrap methods is better than that of Guttman et.al.'s method. Since simulation results for other values of reliabilities are similar, we don't report here.

<Figure 3> represents the plot of interval lengths against reliabilities when $n=m=10$. <Figure 3> illustrates that the bootstrap methods yield slightly longer interval lengths than Guttman et.al.'s method. Also all interval lengths tend to decrease as $R(t, z)$ deviates from 0.5.

<Figure 4> represents the plot of interval lengths against sample size when $R(\bar{t}, \bar{z})=0.9$. <Figure 3> illustrate that interval length discrepancy by every method tend to decrease as sample size increase. Since simulation results for other values of reliabilities are similar, we don't report here.

Bootstrap methods can require even more computing than Guttman et.al.'s method, and up to hundreds to thousands of times more computing time than using Guttman et.al.'s method. However, with high speed computers, even this may not be a severe problem, and the improvement may often be worth the extra cost.

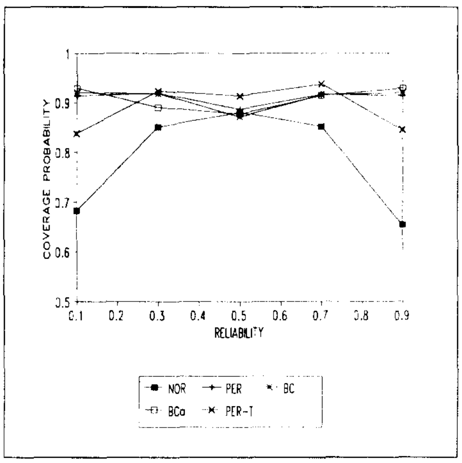
< Table 4.1 > The Actual Coverage Probabilities of the Confidence Intervals

| $n(=m)$ | $R(\bar{t}, \bar{z})$ | Normal | Percentile | BC | BCa | Per-t |
|---|---|---|---|---|---|---|
| 10 | 0.1 | 0.6820 | 0.9130 | 0.9200 | 0.9290 | 0.8380 |
| | 0.3 | 0.8510 | 0.9190 | 0.9180 | 0.8900 | 0.9240 |
| | 0.5 | 0.8800 | 0.8870 | 0.8720 | 0.8770 | 0.9130 |
| | 0.7 | 0.8520 | 0.9170 | 0.9160 | 0.9150 | 0.9380 |
| | 0.9 | 0.6540 | 0.9140 | 0.9200 | 0.9290 | 0.8450 |
| 20 | 0.1 | 0.7170 | 0.8940 | 0.9060 | 0.9180 | 0.8550 |
| | 0.3 | 0.8670 | 0.8940 | 0.8950 | 0.8920 | 0.9070 |
| | 0.5 | 0.8830 | 0.8850 | 0.8790 | 0.8860 | 0.8950 |
| | 0.7 | 0.8720 | 0.8890 | 0.9020 | 0.8940 | 0.9290 |
| | 0.9 | 0.6970 | 0.8780 | 0.8800 | 0.9140 | 0.8580 |
| 30 | 0.1 | 0.7260 | 0.8850 | 0.8960 | 0.9090 | 0.8590 |
| | 0.3 | 0.8700 | 0.9020 | 0.9040 | 0.9020 | 0.8990 |
| | 0.5 | 0.8860 | 0.8890 | 0.8810 | 0.8950 | 0.8950 |
| | 0.7 | 0.8870 | 0.9090 | 0.9150 | 0.9050 | 0.9190 |
| | 0.9 | 0.7120 | 0.8790 | 0.8780 | 0.8870 | 0.8640 |
| 50 | 0.1 | 0.7710 | 0.9160 | 0.9070 | 0.9070 | 0.9040 |
| | 0.3 | 0.8840 | 0.9000 | 0.8930 | 0.8990 | 0.9020 |
| | 0.5 | 0.8980 | 0.8920 | 0.8940 | 0.8980 | 0.8980 |
| | 0.7 | 0.8850 | 0.9130 | 0.9070 | 0.9060 | 0.9060 |
| | 0.9 | 0.7810 | 0.8820 | 0.8850 | 0.8900 | 0.8770 |

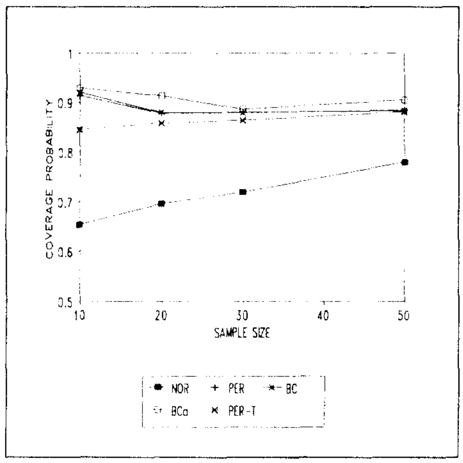
Normal denotes interval based on normal-theory.
Percentile denotes interval based on percentile method.
BC denotes interval based on bias-correct method.
BCa denotes interval based on bias-correct acceleration method.
Per-t denotes interval based on percentile-t method.

< Table 4.2 > The Interval Lengths of the Confidence Intervals

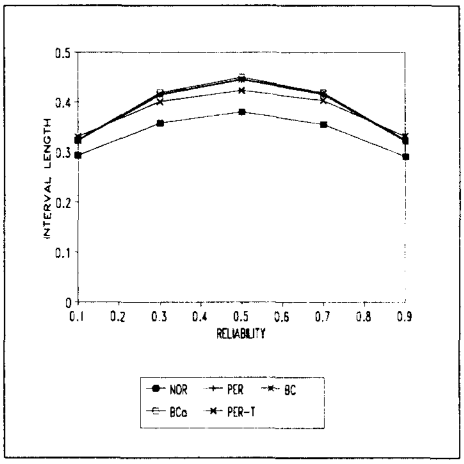
| $n(=m)$ | $R(\bar{t}, \bar{z})$ | Normal | Percentile | BC | BCa | Per-t |
|---|---|---|---|---|---|---|
| 10 | 0.1 | 0.2936 | 0.3232 | 0.3229 | 0.3240 | 0.3314 |
| | 0.3 | 0.3585 | 0.4147 | 0.4153 | 0.4191 | 0.4014 |
| | 0.5 | 0.3809 | 0.4449 | 0.4456 | 0.4500 | 0.4233 |
| | 0.7 | 0.3565 | 0.4160 | 0.4164 | 0.4195 | 0.4032 |
| | 0.9 | 0.2910 | 0.3224 | 0.3221 | 0.3238 | 0.3324 |
| 20 | 0.1 | 0.1875 | 0.2241 | 0.2236 | 0.2242 | 0.2284 |
| | 0.3 | 0.2594 | 0.2898 | 0.2911 | 0.2948 | 0.2843 |
| | 0.5 | 0.2816 | 0.3070 | 0.3081 | 0.3119 | 0.2999 |
| | 0.7 | 0.2591 | 0.2884 | 0.2894 | 0.2928 | 0.2831 |
| | 0.9 | 0.1899 | 0.2262 | 0.2257 | 0.2262 | 0.2304 |
| 30 | 0.1 | 0.1458 | 0.1811 | 0.1807 | 0.1811 | 0.1840 |
| | 0.3 | 0.2128 | 0.2362 | 0.2371 | 0.2405 | 0.2331 |
| | 0.5 | 0.2331 | 0.2448 | 0.2456 | 0.2484 | 0.2414 |
| | 0.7 | 0.2137 | 0.2359 | 0.2370 | 0.2405 | 0.2329 |
| | 0.9 | 0.1470 | 0.1818 | 0.1813 | 0.1817 | 0.1846 |
| 50 | 0.1 | 0.1271 | 0.1383 | 0.1380 | 0.1386 | 0.1399 |
| | 0.3 | 0.1656 | 0.1829 | 0.1836 | 0.1858 | 0.1814 |
| | 0.5 | 0.1825 | 0.1887 | 0.1892 | 0.1910 | 0.1872 |
| | 0.7 | 0.1662 | 0.1826 | 0.1835 | 0.1856 | 0.1812 |
| | 0.9 | 0.1286 | 0.1388 | 0.1386 | 0.1392 | 0.1404 |



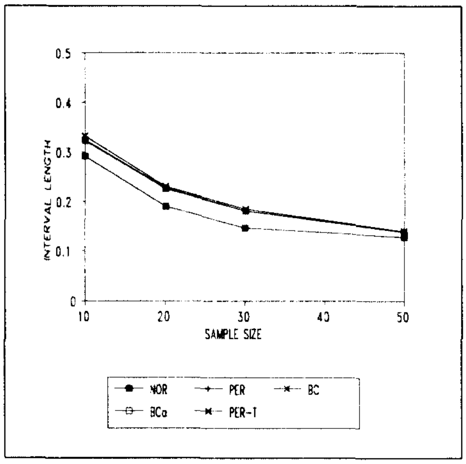

<Figure 1> Plot of Coverage Probabilities against Reliabilities when $n=m=10$




<Figure 2> Plot of Coverage Probabilites against Sample Sizes when $R(\bar{t}, \bar{z})=0.9$




< Figure 3 > Plot of Interval Lengths against Reliabilities when $n=m=10$




< Figure 4 > Plot of Interval Lengths against Sample Sizes when $R(\bar{t}, \bar{z})=0.9$

# References


[ 1 ] Duncan, A.G.(1986), *Quality and Industrial Statistics*(5th ed.), Homewood, IL: Richard D. Irwin.

[ 2 ] Efron, B.(1979), "Bootstrap Methods: Another Look at the Jackknife," *The Annals of Statistics*, 7, pp. 1-26.

[ 3 ] Efron, B.(1981), "Nonparametric Standard Errors and Confidence Intervals (With Discussion)," *Canadian Journal of Statistics*, 9, pp. 139-172.

[ 4 ] Efron, B.(1982), "The Jackknife, The Bootstrap and Other Resampling Plans," *SIAM*, Philadelphia, pp. 38.

[ 5 ] Efron, B.(1987), "Better Bootstrap Confidence intervals(With discussion)," *Journal of the American Statistician Association*, 82, pp. 171-200.

[ 6 ] Efron, B. and Tibshirani.(1993), "An Introduction to the Bootstrap," Chapman and Hall, London.

[ 7 ] Guttman, I., Johnson, R., Bhattacharyya, G.K., and Reisser, B.(1988), "Confidence Limits for Stress-Strength Models with explanatory variables," *Technometrics*, 30, pp. 161-168.

[ 8 ] Hall, P.(1988), "Theoretical Comparison of Bootstrap Confidence Intervals (With Discussion)," *The Annals of Statistics*, 16, pp. 927-953.

[ 9 ] Reisse, B. and Guttman, I.(1986), "Statistical Inference for Pr(Y<X)," *Technometrics*, 28, pp. 253-257.