⊠연구논문

# Strategies for Robust Design with Multiple Responses

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## Abstract

This paper considers robust design strategies for off-line quality control, with the use of experimental design and response surface methodology, in situations where all products have multiple quality characteristics. These strategies can be developed using the desirability concept of desirability functions to determine the settings of the design factors, not only to get the average performances on target but also to minimize variability around the target values.

## 1. Introduction

Quality control is a set of statistical methods used to maintain and improve quality. Quality control can be described as on-line. On-line quality control techniques, such as control charts, can be used to estimate process parameters and to monitor the manufacturing process to detect changes. However, Off-line quality control methods focus directly on quality improvement activities rather than quality monitoring. Compared with on-line quality control activities on a production line, off-line quality control brings quality control activities into the design and development stages. By designing products and processes well and producing items that meet the requirements and specifications of the customer off-line quality control can lead to a wide variety of benefits such as reductions in scrap, rework, inspection and so forth.

Off-line quality control concentrates on the product and process design stage to ensure product robustness. Manufacturers want to make products robust against the variation in the user's environment, and to make the process which produces the product robust against the normal variations in manufacturing and materials. This is the motivating idea behind robust design.

Robust design methods have been developed under an assumption that the quality of a product is assessed with respect to a single quality characteristic. In practice, however, every product has several quality characteristics that are important to the user.

Multiple response robust design problems are difficult because the scale, range and type of target (larger-is-better, smaller-is-better and nominal-is-better) for each response can be different. The use of desirability functions may provide a solution to the problem by combining measurements on individual characteristics into one index. If the values of all the quality characteristics are transformed using desirability functions, then all of the quality characteristics can be represented on a common scale and range. If the combination of the controllable variables that maximizes the value of the overall desirability function can be identified, multiple response robust design problems can be solved.

Previously, researchers have assumed that there was only a single quality characteristic that the customer would consider to be important for manufactured products or that the type of target for each of the quality characteristics were all of the "nominal-is-better". The strategies developed by them have been considered only the relationship between the controllable variables and the quality characteristics. However, the robust strategy suggested by this paper can be used for making the product and process robust against the hard-to-control variables as well as the controllable variables.

## 2. Desirability Functions

The desirability concept was discussed in Harrington(1965). Harrington(1965) presented an optimization scheme utilizing desirability functions to transform multiple responses into a single response function. This approach was adapted by Derringer and Suich(1980), and was used in conjunction with response surface methodology (RSM) for process optimization.

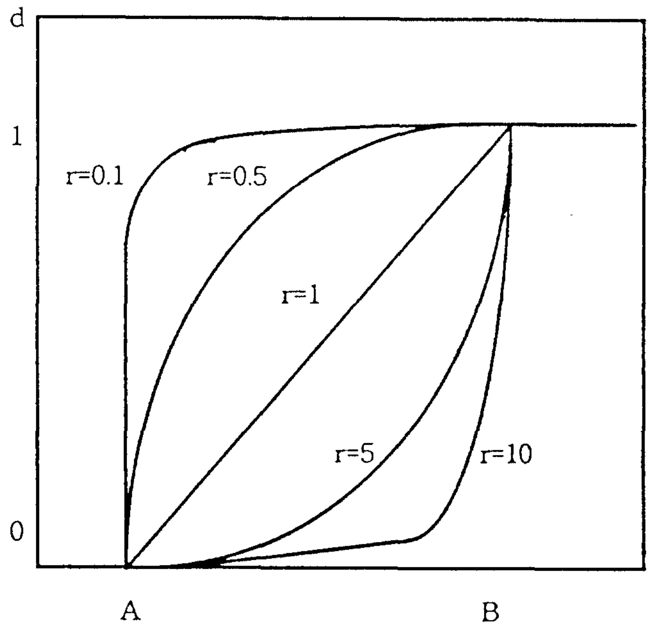
Derringer and Suich(1980) discussed two types of desirability functions: one-sided and two-sided. The one-sided desirability function can be used for those responses for which either minimization or maximization for the mean response is desired. The two-sided desirability function can be used for those responses for which minimization and maximization of the mean response are desired.

There are many one-sided transformations that could be used as desirability functions. Some examples are shown in <Figure 2.1>. We assumed that there are $p$ responses, $y_1$, $y_2$, ..., $y_p$. For any response $y_i$, one specific one-sided transformation (Derringer and Suich, 1980) can be given by:

$$d_i = \begin{cases} 0 & \text{if } y_i \le A_i \\ \left[ \dfrac{B_i - y_i}{B_i - A_i} \right]^r & \text{if } A_i < y_i < B_i \\ 1 & \text{if } y_i \le B_i \end{cases} \qquad (2.1)$$

where $y_i$ is the response variable, the value $A_i$, and $B_i$ are the minimal acceptable bound and the minimum most desirable value on the response $y_i$. The selection of a suitable value of $r$ offers the user flexibility in the definition of desirability functions (Box and Draper, 1988). If one wants to indicate that values considerably above $A_i$ are highly desirable, a large value or $r$ may be chosen. A small value of $r$ indicates that having values of $y_i$ considerably above $A_i$ are not important.

In <Figure 2.1>, several desirability functions with the same $A$ and $B$ values are plotted with different values for $r$. <Figure 2.1> indicates that $A_i$ is the minimum acceptable value and $B_i$ is the minimum target value on the response $y_i$.




< Figure 2.1 > One-sided transformation for r

If any response variable, $y_i$, is greater than $B_i$, then $y_i$ is considered to be perfectly desirable, That is, for any value of $y_i \geq B_i$, the desirability value of $y_i$ has the maximum desirability value, 1. If any response variable, $y_i$, is less than $A_i$, then $y_i$ is considered unacceptable. That is, for any value of $y_i \leq A_i$, the desirability value of $y_i$ has the minimum desirability value, 0.

A two-sided desirability function can be used for situations where there is a specific target either a point or an interval for $y_i$ and where there are upper and lower desirability limits on $y_i$, such as $A_i < T_i < B_i$, where $A_i$ and $B_i$ are the maximum least desired value for a minimum constraint and that for a maximum constraint and $T_i$ is the most desirable value (target). That is, two-sided desirability function arises when the response variable $y_i$ has both a minimum and a maximum constraint.

As for the one-sided desirability functions, there are also many two-sided transformations that could be used as desirability functions. For example, see <Figure 2.2>. One two-sided desirability function(Derringer and Suich, 1980) is given by

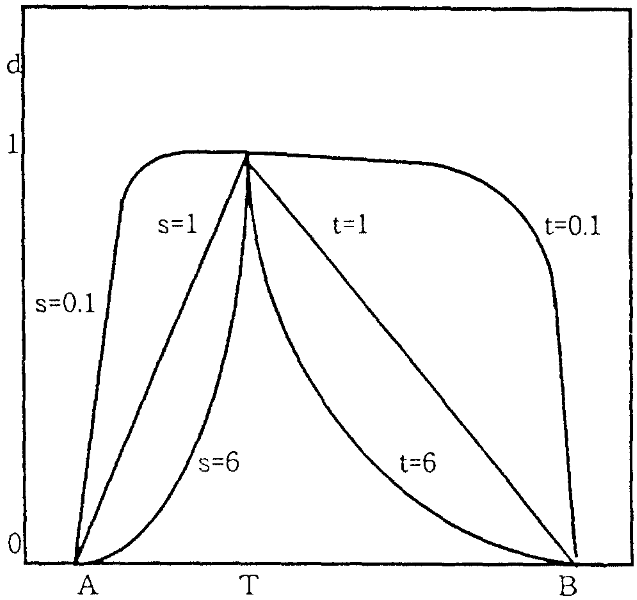
$$d_i = \begin{cases} \left[ \dfrac{y_i - A_i}{T_i - A_i} \right]^s & \text{if} \quad A_i \leq y_i \leq T_i \\ \left[ \dfrac{y_i - B_i}{T_i - B_i} \right]^t & \text{if} \quad T_i < y_i \leq B_i \\ 0 & \textit{otherwise} \end{cases} \tag{2.2}$$

By selecting the powers $s$ and $t$, one can attribute various levels of desirability to $Y$. In <Figure 2.2>, the desirability function given in (2.2) with different values of $s$ and $t$ is plotted.

The powers $s$ and $t$ can be the same or different. As $s$ and/or $t$ increases, more importance is given for having the response near the target, $T$. Conversely, as the exponents decreases, less importance is given for having the response near the target.

The overall assessment for desirability values, $D_1^*$, is calculated by combining the desirability values obtained for the different criteria by using the geometric mean:

$$D_1^* = (d_1 \times d_2 \times \cdots \times d_p)^{1/p}$$




< Figure 2.2 > Two-sided transformation for s and t

If one of the responses has an unacceptable value (that is, if $d_i$ =0), the overall product will also be unacceptable (resulting in $D_1^*$=0), regardless of the value of the remaining responses. On the other hand, if all the responses are acceptable, the value of $D_1^*$ will fall in the interval [0,1] and will increase with increasing $d$-values.

# 3. Methodology

In this section, we will discuss desirability functions for the mean and dispersion as tools for solving multiple response robust design problems. First, we will consider two different types of averages, the geometric average and the arithmetic average. These averages will be considered for use with desirability functions for the mean. Second, we will develop desirability function for dispersion and discuss its use.

Suppose that $y_{ijl}$ is the response associated with the $j$th noise variable for the $l$th quality characteristic at the $i$th product design, and $d_{ijl}$ ( $0 \leq d_{ijl} \leq 1$) is a desirability value for $y_{ijl}$ obtained from the $l$th desirability function like equation (2.1) or (2.2).

The basic procedure for comparing products or processes using desirability functions is as follows: For each response, $y_{ijl}$, transform $y_{ijl}$ into $d_{ijl}$ using the $l$th desirability function and assess the desirability of the product or process based upon some function of the $d_{ijl}$.

## 3.1 The Desirability Function for Location

Desirability functions are used to obtain the desirability values for individual responses of each quality characteristic. Desirability functions for the average response can also be considered. That is, one could consider a desirability function for the location of the average of the responses for each of the quality characteristics. To evaluate responses for each quality characteristic, the geometric average and arithmetic average can be used.

### 3.1.1. The Factor Level Combinations by Geometric Average

Assume that there are $l$ quality characteristics associated with some product. Each quality characteristic has the $m$ observations. Here, we will discuss the factor level combinations.

Let $d_{ijl}$ be a desirability value for the individual response, $y_{ijl}$. That is, $d_{ijl}$ is a desirability value associated with the observed value of the $l$th quality characteristic at the $i$th product design, $x_i$, under the $j$th noise condition, $w_j$.

Let $d_{il}^*$ be a geometric average of $d_{i1l}$, ..., $d_{iml}$; that is,

$$d_{il}^* = (d_{i1l} \times \cdots \times d_{iml})^{1/m}$$

where $i$ = 1, 2, ..., $k$ and $l$ = 1, 2, ..., $p$.

The quantity $d_{il}^*$ is the geometric average, over all noise variables, of the desirability values of the $l$th quality characteristic at the $i$th product design, $x_i$. The geometric average, $d_{il}^*$, penalizes a combination of the controllable factor levels if any response has a desirability value of zero.

### 3.1.2. The Factor Level Combinations by Arithmetic Average

Each quality characteristic consists of $m$ observations. To evaluate each quality characteristic, $m$ observations in each quality characteristic are transformed into their desirability values and those desirability values are calculated as the value of such averages as the arithmetic or geometric average.

Here, we will discuss two different types of arithmetic average for desirability values. One arithmetic average for desirability values is obtained from the desirability value for the arithmetic average of the individual response. Another average is obtained from the arithmetic average of the individual desirability value.

Let $\overline{y_{il}}$ be the arithmetic average of the individual response, $y_{i1l}, \ldots, y_{iml}$, that is,

$$\overline{y_{il}} = \frac{1}{m}(y_{i1l} + \cdots + y_{iml})$$

where $i = 1, 2, \ldots, k$ and $l = 1, 2, \ldots, p$. The average response value, $\overline{y_{il}}$, is the arithmetic average of the response values of the $l$th quality characteristic at the $i$th product design over $m$ noise variables.

Let $\overline{d_{il}}$ be the desirability value of average response value, $\overline{y_{il}}$, for the $l$th quality characteristic at the ith combination of the product design variables. Let $d_{il}^{-}$ be the arithmetic average of the individual desirability value, $d_{i1l}, \ldots, d_{iml}$, for the individual response, $y_{i1l}, \ldots, y_{iml}$, that is,

$$d_{il}^{-} = \frac{1}{m}(d_{i1l} + \cdots + d_{iml})$$

where $i = 1, 2, \ldots, k$ and $l = 1, 2, \ldots, p$.

The difference between two averages, $d_{il}^{-}$ and $\overline{d_{il}}$ is that $d_{il}^{-}$, is obtained from desirability functions for the location of the average response and $d_{il}^{-}$ is obtained from desirability functions for the location of individual response. Using the property of the two different types of arithmetic average, the location effect of each quality characteristic can be calculated differently.

### 3.1.3. The Overall Quality for A Product/Process Using the Average of Quality Characteristics

To evaluate the quality of the product or process, the geometric average can be used to determine the overall quality of a product or process that is comprised of several quality characteristics. Each quality characteristic can be evaluated by desirability values for the arithmetic average of the response over the noise variables. The overall quality of a product or process that is comprised of several quality characteristics can be calculated by the geometric average for the arithmetic averages of quality characteristics.

Let $\overline{d}_{il}$ be the desirability value of average response value, $\overline{y}_{il}$, for the $l$th quality characteristic at the $i$th combination of the product design variables. Let G ( $\overline{d}_{i.}$) be the geometric average of the desirabilities, $\overline{d}_{il}$, that is,

$$G(\overline{d}_{i.}) = (\overline{d}_{i1} \times \overline{d}_{i2} \times \cdots \times \overline{d}_{ip})^{1/p}$$

where $i$ =1, 2, ..., $k$.

The geometric average of overall desirability, G( $\overline{d}_{i.}$), is obtained by calculating the geometric average of desirability values for the average responses of all quality characteristics at the $i$th product design. This is the overall desirability average for all responses of a product (or process) at the $i$th product design, $x_i$.

To evaluate the overall quality for a product or process, the geometric average of desirability values for each quality characteristic can be used. Each quality characteristic of a product or process can be evaluated using the geometric average of desirability values for individual responses.

Let $d_{ijl}$ be a desirability value for the individual response, $y_{ijl}$. Let $d_{il}^*$ be the geometric average of $d_{i1l}$, ...., $d_{iml}$. Let $G(d_{i.}^*)$ be the geometric average of the geometric averages, $d_{il}^*$, for the desirability values in the $l$th response at the $i$th product design over the noise space.

The geometric average can be expressed as

$$G(d_{i.}^*) = ((\prod d_{ij1})^{1/m}(\prod d_{ij2})^{1/m} \cdots (\prod d_{ijp})^{1/m})^{1/p}$$

$$= (d_{i1}^* \times d_{i}^{2} \times \cdots \times d_{ip}^*)^{1/p}$$

where $i$ = 1, 2, ..., $k$, $j$ = 1, 2, ..., $m$, and $l$ = 1, 2, ..., $p$.

The geometric average, G( $d_{i.}^*$), penalizes all products or processes that have unacceptable responses. The geometric average, G( $d_{i.}^*$), is calculated not only the product of the geometric average of each quality characteristic but also the product of desirability values for all individual responses. This means that the combination of controllable variables that has any unacceptable response is rejected.

### 3.2 The Desirability Function for Dispersion

Desirability functions for dispersion will be considered here. Let $s_{il}$ be the estimated standard deviation of the $l$th response at the $i$th experiment point. That is,

$$s_{il}=\sqrt{\frac{\sum_{j=0}^{m}(y_{ijl}-\overline{y_{il}})^2}{m-1}}$$

where $y_{ijl}$ is the response of the $l$th quality characteristic at the $i$th product design under the $j$th noise variable and $\overline{y_{il}}$ is the arithmetic average of the $l$th quality characteristic at the $i$th design over the $m$ noise variables.

Let $e_{il}$ ( $0\le e_{il}\le 1$) be the desirability value of $s_{il}$ for $i$ = 1, 2, ..., $m$ and $l$ = 1, 2, ..., $p$, presented by the desirability function. Let $G(e_{i.})$ be the geometric average for desirability values of the standard deviation of each quality characteristic at the $i$th product design. We let $\sigma_l^2$ represent variance of the original responses over noise variables for the $l$th quality characteristic. We let $e_l^2$ represent the desirability values for the variance, $\sigma_l^2$, of the lth quality characteristic. That is, the desirability values, $e_l^2$, for the variance, $\sigma_l^2$, is $e_{l1}\times e_{l2}\times\cdots\times e_{lp}$. The geometric average, $G(e_{i.})$ is

$$G(e_{i.})=(e_{i1}\times e_{i2}\times\cdots\times e_{ip})^{1/p}$$

where $i$ = 1, 2, ..., $m$. The is an overall evaluation of the dispersion for each combination of controllable variables. This expression also penalizes the combination of controllable variables having a large standard deviation that results in a zero desirability value.

## 4. Optimization Strategies

In this section, we will introduce some strategies based on the concepts of desirability functions for determining the best combination of controllable design variables for multiple response robust design. This section will not only describe concepts for several different robust design strategies that use relationships between location effects and dispersion effects in multiple response design, but also will develop the procedures for finding the optimal robust design point.

Some of the strategies that have been applied successfully to the single response robust design problem can, with appropriate modifications, be extended to accommodate multiple responses under the assumption that all of the quality characteristics are of the "nominal-is-best" variety. For example, multiple response problems are often formulated and solved as constrained optimization problems (Lin and Tu(1995)) or can be formulated and solved as multiple-objective decision making problems. However, when at least one quality characteristic is not of the nominal-is-best type, new strategies for multiple response robust design must be considered.

In this section, we present new strategies for finding the best combination of controllable variables for multiple response robust design problems having controllable variables, $x_i$, $i = 1, 2, \ldots, k$, noise variables, $w_j$, $j = 1, 2, \ldots, m$, and responses $y_l$, $l = 1, 2, \ldots, p$. The strategies described below fall into two strategies: direct, and distance-of-the-desirability-from-one strategies.

## 4.1 Direct Optimization Strategy

The direct strategy is directly to find the best combination of the controllable variables using the concept of the desirability function and response surface methodology. Without the use of derivatives, complicated mathematical analysis or complex analysis procedures, the direct strategies for multiple response robust design (DSMRD) are a straight forward approach for finding the best combination of the controllable variables. The DSMRD is considered here because of simplicity.

In this section, the arithmetic and geometric average of the responses are considered, along with their desirability values. The geometric average for the desirability values of each quality characteristic is represented as $G(d^*)$. That is, $G(d^*) = (d_1^* \times d_2^* \times \cdots \times d_p^*)^{1/p}$. The geometric average of the desirability values for the arithmetic average of each quality characteristic is represented as $G(\overline{d})$. That is, $G(\overline{d}) = (\overline{d}_1 \times \overline{d}_2 \times \cdots \times \overline{d}_p)^{1/p}$. The desirability value, $\overline{d}_i$ ($i = 1, 2, \ldots, p$) is obtained from the arithmetic average, $\overline{y}_i$, using the desirability function for location. The desirability value, $G(e)$, of the standard deviation, $s$, is also considered. The desirability value, $G(e)$, is also obtained from the desirability function for dispersion.

To develop the DSMRD, the use of $G(d^*)$, $G(\overline{d})$and $G(e)$ are considered. These desirability values can be used to evaluate each combination of controllable variables. And these desirability values can be also used to compare with several combinations of controllable variables.

The direct strategy for multiple response robust design is considered to be a one-step procedure. It is used to find the best combination of controllable variables using the geometric average, $(G(\overline{d}) \times G(d^*) \times G(e))^{1/3}$, for $G(d)$, $G(\overline{d})$ and $G(e)$. The optimal design point is selected by the combination of controllable variables that has the largest geometric average value.

The procedure of this strategy is as followed: The first step of this strategy is to design an experiment over an initial region of the design variables. The desirability function for location(for the individual observations), another desirability function for location (for the arithmetic average) and the desirability function for dispersion are also constructed. It could be done in the opposite order.

The second step is to collect data sets at product design points. The data are transformed into measures of desirability using the desirability functions for location and for dispersion.

The third step is to calculate the arithmetic average, $\overline{y}_{il}$, and standard deviation, $s_{il}$, for each quality characteristic at design point $i$ ($i$ = 1, 2, ..., $k$). The averages and standard deviations are transformed into desirability values, $\overline{d}_{il}$, for the arithmetic average, and desirability values, $e_{il}$, for the standard deviation, $s_{il}$, using the desirability functions for the mean and dispersion. The geometric averages, $d^*_{il}$, for the desirability, $d_{ijl}$, of each of the individual reponses are then calculated. After obtaining $\overline{d}_{il}$, $d^*_{il}$ and $e_{il}$, $G(\overline{d}_i)$, $G(d^*_i)$ and $G(e_i)$ are calculated.

The fourth step is to transform three values ( $G(\overline{d}_i)$, $G(d^*_i)$ and $G(e_i)$) to a single measure for the evaluation of the overall desirability, $D_i(x)$, at experimental point $i$ ($i$ = 1, 2, ..., $k$).

That is, $D_i(x) = (G(\overline{d}_{i.}) \times G(d^*_{i.}) \times G(e_{i.}))^{1/3}$.

The fifth step is to fit a predictive response model for $D_i(x)$, $i$ = 1, 2, ..., $k$, over this local region. The next step is to determine the path of the steepest ascent and run additional experiments along that path.

The direction of steepest ascent could be followed until one is sufficiently far from the initial experimentation region or until the predictive response model no longer adequately predicts the response along the ascent direction gradient. In such situations, a new local region for experimentation would be then defined.

On a new local region, a new set of experiments is performed. This procedure continues until little or no further improvement in response can be achieved from the method. Finally in the region where the most desirable response values are suspected to be found, additional experiments are performed to verify that this is so.

## 4.2 Distance-of-the-Desirability-Function-From-One Strategy

The distance-of-the-desirability-function-from-one strategy considers the distance between the minimum most desired values for each quality characteristic and the desirability values for the responses of each quality characteristic. Intuitively, this strategy makes sense since it is to find the best combination of controllable variables based on the difference between the minimum most desired values and the desirability values of responses.

The factors to be considered are the arithmetic average and standard deviation for each quality characteristic. The minimum most desired values of the arithmetic average and standard deviation are one.

Let $\overline{d}_{il}$ denote the desirability value of the arithmetic average for the $l$th quality characteristic at the $i$th design point. Let $e_{il}$ denote the desirability value for the standard deviation of the $l$th quality characteristic at the $i$th design point.

The distance between the minimum most desired values for the arithmetic average and standard deviation of the $l$th quality characteristic is calculated as follows:

$$Dist_i = \left( \sum_{l=1}^{p} [ (1-\overline{d}_{il})^2 + (1-e_{il})^2 ] \right)^{1/2p}$$

If there is no difference between the minimum most desired values and the desirability values for the arithmetic average and standard deviation, then an ideal combination of controllable variables is said to be achieved. Our strategy is to find the best combination of controllable variables which gives the shortest distance between the minimum most desired values and the desirability values for the average and standard deviation.

Each step of this strategy is as followed:

Steps 1-3 are similar to those for the direct strategy described above.

Step 4. Calculate the desirability value, $e_{il}$, for the standard deviation of the $l$th quality characteristic at the $i$th experimental point. Calculate the desirability value, $d_{il}$, for the arithmetic average of the $l$th response at the ith experimental point.

Step 5. Calculate $Dist_i = \left( \sum_{l=1}^{p} [ (1-\overline{d}_{il})^2 + (1-e_{il})^2 ] \right)^{1/2p}$

Step 6. Fit a predictive response model for $Dist_i(x)$ over this local region. The next step is to determine the path of the steepest decent and run additional experiments along that path.

This direction of steepest decent could be followed until one is sufficiently far from the initial experimentation region or until the predictive response model no longer adequately predicts the response along the decent direction gradient. In such situations, a new local region for experimentation would be then defined.

On a new local region, a new set of experiments is performed. This procedure continues until little or no further improvement in response can be achieved from the method. Finally, in the region where the most desirable response values are suspected to be found, additional experiments are performed to verify that this is so.

## 5. Application : Traffic Light System Model

This section investigates the utility of the proposed strategies in another application area. Although the strategies that we have developed can be used for the design of any system, its potential utility can be gauged by demonstrating its application in a variety of different problem area.

This example that is presented here is an adaptation of the single-lane traffic analysis model in Pritsker(1995). To make it suitable for a multiple response robust design problem, the goal of the analysis is reformulated.

### 5.1 The Problem Description

This that we will consider system consists of traffic flow on a two-lane highway, with traffic moving in both directions. One lane of the highway is closed for 500 meters for repairs. A traffic light is at each end of the closed lane to control the traffic flow through the construction zone. The traffic lights have been designed to allow flow in only one direction at a time. When the light turns green, the waiting cars passed the light every two seconds. If a car arrives at a green light(no waiting cars), it passes through without delay.

One cycle through the light sequence has the following four steps: (1) green in direction 1, (2) both directions red, (3) green in direction 2, (4) both directions red. This cycle is continuously repeated. Arrival patterns to the traffic lights are exponential with an average of 12 seconds between cars from direction 1 and 9 seconds in direction 2.

We will suppose that the objective of this problem is to reduce the variability of the waiting time, the queue length of the cars in both directions and to minimize the average waiting time and the average queue length for cars moving in both directions.

## 5.2 Variables

Two response variables for this model are considered. Engineer who designs the system wants that the average waiting time for cars in direction 1 and in direction 2 and the average queue length for cars in both directions are as small as possible to reduce the number of waiting cars on road and to protect neighbors from air pollution caused by waiting cars.

In this example, there are four controllable variables the green time in direction 1, the green time in direction 2, the length of time that both signals are red following a green signal in direction 1, and the length of time that both signals are red following a green signal in direction 2. The range of four controllable variables are as follow:

The green time in direction 1 ($x_1$) is 45 seconds to 75 seconds.

The length of time that both signals are red following a green signal in direction 1 ($x_3$) is 45 to 75 seconds.

The green time in direction 2 ($x_2$) is 45 seconds to 75 seconds.

The length of time that both signals are red following a green signal in direction 2 ($x_4$) is 45 to 75 seconds.

For noise variables, the mean interarrival time of cars in direction 1, and the mean interarrival time of cars in direction 2 are selected. The mean interarrival time of cars in direction 1 ($N_1$) is exponentially distributed with a mean of 8 to 12 seconds. The mean interarrival time of cars in direction 2 ($N_2$) is exponentially distributed with a mean of 5 to 9 seconds.

## 5.3 The Experimental Design Procedure

Using the notation given section 3, several strategies described in section 4 were implemented and executed to find the best design point of the traffic light design. All the variables and the functions are real valued and continuous. The bounds shown on the variables are assumed to be given as follows:

| Variable | Minimum | Maximum |
|---|---|---|
| $x_1$ | 30 | 90 |
| $x_2$ | 30 | 90 |
| $x_3$ | 30 | 90 |
| $x_4$ | 30 | 90 |
| $N_1$ | 8 | 12 |
| $N_2$ | 5 | 9 |

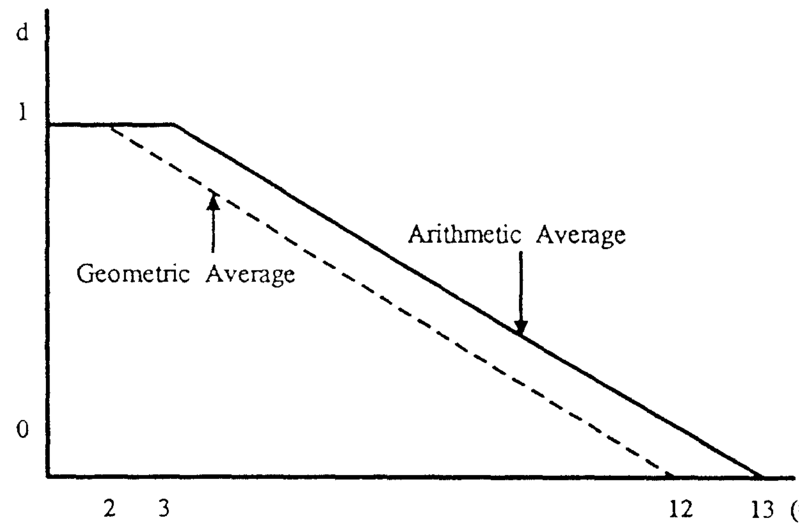
Here $x_i$ ($i$ = 1, 2, 3, 4) refers to the design variables and $N_j$ ($j$ = 1, 2) refers to the noise variables. The objective functions for this problem vary by designs strategy and consists of different combinations of arithmetic averages, geometric averages, and standard deviations for two response variables, the average waiting time ($y_1$) and the average queue length ($y_2$) for cars.

5.3.1 Planning the Experiment

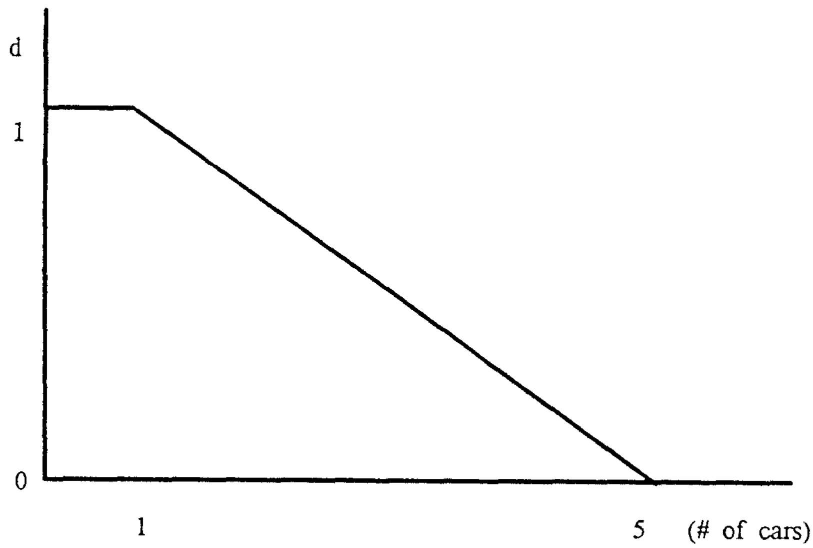
In order to estimate the regression equations for $G(\bar{d}), G(d^*))$, $G(d)$, and $G(e)$, initial starting points in the experimental region need to be specified. The measure the effects of the changes of the $x_i$, a Resolution IV, $2^{4-1}$ fractional factorial design is chosen. A $2^2$ factorial design is the choice for the Noise Matrix. To measure the dispersion effect, both $N_1$ and $N_2$ are perturbed according to a $2^2$ factorial design pattern for each response. The design-noise matrices in <Figure 5.1> is given.

5.3.2 The Setting of the Desirability Function for the Traffic Light Model

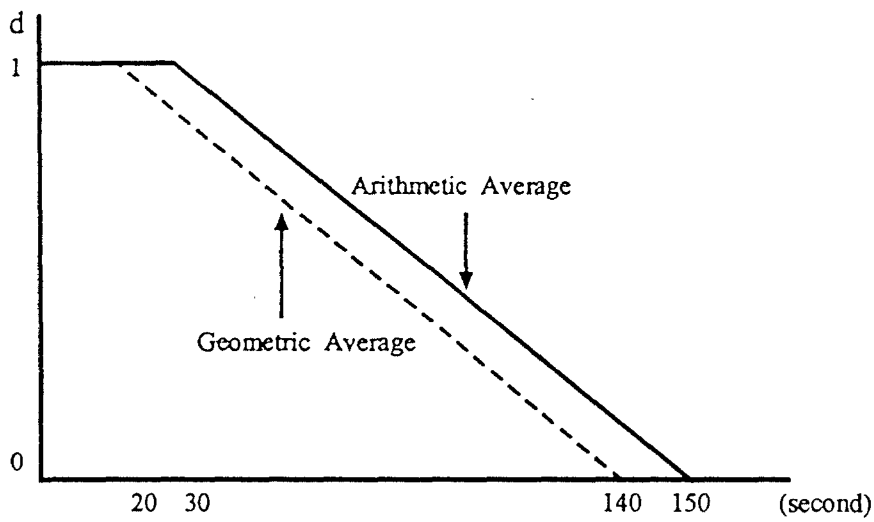
Designer's desire wants to design a traffic system that the average waiting time of all cars passing this road is only in 30 seconds. However, if the average waiting time of cars is over 150 seconds, the system should be redesigned. For the average queue length of cars at the traffic light, designer wants to be as small as possible. The limitation of queue length that can be accepted is the range from 3 to 13. If the waiting line of cars is over 13 cars, this system is also rejected. The desirability functions for each response are shown in Figure 5.2-5.5.



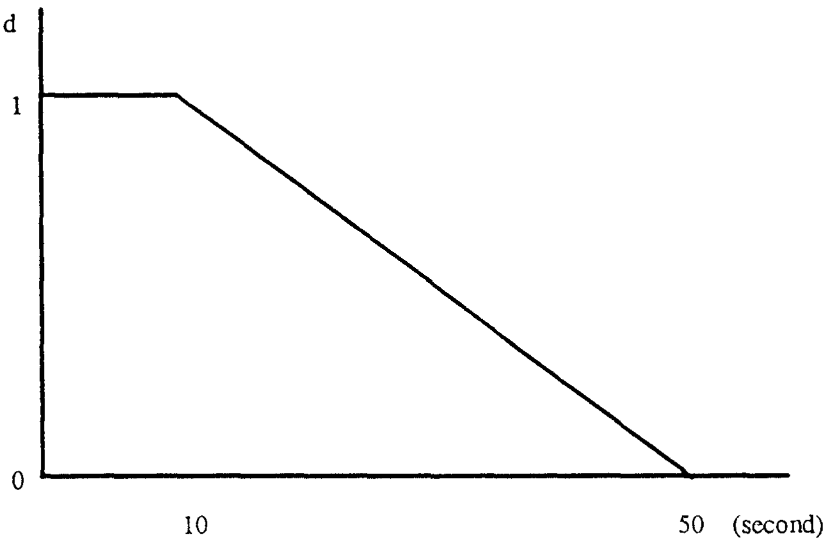

< Figure 5.2 > Desirability function of the expected queue length for the average




< Figure 5.3 > Desirability function of the expected queue length for dispersion




< Figure 5.4 > Desirability function of the expected waiting time for the average




< Figure 5.5 > Desirability function of the expected waiting time for dispersion

## 5.4 The Solution Procedure

The final design point and the results of each strategy for the traffic light model is shown in <Table 5.1>. The direct strategy used the steepest ascent method of response surface methodology to search for the best design point. However, since distance strategy should find the design point having a minimum distance from its target, the steepest decent method of RSM was used.

< Table 5.1 > Comparison the results of RSM with the value of starting point

| Strategy | RSM | Starting Point | Improvement |
|---|---|---|---|
| Direct Desirability Value | 42.89, 35.75, 38.68, 33.67<br>0.8396 | 60, 60, 60, 60<br>0.5964 | 40.33% |
| Distance Desirability Value | 56.66, 30.46, 33.30, 30.02<br>0.2071 | 60, 60, 60, 60<br>0.3256 | 57.22% |

In <Table 5.1>, most of results given are the values between 45 and 60 seconds for the green time in direction 1, the values between 30 and 35 seconds for the red time both directions after a green in direction 1, the values between 30 and 40 seconds for the green time in direction 2, and 30 and 35 seconds for the red time both directions after a green in direction 2. The detailed procedure to obtain the best design point of each strategy for $2^{4-1}$ fractional factorial design is shown in reference [6] or [9].

To compare with the results of two strategies, improvement rate was used. Improvement rate is a criterion that represents how much the final result of RSM is improved more than the desirability value at the initial design point. According to changing the initial design point, improvement rate is also changed. However, as the initial design point, the center of the design, (60, 60, 60, 60), between low level and high level was selected to compare the final solutions of two strategies.

Distance strategy made the maximum improvement rate (57.22%) between two strategies. Direct strategy that was represented by the geometric average for desirability values of the arithmetic average, geometric average, and standard deviation had the improvement of 40.33%. For the results of two strategies, direct and distance strategy, the average waiting time and the average queue length at the final design point were less about 23-35 seconds and 3-5 cars than that at the initial design point.

## 6. Summary and Recommendations

Up to now, there has been no multiple response robust design strategy that dealt with both performance for average and dispersion and with the various types of target. Hence, the new approach developed in this research produces a new paradigm for solving multiple response robust design problems. These new strategies satisfy all conditions that can be brought about by multiple response robust design problems. By explicitly considering robustness as an objective in this problem formulation, this approach forces designers to design products and processes which not only perform well, but which are also robust.

A primary motivating factor for this research is to develop robust designs for products in the quality control domain. Though the proposed strategies have been tested only on system design problems, its generality and concept make it applicable for many types product and process design problems. Consequently, this research presents not only a new method for product designers in the quality control domain but also a new strategy for system designers in the manufacturing domain.

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