Derivation of a Linear PID Control Law from a Fuzzy Control Theory

퍼지 제어기로부터 PID 제어기의 구현에 관한 연구

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ABSTRACT

Proportional-integral-derivative(PID) controllers have been still widely used in industrial processes due to their simplicity, effectiveness, robustness for a wide range of operating conditions, and the familiarity of control engineers. And a number of recent papers in fuzzy systems are showing that fuzzy systems are universal approximators. That is, fuzzy controllers are capable of approximating any real continuous function on a compact set of arbitrary accuracy.

In this paper, we derive the linear PID control law from the fuzzy control algorithm where all fuzzy sets for representing plant state variables and a control variable use common triangular types. We first lead a linear PD control law from a fuzzy logic control with only two fuzzy sets for error and change-of-error. And then we derive the linear PID control law from a fuzzy controller. We here assumed that the intervals of error, change-of-error, and integral error could be partitioned into arbitrary numbers, respectively. As a result, a linear PID controller is only a sort of various fuzzy logic controls.

요 약

여러 가지 고급 제어 이론들에 관한 연구가 심도있게 진행되고 있음에도 불구하고 아직까지 산업현장에는 여러 가지 변형된 형태의 PID 제어기가 널리 사용되고 있다. 이는 PID 제어기 자체가 가진 제어 구조의 단순성, 효율성, 강건성, 그리고 제어 기술자들에 대한 친밀감 등에 기인한다. 또한 요즘 제어 분야에서는 퍼지 이론을 도입하는 연구가 활발히 진행되고 있다. 특히, 퍼지 이론을 사용해서 거의 모든 함수들을 근사화시킬 수 있다는 연구 결과들이 발표되면서 수학적으로 안정성 및 강건성을 명확히 증명하기에 다소 미흡하였던 퍼지 논리 제어에 관한 연구가 활기를 띠고 있다. 본 논문에서는 먼저 간단한 퍼지 제어기로부터 선형 PD 제어기를 유도한다. 그리고 나서 다소 일반적인 경우의 퍼지 제어기를 사용하여 산업 현장에서 가장 널리 사용되고있는 선형 PID 제어기를 유도하여 결국 PID 제어기는 퍼지 제어기의 일종에 불과함을 입증할 것이다.

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Ⅰ. Introduction

It is a fact that proportional-integral-derivative (PID) controllers have been still widely used in industrial processes even if control theory has been developed significantly. This is due to their simplicity, effectiveness, robustness for a wide range of operating conditions, and the familiarity of control engineers. These controllers are also embedded in all sorts of special purpose control systems, and have several important functions: they have the ability to eliminate steady-state errors through the integral action, they can predict some degree of the future through the derivative action, and they can cope with the saturation of an actuator. Also, various proportional-derivative (PD) controllers provide high sensitivity and tend to increase the stability of the overall feedback control system. In addition, they can reduce overshoot. So, they can be effectively used for first or second-order linear systems[1]. While the PID controllers perform poorly for the processes with high nonlinearity, large time delay, and time varying properties.

Recently, there has been growing interest in using fuzzy set theory for control systems. Since L. Zadeh had introduced the fuzzy set theory in 1965, this theory has been applied to many areas such as automatic control, biology and medicine, decision making, economic etc. It provides an effective means of capturing approximate, inexact nature of the real world. Fuzzy logic control has emerged as one of the most active and fruitful areas for research in the application of fuzzy set theory. Fuzzy logic control is particularly the useful control method for the plants with the difficulty in derivation of a mathematical model or with the limitation in performances using conventional linear control scheme. The majority of works in the field of fuzzy control theory use only error (e) and change-of-error (δe or $\dot{e}$) as inputs of a controller. In this case, fuzzy controllers are divided into two categories: one is a PD-type fuzzy control which generates control input (u) from error and change-of-error, the other is a PI(proportional-integral)-type fuzzy control which generates incremental control input (Δu) from error and change-of-error. There also is a PID-type fuzzy control which generates control input from error, change-of-error, and integral error (σe or $\int e\,dt$). In [1] and [3], it is shown that for nonlinear systems PD and PI-type fuzzy controllers are better than the conventional PD and PI controllers, respectively.

A number of recent papers in fuzzy systems are showing that fuzzy systems are universal approximators. That is, fuzzy controllers are capable of approximating any real continuous function on a compact set of arbitrary accuracy[3]. In [3] and [4], they showed that the output of special fuzzy controllers is equivalent to that of a linear PI controller. In [5], a linear PD and PID controllers are realized by a fuzzy controller which the output fuzzy sets are singleton types. In this paper we derive the linear PID control law from the fuzzy control algorithm where all fuzzy sets for representing plant states and a control variable use common triangular types.

In Section Ⅱ we give a short comments on the fuzzy control theory, and we present the derivation of a linear PID control law from fuzzy control theory in Section Ⅲ. In this section we first discuss the realization of a linear PD control law from a fuzzy controller, and then in detail describe the derivation of a linear PID controller from a fuzzy controller. The final Section contains a brief discussion and conclusions.

Ⅱ. Fuzzy Control Theory

The input of fuzzy controllers is mostly one among the set of error and change-of-error, the set of error, change-of-error, and integral error, and another set, where the element of sets is the process state variables representing the contents of the fuzzy rule-antecedent (if-part of a rule). And these controllers use a control input or an incremental control input as the variables representing the contents of the rule-consequent (then-part of a rule), namely, their outputs. Here, we briefly

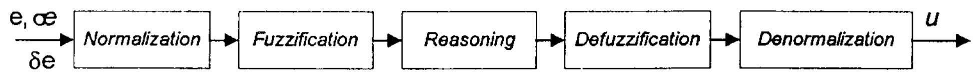


Fig. 1 Computational structure of the FLC.

describe the PD-type fuzzy control theory which generates a control input from error, change-of-error, and integral error.

The computational structure of a fuzzy controller consists of a number of computational steps as depicted in Fig. 1. There are five computational steps:

① Normalization
② Fuzzification
③ Reasoning
④ Defuzzification
⑤ Denormalization

Normalization is the step of input scaling. In other words, the domains of the process state variables in the rule-antecedent are mapped into the same universe of discourse after this step.

Fuzzification transforms the continuous input signal into linguistic variables such as $\tilde{e}$, $\widetilde{\delta e}$, and $\widetilde{\sigma e}$, where $\tilde{e}$, $\widetilde{\delta e}$, and $\widetilde{\sigma e}$ are the linguistic values taken by the process state variables e, δe, and σe, respectively. The meaning of linguistic values is illustrated by membership functions which are the central concept of fuzzy set theory. The membership function of linguistic value $\tilde{e}$ is denoted by $\mu_{\tilde{e}}(e)$, and it represents numerically the degree to which an element or fuzzy variable e belongs to the fuzzy set $\tilde{e}$. Then the meaning of $\tilde{e}$ is given by $\mu_{\tilde{e}}(e): E \rightarrow [0, 1]$, where E is the universe of discourse.

Reasoning or inference is the step of rule firing. The fuzzy engine carries out rule inference where human experience is injected through linguistic rules. For the most part the min-max or the product-sum method is used to infer fuzzy control rules. The result of this step is given by the fuzzy set $\mu_{\tilde{u}'}(u)$.

Defuzzification converts the inferred control action back to continuous control signal that interpolates between simultaneously fired rules. That is, this is the step that obtains a crisp value u from a fuzzy set $\mu_{\tilde{u}'}(u)$. The center-of-gravity, the center-average, or the height method is used for this.

Denormalization is the procedure of output scaling. In other words, the domain of the control variable u in the rule-consequent is mapped into another universe of discourse.

Ⅲ. Derivation of PID Control Law

Now we in detail describe the realization of a PID controller from a fuzzy control algorithm in this section. In order to derive a conventional linear PID control law from a fuzzy control algorithm with nonlinearities it is necessary that the relationship between input and output of a fuzzy controller has to be linear. Therefore the particular types for membership functions, reasoning method, and defuzzification method are required so that a fuzzy controller can produce a linear output with respect to a given input. The membership functions are all triangular types and intersect at the height of 0.5. The product-sum operation rule will be used for reasoning of fuzzy rules, and the center-average method for defuzzification of a inferencing result.

A. Realization of a linear PD controller

For simplicity, we first give the realization of a PD controller by a fuzzy logic control method. A linear PD control law is given in the form of the linear combination of error e and change-of-error δe or $\dot{e}$ as follows:

$$u = K_P \cdot e + K_D \cdot \delta e \qquad (1)$$

Let e_{b1} and e_{b2} be the minimal and maximal values of possible error e, and let δe_{b1} and δe_{b2} be the minimal and maximal values of possible change-of-error δe, namely,

$$e_{b1} \leq e \leq e_{b2}, \qquad \delta e_{b1} \leq \delta e \leq \delta e_{b2}. \tag{2}$$

And we assume that the number of fuzzy sets for error e is two and they are $\tilde{e}_1$ and $\tilde{e}_2$, respectively, and similarly, the number of fuzzy sets for change-of-error δe is two and they are $\widetilde{\delta e}_1$ and $\widetilde{\delta e}_2$, respectively. As shown in Fig. 2 the types of membership functions for error and change-of-error are all right-angled triangles and the types for a control variable are isosceles triangles.

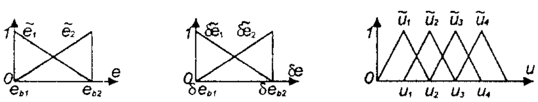


Fig. 2 Fuzzy sets for error e, change-of-error δe, and a control variable u.

Fuzzy control rules which realizes a PD controller are as follows:

R^1: if e is $\tilde{e}_1$ and δe is $\widetilde{\delta e}_1$ then u is $\tilde{u}_1$
R^2: if e is $\tilde{e}_1$ and δe is $\widetilde{\delta e}_2$ then u is $\tilde{u}_2$
R^3: if e is $\tilde{e}_2$ and δe is $\widetilde{\delta e}_1$ then u is $\tilde{u}_3$
R^4: if e is $\tilde{e}_2$ and δe is $\widetilde{\delta e}_2$ then u is $\tilde{u}_4$.

As the facts are e and δe, the conclusion $\tilde{u}'$ is computed as follows. We use the product-sum operation rule for inferencing of fuzzy control rules, then the result which is inferred from the facts and fuzzy ruies is given by

$$\mu_{\tilde{u}'}(u) = \mu_{\tilde{u}_1'}(u) + \mu_{\tilde{u}_2'}(u) + \mu_{\tilde{u}_3'}(u) + \mu_{\tilde{u}_4'}(u), \tag{3}$$

where

$$\begin{aligned} \mu_{\tilde{u}_1'}(u) &= \mu_{\tilde{e}_1}(e) \cdot \mu_{\widetilde{\delta e}_1}(\delta e) \cdot \mu_{\tilde{u}_1}(u) \\ \mu_{\tilde{u}_2'}(u) &= \mu_{\tilde{e}_1}(e) \cdot \mu_{\widetilde{\delta e}_2}(\delta e) \cdot \mu_{\tilde{u}_2}(u) \\ \mu_{\tilde{u}_3'}(u) &= \mu_{\tilde{e}_2}(e) \cdot \mu_{\widetilde{\delta e}_1}(\delta e) \cdot \mu_{\tilde{u}_3}(u) \\ \mu_{\tilde{u}_4'}(u) &= \mu_{\tilde{e}_2}(e) \cdot \mu_{\widetilde{\delta e}_2}(\delta e) \cdot \mu_{\tilde{u}_4}(u) \end{aligned} \tag{4}$$

and where membership functions $\mu_{\tilde{e}_1}(e)$, $\mu_{\tilde{e}_2}(e)$, $\mu_{\widetilde{\delta e}_1}(\delta e)$, and $\mu_{\widetilde{\delta e}_2}(\delta e)$ are computed as follows:

$$\begin{aligned} \mu_{\tilde{e}_1}(e) &= \frac{e_{b2} - e}{e_{b2} - e_{b1}}, \qquad \mu_{\tilde{e}_2}(e) = 1 - \mu_{\tilde{e}_1}(e), \\ \mu_{\widetilde{\delta e}_1}(\delta e) &= \frac{\delta e_{b2} - \delta e}{\delta e_{b2} - \delta e_{b1}}, \qquad \mu_{\widetilde{\delta e}_2}(\delta e) = 1 - \mu_{\widetilde{\delta e}_1}(\delta e), \end{aligned} \tag{5}$$

also where, $+$ and $\cdot$ represent algebraic sum and product operations, respectively. Let the center of a fuzzy set $\tilde{u}_k'$ $(k=1, 2, 3, 4)$ be u_k. Then the value of u_k becomes the center value of a fuzzy set $\tilde{u}_k'$ which is generated by the product operation rule, and from the fuzzy control rules u_k can be expressed as follows [5]:

$$\begin{aligned} u_1 &= K_P \cdot e_{b1} + K_D \cdot \delta e_{b1} \\ u_2 &= K_P \cdot e_{b1} + K_D \cdot \delta e_{b2} \\ u_3 &= K_P \cdot e_{b2} + K_D \cdot \delta e_{b1} \\ u_4 &= K_P \cdot e_{b2} + K_D \cdot \delta e_{b2}. \end{aligned} \tag{6}$$

These are the values of Eq. (1) at points $(e_{b1}, \delta e_{b1})$, $(e_{b1}, \delta e_{b2})$, $(e_{b2}, \delta e_{b1})$, and $(e_{b2}, \delta e_{b2})$.

For defuzzification we use the center-average method as follows:

$$u = \frac{\sum_{k=1}^{4} \mu_{\tilde{u}'}(u_k) \cdot u_k}{\sum_{k=1}^{4} \mu_{\tilde{u}'}(u_k)}. \tag{7}$$

From the definition of the center value u_k, the following equation holds.

$$\mu_{\tilde{u}_k'}(u_k) = 1, \qquad k = 1, 2, 3, 4. \tag{8}$$

Substituting Eq. (8) into (7), the denominator of Eq. (7) is expressed as follows:

$$\begin{aligned} \sum_{k=1}^{4} \mu_{\tilde{u}'}(u_k) &= \mu_{\tilde{u}_1'}(u_1) + \mu_{\tilde{u}_2'}(u_2) + \mu_{\tilde{u}_3'}(u_3) + \mu_{\tilde{u}_4'}(u_4) \\ &= \mu_{\tilde{e}_1}(e) \cdot \mu_{\widetilde{\delta e}_1}(\delta e) + \mu_{\tilde{e}_1}(e) \cdot \mu_{\widetilde{\delta e}_2}(\delta e) \end{aligned}$$

$$+\mu_{\tilde{e}_2}(e)\cdot\mu_{\widetilde{\delta e}_1}(\delta e)+\mu_{\tilde{e}_2}(e)\cdot\mu_{\widetilde{\delta e}_2}(\delta e)$$
$$=\omega_1+\omega_2+\omega_3+\omega_4, \quad (9)$$

where ω_k represents the degree of fitness of the facts e and δe to the antecedent part of the rule, and is expressed as follows:

$$\begin{aligned}\omega_1&=\mu_{\tilde{e}_1}(e)\cdot\mu_{\widetilde{\delta e}_1}(\delta e)\\ \omega_2&=\mu_{\tilde{e}_1}(e)\cdot\mu_{\widetilde{\delta e}_2}(\delta e)\\ \omega_3&=\mu_{\tilde{e}_2}(e)\cdot\mu_{\widetilde{\delta e}_1}(\delta e)\\ \omega_4&=\mu_{\tilde{e}_2}(e)\cdot\mu_{\widetilde{\delta e}_2}(\delta e).\end{aligned} \quad (10)$$

So, Eq. (7) can be rewritten as the following equation.

$$u=\frac{\sum_{k=1}^{4}\omega_k\cdot u_k}{\sum_{k=1}^{4}\omega_k}, \quad (11)$$

that is, the final consequence is obtained by the weighted average of u_k by the degree of ω_k. Substituting Eq.'s (5) and (10) into Eq. (11), the denominator and the numerator of Eq. (11) are reduced as the following Eq.'s (12) and (13), respectively.

$$\begin{aligned}\sum_{k=1}^{4}\omega_k=&\frac{e_{b2}-e}{e_{b2}-e_{b1}}\cdot\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\\ &+\frac{e_{b2}-e}{e_{b2}-e_{b1}}\cdot\left(1-\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\right)\\ &+\left(1-\frac{e_{b2}-e}{e_{b2}-e_{b1}}\right)\cdot\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\\ &+\left(1-\frac{e_{b2}-e}{e_{b2}-e_{b1}}\right)\cdot\left(1-\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\right)\\ =&1.\end{aligned} \quad (12)$$

$$\begin{aligned}&\sum_{k=1}^{4}\omega_k\cdot u_k\\ &=\frac{e_{b2}-e}{e_{b2}-e_{b1}}\cdot\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\cdot(K_P\cdot e_{b1}+K_D\cdot\delta e_{b1})\\ &+\frac{e_{b2}-e}{e_{b2}-e_{b1}}\cdot\left(1-\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\right)\cdot(K_P\cdot e_{b1}+K_D\cdot\delta e_{b2})\\ &+\left(1-\frac{e_{b2}-e}{e_{b2}-e_{b1}}\right)\cdot\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\cdot(K_P\cdot e_{b2}+K_D\cdot\delta e_{b1})\\ &+\left(1-\frac{e_{b2}-e}{e_{b2}-e_{b1}}\right)\cdot\left(1-\frac{\delta e_{b2}-\delta e}{\delta e_{b2}-\delta e_{b1}}\right)\cdot(K_P\cdot e_{b2}+K_D\cdot\delta e_{b2})\\ &=K_P\cdot e+K_D\cdot\delta e.\end{aligned} \quad (13)$$

Therefore, Eq. (11) is reduced by the same equation to Eq. (1) as follows:

$$u=K_P\cdot e+K_D\cdot\delta e$$

Consequently, we could derive the a PD control law from a fuzzy control algorithm. Here, we used right-angled triangles with intersection points at the height 0.5 as membership functions for fuzzy sets $\tilde{e}$ and $\widetilde{\delta e}$, isosceles triangles as membership functions for a fuzzy set $\tilde{u}$, the product-sum operation rule for approximate reasoning, and the center-average method for defuzzification. In addition to these, we also assumed that the number of fuzzy sets $\tilde{e}$ and $\widetilde{\delta e}$ is two, respectively.

In continuous section, we will derive the linear PID control law from fuzzy control theory. Additionally, we will lead more general case than that of a previous section, namely, we will not restrict the number of fuzzy sets representing plant state variables. In other words, we will derive the case that the ranges of error, change-of-error, and integral error are partitioned into several regions. Furthermore, we will use common triangular types instead of isosceles triangles as membership functions for a fuzzy set $\tilde{u}$.

B. Derivation of a linear PID controller

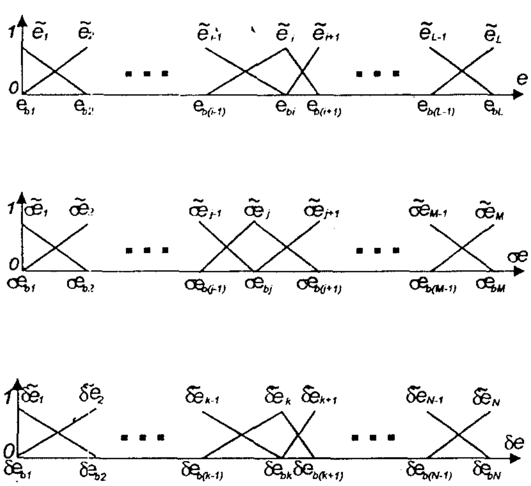
Now we consider the realization of a conventional linear PID control law from a fuzzy control algorithm. As well known, the control law for a PID controller is the form which is linearly combined by error e, change-of-error δe, and integral error σe or $\int e\,dt$ as follows:

$$u=K_P\cdot e+K_D\cdot\delta e+K_I\cdot\sigma e \quad (14)$$

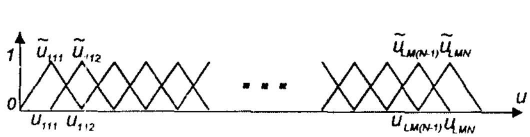
And we assume that the numbers of fuzzy sets representing process state variables are L, M, and N for error, integral error, and change-of-error, respectively. In other words, as shown in Fig. 3 the ranges of possible error e, integral error σe, and change-of-error δe are divided by L, M, and N fuzzy sets, respectively. Also, we let the maximal values of error e, integral error σe, and change-of-error δe be e_{bL}, σe_{bM}, and δe_{bN}, respectively, that is,

$$\begin{aligned} &e_{b1} \le e \le e_{bL} \\ &\sigma e_{b1} \le \sigma e \le \sigma e_{bM} \\ &\delta e_{b1} \le \delta e \le \delta e_{bN}, \end{aligned} \tag{15}$$

where σe_{b1} represents the minimal value of the possible integral error σe.



(a) Fuzzy sets for error, integral error, and change-of-error.



(b) Fuzzy sets for a control variable.

Fig. 3 Fuzzy sets for error e, integral error σe, change-of-error δe, and a control variable u.

As shown in Fig. 3, membership functions for fuzzy sets $\tilde{e}$, $\widetilde{\sigma e}$, and $\widetilde{\delta e}$ are common triangular types of which neighboring fuzzy sets always intersect at the height of 0.5, and we first assume that membership functions for a fuzzy set $\tilde{u}$ is isosceles triangular types. Then fuzzy control rules which realizes a PID control law have the following form:

R^{ijk}: if e is $\tilde{e}_i$ and σe is $\widetilde{\sigma e}_j$ and δe is $\widetilde{\delta e}_k$ then u is $\tilde{u}_{ijk}$,

where

$$\begin{aligned} &i = 1, 2, \cdots, L \\ &j = 1, 2, \cdots, M \\ &k = 1, 2, \cdots, N. \end{aligned} \tag{16}$$

As the facts are e, σe, and δe, the conclusion $\tilde{u}_k'$ is computed as follows. In the same way we use the product-sum method for reasoning of fuzzy rules, then the inference result is given by

$$\mu_{\tilde{u}'}(u) = \sum_{i=l}^{l+1} \sum_{j=m}^{m+1} \sum_{k=n}^{n+1} \mu_{\tilde{u}_{ijk}'}(u), \tag{17}$$

where l, m, and n are arbitrary positive integers with the following ranges:

$$\begin{aligned} &1 \le l \le L-1 \\ &1 \le m \le M-1 \\ &1 \le n \le N-1, \end{aligned} \tag{18}$$

and

$$\mu_{\tilde{u}_{ijk}'}(u) = \mu_{\tilde{e}_i}(e) \cdot \mu_{\widetilde{\sigma e}_j}(\sigma e) \cdot \mu_{\widetilde{\delta e}_k}(\delta e) \cdot \mu_{\tilde{u}_{ijk}}(u). \tag{19}$$

And membership functions $\mu_{\tilde{e}_l}(e)$, $\mu_{\tilde{e}_{(l+1)}}(e)$, $\mu_{\widetilde{\sigma e}_m}(\sigma e)$, $\mu_{\widetilde{\sigma e}_{(m+1)}}(\sigma e) \cdot \mu_{\widetilde{\delta e}_n}(\delta e)$, and $\mu_{\widetilde{\delta e}_{(n+1)}}(\delta e)$ are computed as follows:

$$\mu_{\tilde{e}_l}(e) = \frac{e_{b(l+1)} - e}{e_{b(l+1)} - e_{bl}}, \qquad \mu_{\tilde{e}_{(l+1)}}(e) = 1 - \mu_{\tilde{e}_l}(e)$$

$$\mu_{\widetilde{\sigma e}_m}(\sigma e) = \frac{\sigma e_{b(m+1)} - \sigma e}{\sigma e_{b(m+1)} - \sigma e_{bm}}, \quad \mu_{\widetilde{\sigma e}_{(m+1)}}(\sigma e) = 1 - \mu_{\widetilde{\sigma e}_m}(\sigma e)$$

$$\mu_{\tilde{\delta e}_n}(\delta e)=\frac{\delta e_{b(n+1)}-\delta e}{\delta e_{b(n+1)}-\delta e_{bn}}\ ,\quad \mu_{\tilde{\delta e}_{(n+1)}}(\delta e)=1-\mu_{\tilde{\delta e}_n}(\delta e). \tag{20}$$

Let the center value of a fuzzy set $\tilde{u}_{ijk}'$ be $\tilde{u}_{ijk}$. Then the center value, $\tilde{u}_{ijk}$, is also the peak value which is generated by the product operation rule, and from the fuzzy control rules $\tilde{u}_{ijk}$ can be expressed as follows [5]:

$$\tilde{u}_{ijk}=K_P\cdot e_{bi}+K_I\cdot \sigma e_{bj}+K_D\cdot \delta e_{bk} \tag{21}$$

This is the value of Eq. (14) at points (e_{bi}, σe_{bj}, δe_{bk}). For defuzzification we use the center-average method as follows:

$$u=\frac{\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\mu_{\tilde{u}_{ijk}'}(u)\cdot u_{ijk}}{\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\mu_{\tilde{u}_{ijk}'}(u_{ijk})} \tag{22}$$

From the definition of u_{ijk}, the following equation holds.

$$\mu_{\tilde{u}_{ijk}}(u_{ijk})=1, \tag{23}$$

where i, j, and k are the same to Eq. (16).

Substituting Eq. (23) into (22), the denominator of Eq. (22) is expressed as follows:

$$\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\mu_{\tilde{u}_{ijk}'}(u_{ijk})=\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\omega_{ijk}. \tag{24}$$

where ω_{ijk} represents the degree of fitness of the facts e, σe, and δe to the antecedent part of the rule, and is expressed as follows:

$$\omega_{ijk}=\mu_{\tilde{e}_i}(e)\cdot\mu_{\tilde{\sigma e}_j}(\sigma e)\cdot\mu_{\tilde{\delta e}_k}(\delta e). \tag{25}$$

Applying the above procedure to the numerator of Eq. (22), the following equation is obtained.

$$u=\frac{\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\omega_{ijk}\cdot u_{ijk}}{\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\omega_{ijk}} \tag{26}$$

that is, the final consequence is obtained by the weighted average of u_{ijk} by the degree of ω_{ijk}. Substituting Eq.'s (20) and (25) into (26), the denominator of Eq. (26) is reduced as followings.

$$\begin{aligned}
&\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\omega_{ijk}\\
&=\mu_{\tilde{e}_l}(e)\cdot\mu_{\tilde{\sigma e}_m}(\sigma e)\cdot\mu_{\tilde{\delta e}_n}(\delta e)+\mu_{\tilde{e}_l}(e)\cdot\mu_{\tilde{\sigma e}_m}(\sigma e)\cdot\mu_{\tilde{\delta e}_{(n+1)}}(\delta e)\\
&\quad+\mu_{\tilde{e}_l}(e)\cdot\mu_{\tilde{\sigma e}_{(m+1)}}(\sigma e)\cdot\mu_{\tilde{\delta e}_n}(\delta e)\\
&\quad+\mu_{\tilde{e}_l}(e)\cdot\mu_{\tilde{\sigma e}_{(m+1)}}(\sigma e)\cdot\mu_{\tilde{\delta e}_{(n+1)}}(\delta e)\\
&\quad+\mu_{\tilde{e}_{(l+1)}}(e)\cdot\mu_{\tilde{\sigma e}_m}(\sigma e)\cdot\mu_{\tilde{\delta e}_n}(\delta e)\\
&\quad+\mu_{\tilde{e}_{(l+1)}}(e)\cdot\mu_{\tilde{\sigma e}_m}(\sigma e)\cdot\mu_{\tilde{\delta e}_{(n+1)}}(\delta e)\\
&\quad+\mu_{\tilde{e}_{(l+1)}}(e)\cdot\mu_{\tilde{\sigma e}_{(m+1)}}(\sigma e)\cdot\mu_{\tilde{\delta e}_n}(\delta e)\\
&\quad+\mu_{\tilde{e}_{(l+1)}}(e)\cdot\mu_{\tilde{\sigma e}_{(m+1)}}(\sigma e)\cdot\mu_{\tilde{\delta e}_{(n+1)}}(\delta e)
\end{aligned}$$

$$\begin{aligned}
&=\frac{e_{b(l+1)}-e}{e_{b(l+1)}-e_{bl}}\cdot\frac{\sigma e_{b(m+1)}-\sigma e}{\sigma e_{b(m+1)}-\sigma e_{bm}}\cdot\frac{\delta e_{b(n+1)}-\delta e}{\delta e_{b(n+1)}-\delta e_{bn}}\\
&\quad+\frac{e_{b(l+1)}-e}{e_{b(l+1)}-e_{bl}}\cdot\frac{\sigma e_{b(m+1)}-\sigma e}{\sigma e_{b(m+1)}-\sigma e_{bm}}\cdot(1-\mu_{\tilde{\delta e}_n}(\delta e))\\
&\quad+\frac{e_{b(l+1)}-e}{e_{b(l+1)}-e_{bl}}\cdot(1-\mu_{\tilde{\sigma e}_m}(\sigma e))\cdot\frac{\delta e_{b(n+1)}-\delta e}{\delta e_{b(n+1)}-\delta e_{bn}}\\
&\quad+\frac{e_{b(l+1)}-e}{e_{b(l+1)}-e_{bl}}\cdot(1-\mu_{\tilde{\sigma e}_m}(\sigma e))\cdot(1-\mu_{\tilde{\delta e}_n}(\delta e))\\
&\quad+(1-\mu_{\tilde{e}_l}(e))\cdot\frac{\sigma e_{b(m+1)}-\sigma e}{\sigma e_{b(m+1)}-\sigma e_{bm}}\cdot\frac{\delta e_{b(n+1)}-\delta e}{\delta e_{b(n+1)}-\delta e_{bn}}\\
&\quad+(1-\mu_{\tilde{e}_l}(e))\cdot\frac{\sigma e_{b(m+1)}-\sigma e}{\sigma e_{b(m+1)}-\sigma e_{bm}}\cdot(1-\mu_{\tilde{\delta e}_n}(\delta e))\\
&\quad+(1-\mu_{\tilde{e}_l}(e))\cdot(1-\mu_{\tilde{\sigma e}_m}(\sigma e))\cdot\frac{\delta e_{b(n+1)}-\delta e}{\delta e_{b(n+1)}-\delta e_{bn}}\\
&\quad+(1-\mu_{\tilde{e}_l}(e))\cdot(1-\mu_{\tilde{\sigma e}_m}(\sigma e))\cdot(1-\mu_{\tilde{\delta e}_n}(\delta e))\\
&=1.
\end{aligned}$$

Similarly, by repeating the above procedure the numerator of Eq. (26) is reduced as followings:

$$\begin{aligned}
&\sum_{i=l}^{l+1}\sum_{j=m}^{m+1}\sum_{k=n}^{n+1}\omega_{ijk}\cdot u_{ijk}\\
&=\omega_{lmn}\cdot u_{lmn}+\omega_{lm(n+1)}\cdot u_{lm(n+1)}+\omega_{l(m+1)n}\cdot u_{l(m+1)n}\\
&\quad+\omega_{l(m+1)(n+1)}\cdot u_{l(m+1)(n+1)}+\omega_{(l+1)mn}\cdot u_{(l+1)mn}\\
&\quad+\omega_{(l+1)m(n+1)}\cdot u_{(l+1)m(n+1)}+\omega_{(l+1)(m+1)n}\cdot u_{(l+1)(m+1)n}
\end{aligned}$$

$$+\omega_{(l+1)(m+1)(n+1)} \cdot u_{(l+1)(m+1)(n+1)}$$
$$= K_P \cdot e + K_I \cdot \sigma e + K_D \cdot \delta e.$$

That is, Eq. (26) is reduced by the same equation to Eq. (14) of a linear PID control law as follows:

$$u = K_P \cdot e + K_I \cdot \sigma e + K_D \cdot \delta e.$$

Proposition. As shown in Fig. 4, when the fuzzy sets for representing a control variable have common triangular types instead of isosceles triangular types, we can also derive the linear PID control law using the following height defuzzification method instead of the center-average method.

$$u = \frac{\sum_{i=l}^{l+1} \sum_{j=m}^{m+1} \sum_{k=n}^{n+1} \bar{\omega}_{ijk} \cdot \bar{u}_{ijk}}{\sum_{i=l}^{l+1} \sum_{j=m}^{m+1} \sum_{k=n}^{n+1} \bar{\omega}_{ijk}} \tag{27}$$

where $\bar{u}_{ijk}$ is the peak value instead of the center value of a fuzzy set $\tilde{u}_{ijk}$ for representing a control variable u and is also expressed by following equation,

$$\bar{u}_{ijk} = K_P \cdot e_{bi} + K_I \cdot \sigma e_{bj} + K_D \cdot \delta e_{bk}, \tag{28}$$

and $\bar{\omega}_{ijk}$ is given by

$$\bar{\omega}_{ijk} = \mu_{\tilde{e}_i}(e) \cdot \mu_{\widetilde{\sigma e}_j}(\sigma e) \cdot \mu_{\widetilde{\delta e}_k}(\delta e) \cdot \mu_{\tilde{u}_{ijk}}(\bar{u}_{ijk}). \tag{29}$$

Proof. From the similar procedure to above, the linear PID control law is easily derived so we here omit the proof.

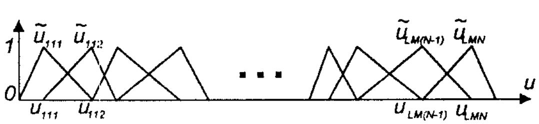


Fig. 4 Fuzzy sets for a control variable u of Proposition.

Ⅳ. Conclusions

In this paper, we minutely described the derivation of a linear PID control law from a fuzzy control theory. We first lead a linear PD control law from a fuzzy control algorithm with only two fuzzy sets for error and change-of-error, respectively. And then we derived a linear PID control law from a fuzzy controller. In this case we assumed that the intervals of error, change-of-error, and integral error could be partitioned into arbitrary numbers, respectively. And we used common triangular types with intersection points at the height 0.5 as membership functions for fuzzy sets of error, change-of-error, and integral error. We also showed that a linear PID control law can be derived for two cases: one is the case that membership functions for a fuzzy set of a control variable have isosceles triangular types and the other is the case that membership functions have common triangular types. In the first case we used the center-average method for defuzzification and the height method for the second case. Both cases used the product-sum method for reasoning of fuzzy rules.

Consequently a linear PID controller is only a sort of various fuzzy logic controllers. Furthermore, the control performance of fuzzy controller is better than that of a linear PID controller because the former can treat with a nonlinearity and some degree of uncertainty of a plant.

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