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Design of a SMC-type FLC and Its Equivalence

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ABSTRACT

This paper proposes a new design method for the SMC-type FLC and shows that a SMC-type FLC is an extension of the SMC with BL. The conventional SMC-type FLC uses error and change-of-error as inputs of the FLC and generates the absolute value of a switching magnitude. Then, the fuzzy rule table is constructed on a two-dimensional space of the phase plane and has commonly the skew symmetric property. In this paper, we introduce a new variable, signed distance, from the skew symmetric property of the rule table. And the variable becomes only a fuzzy variable that is used to generate the control input of a SMC-type FLC. That is, we design a new SMC-type FLC that uses a signed distance and a control input as the variables representing the contents of the rule-antecedent and the rule-consequent, respectively. Then, the number of total rules is reduced and the control performance is almost the same as that of the conventional SMC-type FLC. Additionally, we derive the control law of the ordinary SMC with BL from a new SMC-type FLC. Namely, we show that a FLC is an extension of the SMC with BL.

Key words: sliding mode control, fuzzy logic control, SMC-type FLC, signed distance.

1. Introduction

The sliding mode control(SMC) is a powerful nonlinear control. It has been used to control imprecise nonlinear plants. The SMC is based on the control law with a sliding regime which not only is independent of changes in the plant parameters and the external disturbances, but also has the property of reduction of the system order[1]. While in the sliding mode, accordingly, this control algorithm guarantees stability and robustness in itself. However, it has several drawbacks. One of them is the high frequency chattering of a control input. The chattering phenomenon is generally undesirable in practice because it involves extremely high control activity[2]. Many results which lessen the chattering phenomenon have been proposed. There is a method that introduces a thin boundary layer (BL) to the sliding surface([2] and [3]). It eliminates the chattering phenomenon by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. But the method leads to the steady state error, furthermore the error is increased with the extension of a boundary layer. So, it requires the compromise between the steady state error and the chattering amplitude. There also is the sliding mode fuzzy control (SMFC) [4]. We call it the conventional SMC-type fuzzy logic control(FLC). It is the fuzzy logic-based SMC that generates the absolute value of a switching magnitude from error and change-of-error. Since it uses two fuzzy variables in the rule-antecedent, the rule table is constructed on a two-dimensional space and the number of rules is many. So, the tuning of rules is difficult. Also, it is difficult to generate a control input for the SMC because it uses the absolute value of a switching magnitude instead of a control input as a variable representing the content of the rule-consequent.

In this paper, we design a new SMC-type FLC. Firstly, we derive a variable, signed distance, from the skew symmetric property of a two-dimensional rule table. And it is used as only a fuzzy variable representing the content of the rule-antecedent. And we use a control input instead of the absolute value of a switching magnitude as a fuzzy variable of the rule-consequent. So, a control input is directly generated by a single fuzzy variable and the fuzzy rule table for a SMC-type FLC is established on one-dimensional space. Thus, the computational complexity for a control input and the effort for tuning the rules are reduced. We also derive the control law of the ordinary SMC with BL from a new SMC-type FLC. That is, we present the equivalence between the SMC with BL and a SMC-type FLC. As a result, we can conclude that a FLC is an extension of the SMC with BL. And we confirm through computer simulations that both SMC-type FLC's have similar control performances.

In Section II and III, we give a short review of the ordinary SMC with BL and the conventional SMC-type FLC, respectively. And then we explain the design of a SMC-type FLC and the equivalence between the SMC with BL and a SMC-type FLC in Section IV. Continuously, we discuss the control performance of the conventional and a new SMC-type FLC's through computer simulations in Section V. Concluding remarks will be offered in Section VI.

2. Sliding Mode Control

The central idea of the SMC is switching to a different structure at each side of a given switching surface. The term SMC is used for emphasizing the importance of the sliding mode or the sliding regime[2]. The remarkable property of the SMC is that the sliding mode occurs on the switching surface and while in this mode the system remains insensitive to parameter uncertainties and external disturbances[2].

For simplicity, we consider the second order system as follows :

$$\ddot{x}(t)=f(\mathrm{x},t)+u(t)+d(t) \tag{1}$$

where $\mathrm{x}=[x, \dot{x}]^T$ is the state vector and $d(t)$ and $u(t)$ are the disturbance and the control input, respectively. The function $f(\mathrm{x}, t)$ (in general nonlinear) is not exactly known, but the extent of the imprecision on $f(\mathrm{x}, t)$ is upper bounded by a known continuous function of x. Similarly, the disturbance $d(t)$ is bounded by a known continuous function of x,

$$\begin{aligned} &|\Delta f(x,t)| \le F(\mathrm{x},t) \\ &|d(t)| \le D(\mathrm{x},t), \end{aligned} \tag{2}$$

where the model uncertainty $\Delta f(\mathrm{x}, t)$ is expressed as follows :

$$\Delta f(\mathrm{x},t)=f(\mathrm{x},t)-\hat{f}(\mathrm{x},t), \tag{3}$$

where $\hat{f}(\mathrm{x}, t)$ represents the estimation of $f(\mathrm{x}, t)$.

Let $e(t)$ be the tracking error of a state x

$$e(t)=x(t)-x_d(t), \tag{4}$$

where $x_d(t)$ is a desired state. Furthermore, let us define a surface $s(t)$ in the state-space R^2 by the scalar equation $s(\mathrm{x}, t)=0$ as follows :

$$s(\mathrm{x},t)=\left(\frac{d}{dt}+\lambda\right)e(t)=\dot{e}(t)+\lambda e(t), \tag{5}$$

where λ is a strictly positive constant. The surface $s(\mathrm{x}, t)=0$ is called the sliding line. Then the control problem is for the state x to track the desired state $x_d(t)$ even under the model uncertainties and the disturbances. The control input is made to satisfy the following sliding condition,

$$\frac{1}{2}\frac{d}{dt}[s^2(\mathrm{x},t)] \le -\eta|s|,\ \ \eta \ge 0. \tag{6}$$

Then, the control input is as follows (2 and 3) :

$$u=\hat{u}-K(\mathrm{x},t)sat\left(\frac{s}{\Phi}\right) \tag{7}$$

$$sat\left(\frac{s}{\Phi}\right)=\begin{cases} \frac{s}{\Phi} & for\ |s| \le \Phi \\ 1 & for\ s>\Phi \\ -1 & for\ s<-\Phi \end{cases} \tag{8}$$

where Φ represents the boundary layer thickness which is well described in Fig. 1, and ε is the boun-

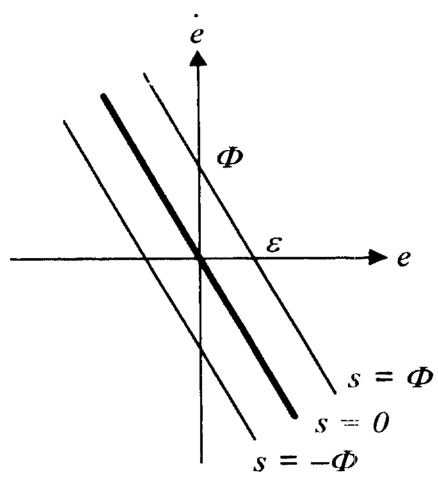


Fig. 1. The boundary layer with thickness Φ.

dary layer width. $\hat{u}$ and $K(x, t)$ is obtained by the followings :

$$\hat{u} = \ddot{x}_d(t) - \hat{f}(x, t) - \lambda \dot{e} \qquad (9)$$

$$K(x, t) \geq F(x, t) + D(x, t) + \eta. \qquad (10)$$

That is, in the SMC with BL, the chattering phenomenon of the SMC is smoothed out in a thin boundary layer $B(t)$ neighboring the sliding surface (Fig. 1).

$$B(t) = \left\{ x, \ |s(x;t)| \leq \Phi \right\}, \quad \Phi > 0. \qquad (11)$$

3. Conventional SMC-type FLC

For a large class of systems, FLC's are designed with respect to the phase plane of error and change-of-error. In the SMC-type FLC, the rules are conditioned in such a way that above the switching line the control variable has a negative value and below it a positive value[5]. Let K_f be the absolute value of the crisp control output of the SMC-type FLC. Then K_f is generally a nonlinear, non-continuous and positive function of e and $\dot{e}$. And the working principle of the SMC-type FLC can be represented by

$$u = -K_f(e, \dot{e}) \cdot sgn(s). \qquad (12)$$

That is, the output of the SMC-type FLC is obtained from rules of Table 1. In Table 1, the abbreviations of linguistic values are as follows : NB (Negative Big), NS(Negative Small), ZR(ZeRo), PS (Positive Small) and PB(Positive Big). And the rule form for the SMC-type FLC is the same as follows :

$R^{(i)}_{SMC}$: *If* e *is* $LE^{(i)}$ and $\dot{e}$ *is* $LDE^{(i)}$
then K_f *is* $LK^{(i)}$,

Table 1. Rules for the conventional SMC-type FLC.

e \ ė	NB	NS	ZR	PS	PB
PB	ZR	NS	NS	NB	NB
PS	PS	ZR	NS	NS	NB
ZR	PS	PS	ZR	NS	NS
NS	PB	PS	PS	ZR	NS
NB	PB	PB	PS	PS	ZR

where

$i = 1, 2, \cdots, I$.

Here, I is the number of rules and $LE^{(i)}$, $LDE^{(i)}$ and $LK^{(i)}$ is the linguistic values of error, change-of-error, and control input in the ith rule, respectively.

4. Design of a SMC-type FLC and Its Equivalence

4.1 Design of a SMC-type FLC

By substituting Table 1 into (12), we can represent rules for the conventional SMC-type FLC in the phase plane. Here, we assume that the number of rules is nearly infinite. Then the boundary between two neighboring linguistic values can be replaced by a direct line (Fig. 2).

From Table 1 and Fig. 2, we know that the more actual states come close to the switching line, the more a control input has a small size. Also, we see that Table 1 has the skew symmetric property. That is, above the switching line control inputs have negative values, below it positive ones, also the more the distance from the switching line goes far, the more control actions become strong. From this property, we now introduce a new variable that can be used as only a single variable representing the rule-antecedent. It is defined by a perpendicular distance

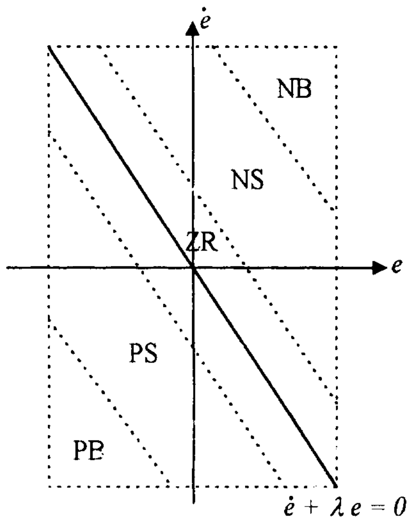


Fig. 2. Rules for the conventional SMC-type FLC in the phase plane.

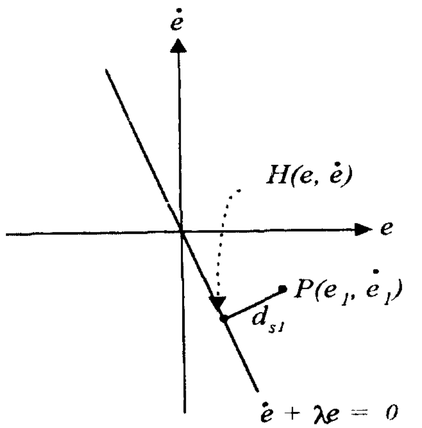


Fig. 3. Depiction of a signed distance.

between an arbitrary operating point and the switching line (Fig. 3), which is denoted by d_s. We call it a signed distance because it has a sign or direction. Namely, at each side of the switching line it has an opposite sign. A variable is gotten by the following procedures(Fig. 3).

Let $H(e, \dot{e})$ be an intersection point of the switching line and the line perpendicular to the switching line from an operating point $P(e_1, \dot{e}_1)$. Then a signed distance is expressed by the following equation.

$$d_{s_1} = [(e - e_1)^2 + (\dot{e} - \dot{e}_1)^2]^{1/2} \tag{13}$$

Since (13) represents a perpendicular distance between a direct line and a point, d_{s1} is derived as the function of only the slope of a direct line and the coordinate values of a point. That is,

$$d_{s_1} = \frac{\dot{e}_1 + \lambda e_1}{\sqrt{1+\lambda^2}}. \tag{14}$$

Without loss of generality, we can rewrite (13) as the following equation.

$$d_s = \frac{\dot{e} + \lambda e}{\sqrt{1+\lambda^2}}. \tag{15}$$

From Fig. 2, we can see that the control action is almost proportional to the negative value of a signed distance, namely, the following relation holds.

$$u \propto -d_s. \tag{16}$$

Now, we design a new SMC-type FLC whose control input is determined by only a signed distance. That is, a fuzzy rule table for the SMC-type FLC is established on a one-dimensional space. Then, a rule form is as follows :

$R^{(j)}_{NSMC}$: *If* d_s *is* $LSD^{(j)}$ *then* u_f *is* $LU^{(j)}$,

where $LSD^{(j)}$ and $LU^{(j)}$ represent the linguistic values for a signed distance and a part (u_f) of the control law of the SMC in the jth control rule, respectively. So, the control law for a new SMC-type FLC is given by the following equation :

$$u = \hat{u} - u_f(d_s), \tag{17}$$

where u_f is obtained from the following rule table.

4.2 Equivalence between a SMC-type FLC and the SMC with BL

Now, we discuss the equivalence between a new SMC-type FLC and the ordinary SMC with BL. Here we derive the SMC with BL from a new SMC-type FLC. From Table 2, the following relation holds.

$$u_f \propto d_s. \tag{18}$$

For simplicity, we neglect the nominal compensation term and assume a linear FLC. Then, (17) can be rewritten as follows :

$$u = -u_f(d_s) = -K_C \cdot d_s, \tag{19}$$

where, K_C is a constant that is obtained by a linear FLC. By substituting (15) into (19), the following equation is obtained.

$$u = -K_C \cdot \frac{\dot{e} + \lambda e}{\sqrt{1+\lambda^2}} = -K_\Phi \cdot s, \tag{20}$$

where

$$K_\Phi = \frac{K_C}{\sqrt{1+\lambda^2}}. \tag{21}$$

Also, in a practical system the magnitude of a control input is restricted by minimum and maximum values. So, we can rewrite (20) as follows :

Table 2. Rule table for a new SMC-type FLC.

d_s	NB	NS	ZR	PS	PB
u_f	NB	NS	ZR	PS	PB

$$u = \begin{cases} -K_m \ , \ if \ u < -K_m \\ K_m \ , \ if \ u > K_m \\ -K_{\Phi} \cdot s \ , \ otherwise \ , \end{cases} \tag{22}$$

where $K_m > 0$ is the maximum value of a control input. By substituting (20) into (22), we can derive the similar equation to the control law of the SMC with BL. That is, from a SMC-type FLC we can lead to the similar form to the control law of the SMC with BL. Thus, we can conclude that a new SMC-type FLC is an extension of the ordinary SMC with BL.

5. Simulation Example

In this section, we compare the performance of two SMC-type FLC's through computer simulations. We use an inverted pendulum as the plant to be controlled. Fig. 4 shows the plant that consists of a pole and a cart. The cart moves on the rail tracks to the horizontal direction left or right. The control objective is to balance the pole starting from arbitrary conditions by supplying a suitable force to the cart. The plant dynamics is expressed as

$$\ddot{\theta} = \frac{g \sin\theta + a \cos\theta - \mu_p \, w^2 l \cos\theta \sin\theta}{l\,(4/3 - \mu_p \cos^2\theta)} \tag{23}$$

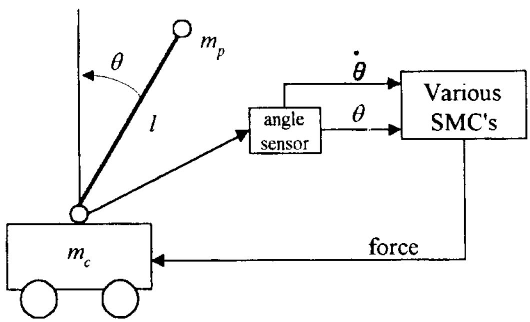


Fig. 4. The inverted pendulum system.

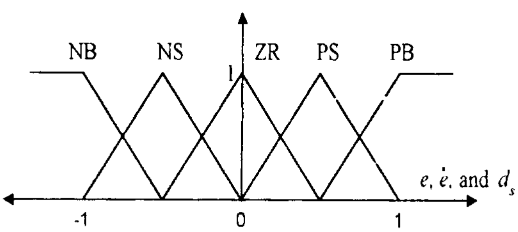
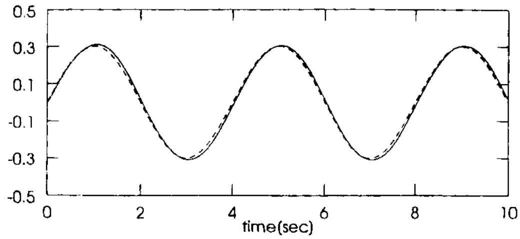


Fig. 5. The fuzzy sets for error, change-of-error and signed distance.

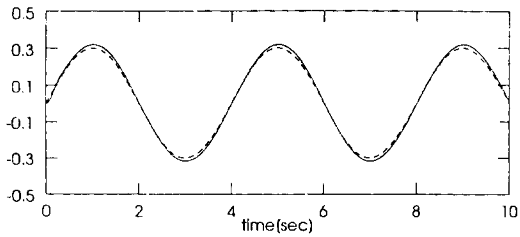
$$\mu_p = \frac{m_p}{m_p + m_c} \ , \ a = \frac{F}{m_p + m_c} \ , \tag{24}$$

where g is an acceleration due to gravity (=9.8 m/sec^2) and F is an applied force. m_c (=1.0 kg) and m_p (=0.1 kg) are masses and l (=0.5 m) is a pole length.

Fig. 5 represents the fuzzy sets for error, change-of-error and signed distance. As shown in Fig. 5, we normalize fuzzy variables for the convenience of

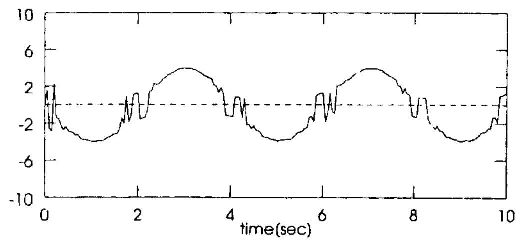


(a) The conventional SMC-type FLC.



(b) The proposed SMC-type FLC.

Fig. 6. The response of comparing the tracking performance.



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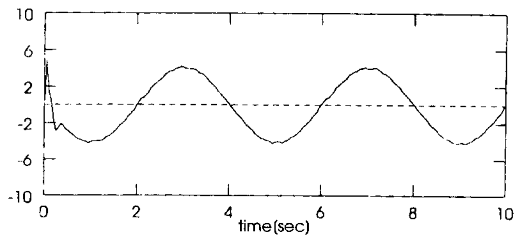
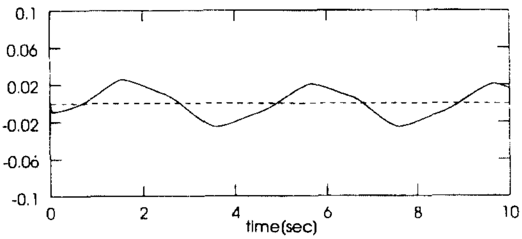
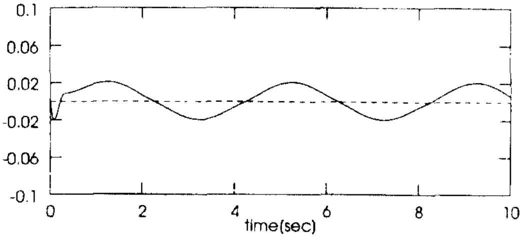


(b) The proposed SMC-type FLC.

Fig. 7. The response of control inputs.



(a) The conventional SMC-type FLC.



(b) The proposed SMC-type FLC.

Fig. 8. The response of tracking errors.

simulations. The fuzzy sets for control variables of both SMC-type FLC's are isosceles triangles too. And we use product-sum and center-average methods for rule inferencing and defuzzification, respectively.

Fig. 6, 7 and 8 show the simulation results of tracking performances, control inputs and tracking errors, respectively. Here (a) and (b) are the cases of the conventional SMC-type FLC and a new SMC-type FLC, respectively. As shown in Fig.'s, the control performances are almost the same even if (b) using proposed scheme has only 5 control rules instead of 25. So, a new SMC-type FLC has several advantages.

6. Concluding Remarks

We have described the conventional and a new SMC-type FLC's and showed a SMC-type FLC is an extension of the SMC with BL. And then we performed computer simulations using an inverted pendulum.

We firstly designed a new SMC-type FLC. Here, we derived a new variable from the skew symmetric property of a two-dimensional rule table using error and change-of-error as the fuzzy variables. And it was used as only a single fuzzy variable for a new SMC-type FLC. Thus, the rule table for the proposed SMC-type FLC had an one-dimensional structure. So, the number of total rules is decreased and the tuning of rules is easy. Nevertheless, the control performance was almost the same as that of the conventional SMC-type FLC. We also showed the control law of the ordinary SMC with BL could derive from that of a SMC-type FLC. Namely, we ensured that a SMC-type FLC is an extension of the SMC with BL. From these results, we might say that the stability of a FLC is implicitly guaranteed from the property of the stability of the SMC with BL.

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