The Korean Communications
in Statistics Vol. 4, No. 3, 1997
pp. 663 - 676

Simultaneous Unit Roots Tests for Both Regular and Seasonal Unit Roots[1)]

Sinsup Cho[2)], Jeong Hyeong Lee[3)], Young Jin Park[4)] and Heon Jin Park[5)]

Abstract

We obtain the simultaneous unit roots test statistics for both regular and seasonal unit roots in a time series with possible seasonal deterministic trends. The limiting distributions of the proposed test statistics are derived and empirical percentiles of the test statistics are tabulated for some seasonal periods. The power and size of the test statistics are examined for finite samples through a Monte Carlo simulation and compared with those of the Lagrange multiplier test.

1. Introduction

Let us consider the time series model of the following form:

$$(1-\rho B)(1-\rho_d B^d) Y_t = \varepsilon_t, \tag{1.1}$$

where $Y_{-d}, \cdots, Y_0$ are initial conditions and the ε_t are independent random variables with mean 0 and variance σ^2. When $\rho = \rho_d = 1$, model (1.1) represents a nonstationary seasonal model with both a regular and a seasonal unit roots. For testing the null hypothesis, $H_0: \rho = \rho_d = 1$, Li (1991) suggested a test based on the LM principle and Hasza and Fuller (1982) considered the F-type test using the transformed regression model:

$$Y_t = \xi_1 Y_{t-1} + \xi_2 Y_{t-d} + \xi_3 Y_{t-d-1} + \varepsilon_t. \tag{1.2}$$

It is noted that (1.1) is the special case of (1.2) with $\xi_1 = \rho$, $\xi_2 = \rho_d$ and $\xi_3 = -\rho\rho_d$. Hence the null hypothesis of both a regular and a seasonal unit roots for model (1.2) is $(\xi_1, \xi_2, \xi_3) = (1, 1, -1)$. For convenience, model (1.2) is reparameterized in Hasza and Fuller

1) The present study was supported (in part) by the Basic Science Research Institute Program, Ministry of Education, 1995 Project No. 1418.

2) Professor, Department of Statistics, Seoul National University, Seoul, 151-742, Korea.
3) The Research Institute for Basic Sciences, Seoul National University, Seoul, 151-742, Korea.
4) Post-Doc., Department of Statistics, North Carolina State University, Raleigh, U.S.A.
5) Assistant Professor, Department of Statistics, Inha University, Incheon, 402-751, Korea.

(1982) as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 (Y_{t-1} - Y_{t-d-1}) + \phi_3 (Y_{t-d} - Y_{t-d-1}) + \varepsilon_t. \tag{1.3}$$

For model (1.3) the null hypothesis is $(\phi_1, \phi_2, \phi_3) = (1, 0, 1)$.

In this paper we consider the seasonal model with seasonal deterministic trends

$$Y_t = \sum_{j=1}^{d} (\alpha_j + \beta_j \tau) \delta_{jt} + \xi_1 Y_{t-1} + \xi_2 Y_{t-d} + \xi_3 Y_{t-d-1} + \varepsilon_t, \tag{1.4}$$

where $\tau = [(t-1)/d + 1]$ with $[x]$ denoting the largest integer no larger than x and δ_{jt} are seasonal indicator variables such that

$$\delta_{jt} = \begin{cases} 1 & \text{if } \ j \equiv (t-1) (\bmod d) + 1 \\ 0 & \text{otherwise.} \end{cases}$$

In the following sections we obtain the test statistics for simultaneous unit roots test for model (1.4) and derive their limiting distributions. The performance of the proposed test statistics will be compared with that of the test statistic based on the Lagrange multiplier principle, Park and Cho (1995). For this purpose we briefly outline the LM test. Consider the following seasonal model with possible seasonal deterministic trends

$$(1 - \rho B)(1 - \rho_d B^d) Y_t = \sum_{j=1}^{d} (\alpha_j + \beta_j \tau) \delta_{jt} + \varepsilon_t. \tag{1.5}$$

This model is also the special case of model (1.4) with $\xi_1 = \rho$, $\xi_2 = \rho_d$ and $\xi_3 = \rho \rho_d$. Park and Cho (1995) suggested the LM test statistic which does not depend on the nuisance parameters α_j's and β_j's, using the restricted maximum likelihood estimators $\tilde{\alpha}_j$'s and $\tilde{\beta}_j$'s obtained under the null hypothesis $(\rho, \rho_d) = (1, 1)$. LM test is based on the approximate log-likelihood function with $\sum_{t=1}^{n} \varepsilon_t^2$ in (1.5).

Let the parameter sets for model (1.5) be $\boldsymbol{\Lambda}^T = (\alpha_1, \cdots, \alpha_d, \beta_1, \cdots, \beta_d, \rho, \rho_d)$ and $\tilde{\boldsymbol{\Lambda}}^T = (\tilde{\alpha}_1, \ldots, \tilde{\alpha}_d, \tilde{\beta}_1, \ldots, \tilde{\beta}_d, 1, 1)$. Then the LM test statistic is given by $\tilde{\boldsymbol{P}}^T \tilde{\boldsymbol{Q}}^{-1} \tilde{\boldsymbol{P}}$ where $\tilde{\boldsymbol{P}} = \boldsymbol{P}(\tilde{\boldsymbol{\Lambda}})$ and $\tilde{\boldsymbol{Q}} = \boldsymbol{Q}(\tilde{\boldsymbol{\Lambda}})$ are the score vector and the Hessian matrix obtained subject to $\tilde{\boldsymbol{\Lambda}}$, respectively.

Following Hasza and Fuller (1982), model (1.4) can be reparameterized as

$$Y_t = \sum_{j=1}^{d} (\alpha_j + \beta_j \tau) \delta_{jt} + \phi_1 Y_{t-1} + \phi_2 (Y_{t-1} - Y_{t-d-1}) + \phi_3 (Y_{t-d} - Y_{t-d-1}) + \varepsilon_t. \tag{1.6}$$

In the followings we use model (1.6) instead of (1.4) for convenience.

In Section 2, we obtain the test statistics for simultaneous unit roots test of model (1.6). The limiting distributions are obtained in Section 3. To compare the performances of the proposed test statistics with the LM test statistic of Park and Cho (1995) we tabulate the empirical percentiles and powers in Section 4. Finally Section 5 contains an example.

2. Test Statistics and Preliminaries

To obtain the test statistic we consider regression type estimators and derive the limiting distributions under the following null hypotheses

$$H_{01}: (\phi_1, \phi_2, \phi_3) = (1, 0, 1), \; \alpha_j = \beta_j = 0, \text{ for } j = 1, \cdots, d. \tag{2.1}$$

$$H_{02}: (\phi_1, \phi_2, \phi_3) = (1, 0, 1) \tag{2.2}$$

To test the presence of both regular and seasonal unit roots, Li (1991) considered the null hypothesis H_{01} where the nuisance parameters are zeros. Unlike Li (1991), we also consider H_{02} as Hasza and Fuller (1979) did when $d=1$. We first concentrate on the case of H_{01} based on F-type statistic.

Let the parameter sets in (1.6) be

$$\Pi^T = (\alpha_1, \cdots, \alpha_d, \beta_1, \cdots, \beta_d, \phi_1, \phi_2, \phi_3),$$

and the design matrix be

$$\Psi_t^T = (\delta_{1t}, \cdots, \delta_{dt}, \tau\delta_{1t}, \cdots, \tau\delta_{dt}, Y_{t-1}, M_{t-1}, N_{t-d}),$$

where we define $N_t = Y_t - Y_{t-1}$ and $M_t = Y_t - Y_{t-d}$ as in Hasza and Fuller (1982). Then model (1.6) can be written as $Y_t = \Psi_t^T \Pi + \varepsilon_t$ and the OLS estimator of Π is given by

$$\hat{\Pi} = \left(\sum_{t=1}^{n} \Psi_t \Psi_t^T\right)^{-1} \sum_{t=1}^{n} \Psi_t Y_t.$$

We have under the null hypothesis

$$(\hat{\Pi} - \Pi) = H_n^{-1} h_n,$$

where $H_n = \sum_{t=1}^{n} \Psi_t \Psi_t^T$ and $h_n = \sum_{t=1}^{n} \Psi_t \varepsilon_t$.

Define the test statistic $\Phi^c_{a,b}$ using the same notation as in Hasza and Fuller (1982) where a corresponds to one of H_{01} and H_{02}, b stands for n-(the number of parameters in Π) and c stands for the number of parameters appeared in the null hypotheses H_{01} and H_{02}. Then

$$\Phi^{2d+3}_{1, n-2d-3} = \{(2d+3)\, \hat{\sigma}^2\}^{-1} h_n^T H_n^{-1} h_n, \tag{2.3}$$

where $\hat{\sigma}^2 = (n-2d-3)^{-1} \sum_{t=1}^{n} (Y_t - \hat{\Pi}^T \Psi_t)^2$. Details of (2.3) can be obtained in Dickey and Fuller (1981).

Since we compare the performance of the test for H_{02} with that of the LM test by Park and Cho (1995), we also briefly discuss how the test statistic for H_{02} is obtained. For this purpose we partition the matrix H_n^{-1} into 4 block matrices.

$$H_n^{-1} = \begin{bmatrix} G^{(11)} & G^{(12)} \\ G^{(12)T} & G^{(22)} \end{bmatrix},$$

where $G^{(11)}$ is a $2d \times 2d$ matrix, $G^{(12)}$ is a $2d \times 3$ matrix, and $G^{(22)}$ is a 3×3 matrix. Also we partition $\widehat{\Pi}$ as follows:

$$\widehat{\Pi} = \begin{bmatrix} \widehat{\Pi}^{(1)T} \\ \widehat{\Pi}^{(2)T} \end{bmatrix},$$

where $\widehat{\Pi}^{(1)}$ contains upper $2d$ rows and $\widehat{\Pi}^{(2)}$ contains lower 3 rows. Let $h_n^{(2)} = \widehat{\Pi}^{(2)} - (1, 0, 1)^T$. Then, the test statistic for H_{02} is

$$\Phi^3_{2, n-2d-3} = (3\, \hat{\sigma}^2)^{-1}\, h_n^{(2)T} G^{(22)}\, h_n^{(2)} \tag{2.4}$$

The limiting distributions of Φ-statistics are obtained following Hasza and Fuller (1979,1982) and are expressed as the functional forms of Brownian motions using Chan and Wei (1988).

Assume that Y_t satisfy

$$Y_t = Y_{t-1} + Y_{t-d} - Y_{t-d-1} + \varepsilon_t. \tag{2.5}$$

Then $N_t = Y_t - Y_{t-1}$ and $M_t = Y_t - Y_{t-d}$ can be expressed as follows:

$$N_t = N_{t-d} + \varepsilon_t = \sum_{k=0}^{[t/d]} \varepsilon_{t-dk}, \quad M_t = M_{t-1} + \varepsilon_t = \sum_{k=1}^{t} \varepsilon_k. \tag{2.6}$$

We need the following asymptotic behavior of sample moments of functions of N_t, M_t and Y_t to obtain the limiting distributions of the proposed test statistics (2.3) and (2.4). Details are given in the Appendix 2 of Park and Cho (1995). The followings are obtained, where $m = [n/d]$.

$$n^{-3/2} \sum_{k=1}^{m} N_{(k-1)d+j} \xrightarrow{L} d^{-3/2} \sigma \int_0^1 W_j(r)\,dr$$

$$n^{-1} \sum_{t=1}^{n} N_{t-d}\varepsilon_t \xrightarrow{L} d^{-1}\sigma^2 \sum_{j=1}^{d} (W_j(1)^2 - 1)/2$$

$$n^{-2} \sum_{t=1}^{n} N^2_{t-d} \xrightarrow{L} d^{-2}\sigma^2 \sum_{j=1}^{d} \int_0^1 W_j(r)^2\,dr$$

$$n^{-5/2} \sum_{k=1}^{m} kN_{(k-1)d+j} \xrightarrow{L} d^{-5/2} \sigma \int_0^1 rW_j(r)\,dr$$

$$n^{-3/2} \sum_{k=1}^{m} M_{(k-1)d+j} \xrightarrow{L} d^{-3/2} \sigma \sum_{i=1}^{j} \int_0^1 W_i(r)\,dr$$

$$n^{-5/2} \sum_{k=1}^{m} kM_{(k-1)d+j} \xrightarrow{L} d^{-5/2} \sigma \sum_{i=1}^{j} \int_0^1 rW_i(r)\,dr$$

$$n^{-1}\sum_{t=1}^{n} M_{t-1}\varepsilon_t \xrightarrow{L} \sigma^2(W(1)^2-1)/2$$

$$n^{-2}\sum_{t=1}^{n} M_{t-1}^2 \xrightarrow{L} \sigma^2\int_0^1 W(r)^2dr$$

$$n^{-2}\sum_{t=1}^{n} M_{t-1}N_{t-d} \xrightarrow{L} d^{-2}\sigma^2\sum_{j=1}^{d}\sum_{i=1}^{j}\int_0^1 W_i(r)W_j(r)dr$$

$$n^{-5/2}\sum_{k=1}^{m} Y_{(k-1)d+j} \xrightarrow{L} d^{-5/2}\sigma\sum_{i=1}^{j}\int_0^1\int_0^{r_2} W_i(r_1)dr_1dr_2$$

$$n^{-7/2}\sum_{k=1}^{m} kY_{(k-1)d+j} \xrightarrow{L} d^{-7/2}\sigma\sum_{i=1}^{j}\int_0^1 r_2\int_0^{r_2} W_i(r_1)dr_1dr_2$$

$$n^{-2}\sum_{t=1}^{n} Y_{t-1}\varepsilon_t \xrightarrow{L} d^{-2}\sigma^2\sum_{j=1}^{d}\sum_{i=1}^{j}\int_0^1\int_0^{r_2} W_i(r_1)dr_1dW_j(r_2)$$

$$n^{-4}\sum_{t=1}^{n} Y_{t-1}^2 \xrightarrow{L} d^{-4}\sigma^2\int_0^1\sum_{j=1}^{d}\{\sum_{i=1}^{j}\int_0^{r_2} W_i(r_1)dr_1\}^2dr_2$$

$$n^{-3}\sum_{t=1}^{n} Y_{t-1}N_{t-d} \xrightarrow{L} d^{-3}\sigma^2\sum_{j=1}^{d}\int_0^1\{\sum_{i=1}^{j}\int_0^{r_2} W_i(r_1)dr_1\}W_j(r_2)dr_2$$

$$n^{-3}\sum_{t=1}^{n} Y_{t-1}M_{t-1} \xrightarrow{L} d^{-2}\sigma^2\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{j}\{\int_0^{r_2} W_i(r_1)dr_1\}W_j(r_2)dr_2.$$

3. Limiting Distributions

To obtain the limiting distributions of (2.3) and (2.4), we premultiply $\boldsymbol{H}_n$ and $\boldsymbol{h}_n$ by the scaling matrix $D_n = diag(D_{1n}, D_{2n})$ where $D_{1n} = diag(n^{-1/2}, \dots, n^{-1/2}, n^{-3/2}, \dots, n^{-3/2})$ and $D_{2n} = diag(n^{-2}, n^{-1}, n^{-1})$.

$$D_n(\sum_{t=1}^{n}\boldsymbol{\Psi}_t\varepsilon_t) = (D_{1n}(\sum_{t=1}^{n}\boldsymbol{\Psi}_{1t}\varepsilon_t), n^{-2}\sum_{t=1}^{n} Y_{t-1}\varepsilon_t, n^{-1}\sum_{t=1}^{n} M_{t-1}\varepsilon_t, n^{-1}\sum_{t=1}^{n} N_{t-d}\varepsilon_t) \quad (3.1)$$

where

$$D_{1n}(\sum_{t=1}^{n}\boldsymbol{\Psi}_{1t}\varepsilon_t) = (n^{-1/2}\sum_{k=1}^{m}\varepsilon_{(k-1)d+1}, \dots, n^{-1/2}\sum_{k=1}^{m}\varepsilon_{kd}, n^{-3/2}\sum_{k=1}^{m}k\varepsilon_{(k-1)d+1}, \dots, n^{-3/2}\sum_{k=1}^{m}k\varepsilon_{kd}).$$

Also we have

$$D_n\left(\sum_{t=1}^{n} \Psi_t \Psi_t^T\right)D_n = \begin{bmatrix} A_n & B_n & \vdots & C_n \\ B_n^T & G_n & \vdots & E_n \\ \cdots & \cdots & \cdots & \cdots \\ C_n^T & E_n^T & \vdots & F_n \end{bmatrix} \tag{3.2}$$

where I is a $d \times d$ identity matrix,

$$A_n = n^{-1} \sum_{k=1}^{m} 1I, \quad B_n = n^{-2} \sum_{k=1}^{m} kI, \quad G_n = n^{-2} \sum_{k=1}^{m} k^2 I,$$

$$C_n = \begin{bmatrix} n^{-5/2} \sum_{k=1}^{m} Y_{(k-1)d+1} & n^{-3/2} \sum_{k=1}^{m} M_{(k-1)d+1} & n^{-3/2} \sum_{k=1}^{m} N_{(k-1)d+1} \\ \vdots & \vdots & \vdots \\ n^{-5/2} \sum_{k=1}^{m} Y_{kd} & n^{-3/2} \sum_{k=1}^{m} M_{kd} & n^{-3/2} \sum_{k=1}^{m} N_{kd} \end{bmatrix}$$

$$E_n = \begin{bmatrix} n^{-7/2} \sum_{k=1}^{m} kY_{(k-1)d+1} & n^{-5/2} \sum_{k=1}^{m} kM_{(k-1)d+1} & n^{-5/2} \sum_{k=1}^{m} kN_{(k-1)d+1} \\ \vdots & \vdots & \vdots \\ n^{-7/2} \sum_{k=1}^{m} kY_{kd} & n^{-5/2} \sum_{k=1}^{m} kM_{kd} & n^{-5/2} \sum_{k=1}^{m} kN_{kd} \end{bmatrix}$$

and

$$F_n = \begin{bmatrix} n^{-4} \sum_{t=1}^{n} Y_t^2 & n^{-3} \sum_{t=1}^{n} Y_{t-1} M_{t-1} & n^{-3} \sum_{t=1}^{n} Y_{t-1} N_{t-d} \\ & n^{-2} \sum_{t=1}^{n} M_{t-1}^2 & n^{-2} \sum_{t=1}^{n} M_{t-1} N_{t-d} \\ & & n^{-2} \sum_{t=1}^{n} N_{t-d}^2 \end{bmatrix}.$$

The first block of matrix (3.2) represents $D_{1n}\left(\sum_{t=1}^{n} \Psi_{1t} \Psi_{1t}^T\right)D_{1n}$.

Theorem 1 Let Y_t satisfy (2.5), and N_t and M_t be defined by (2.6). Then we have

$$D_n\left(\sum_{t=1}^{n} \Psi_t \varepsilon_t\right) \xrightarrow{L} h \quad \text{and} \quad D_n\left(\sum_{t=1}^{n} \Psi_t \Psi_t^T\right)D_n \xrightarrow{L} H, \tag{3.3}$$

$$D_{1n}\left(\sum_{t=1}^{n} \Psi_{1t} \varepsilon_t\right) \xrightarrow{L} r \quad \text{and} \quad D_{1n}\left(\sum_{t=1}^{n} \Psi_{1t} \Psi_{1t}^T\right)D_{1n} \xrightarrow{L} R, \tag{3.4}$$

where

$$h^T = (r^T, d^{-2}\sigma^2 \sum_{j=1}^{d} \sum_{i=1}^{j} \int_0^1 r_2 \int_0^{r_2} W_i(r_1)dr_1 dW_j(r_2), \frac{W(1)^2-1}{2}, d^{-1} \sum_{j=1}^{d} \frac{W_j(1)^2-1}{2})$$

and

$$r^T = (d^{-1/2}W_1(1), \dots, d^{-1/2}W_d(1), \quad d^{-3/2}\int_0^1 rW_1(r)dr, \dots, d^{-3/2}\int_0^1 rW_d(r)dr)$$

and the components of H consist of terms which are the limiting distributions of A_n, B_n, C_n, E_n, F_n and G_n, and the components of R consist of terms which are the limiting distributions of A_n, B_n and G_n.

Proof : The results can be obtained by the application of the asymptotic behaviour of the sample moments given in Section 3.

We obtain the following theorem for $\Phi^c_{a,b}$ under H_{01} and H_{02}.

Theorem 2 Let the assumptions of Theorem 1 hold. Then

(1) $\Phi^{2d+3}_{1,n-2d-3} \xrightarrow{L} \frac{1}{(2d+3)\sigma^2} h^T H^{-1} h$

(2) $\Phi^{3}_{2,n-2d-3} \xrightarrow{L} \frac{1}{3\sigma^2}(h^T H^{-1} h - R_1)$,

where h and H are defined by (3.3) and $R_1 = r' R^{-1} r$ since r and R are defined by (3.4). The explicit form of R_1 is given by

$$R_1 = 4 \sum_{j=1}^{d} \{W_j(1)^2 + 3(\int_0^1 W_j(r)dr)^2 - 3\,W_j(1)\int_0^1 W_j(r)dr\}.$$

The proof of Theorem 2 is obtained using Theorem 1 and the results of Chan and Wei (1988). It is noted that when $d=1$, (1) and (2) are equivalent to (v) and (iv) of Corollary 3.1 of Hasza and Fuller (1979), respectively.

Since the test statistics depend on the null hypotheses, we need a priori information about nuisance parameters to choose a proper statistic. Otherwise, it will be safe to use the test statistic (2) of Theorem 2, which does not depend on the presence of nuisance parameters.

Hasza and Fuller (1982) extended model (1.3) by including the lagged variables $(1-B)(1-B^d)Y_{t-k}$ and showed that the percentage points of the test statistics for model (1.3) are still applicable for the extended model. Similarly we can extend model (1.6) by including the lagged variables $(1-B)(1-B^d)Y_{t-k}$ in (3.5).

$$\begin{aligned} Y_t &= \sum_{j=1}^{d}(\alpha_j + \beta_j t)\delta_{jt} + \phi_1 Y_{t-1} + \phi_2(Y_{t-1} - Y_{t-d-1}) + \phi_3(Y_{t-d} - Y_{t-d-1}) \\ &\quad + \sum_{k=1}^{p} \theta_k (1-B)(1-B^d) Y_{t-k} + \varepsilon_t. \end{aligned} \tag{3.5}$$

It is not difficult to show that the asymptotic distributions of the test statistics for (2.1) and (2.2) in model (3.5) are equivalent to those of the asymptotic distributions of Theorem 2.

4. Simulation Results

Empirical percentiles of the test statistics in (1)-(2) of Theorem 2 have been obtained for $d=2,4,12$ and m=10, 15, 20, 25, 50, 100, 200 in Tables 1 and 2. These results have been obtained from simulation, where the ε_t are generated by the RNNOA subroutine of IMSL and are based on 30,000 replications. The initial conditions are set to zeros as in Dickey and Fuller (1979). The empirical percentiles were smoothed by regression on $\exp(-x)$.

To examine the power of the proposed test statistics under the various alternatives we perform the simulation study. For the simulation we consider the following model

$$Y_t = \sum_{j=1}^{4} b_j \tau \delta_{jt} + N_t \qquad (4.1)$$
$$(1-\rho B)(1-\rho_4 B^4) N_t = \varepsilon_t .$$

By applying the filter $(1-\rho B)(1-\rho_4 B^4)$ to Y_t in (4.1), we can see that the model (4.1) can be represented in the form of (1.6).

For model (1.6) the powers of the tests for null hypotheses H_{01} and H_{02} are summarized in Tables 3-5 for all combinations of ρ and ρ_4=0.8, 0.9, 0.95, 0.98, 0.99, 0,995, 1.0. Here $\rho=1$ and $\rho_4=1$ are considered in order to study the size of the test for finite samples. For the values of nuisance parameters, power is evaluated at $\beta_1=1$, $\beta_2=3, \beta_3=4$ and $\beta_4=2$. Samples of size $n=100$ are generated using the RNNOA subroutine of IMSL to generate pseudo random samples of the ε_t from a standard normal distribution. Based on 10,000 replications, the power of the test is obtained at the significance level 0.05. We include the result of Park and Cho(1995) in Table 5 so that the performance of the proposed test statistic (2) in Theorem 2 can be compared to that of the LM test statistic.

In Tables 3 and 5 we observe that the power decreases as the value of ρ_4 gets smaller when ρ is less than or equal to 0.95. But when ρ is larger than 0.95 the tendency is reversed, i.e., the power decreases as the value of ρ_4 decreases.

For Table 4, the power tends to increase as both ρ and ρ_4 move away from one. But, for Table 3, the power tends to increase when one of ρ and ρ_4 gets close to one and another moves away from one. The result may be due to a relatively large initial value. For a large initial values, the model acts like the model with non-zero mean. But when ρ or ρ_4 is one or close to one, the mean effect is diminished and the effect of the large initial value may be reduced.

Table 1 Percentiles of $\Phi^{2d+3}_{1,n-2d-3}$ Statistic for the Seasonal Trends Model

		Probability of a Smaller Value								
d	$n=md$	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
2	20	0.9951	1.1426	1.2943	1.5197	2.7043	4.8651	5.6520	6.4423	7.4691
2	30	1.0791	1.2231	1.3671	1.5574	2.4943	4.3629	5.0940	5.8414	6.8169
2	40	1.1074	1.2471	1.3860	1.5643	2.4146	4.0320	4.7040	5.4009	6.3197
2	50	1.1177	1.2540	1.3911	1.5660	2.3846	3.8151	4.4306	5.0777	5.9409
2	100	1.1220	1.2574	1.3929	1.5660	2.3675	3.4509	3.9009	4.3766	5.0366
2	200	1.1220	1.2574	1.3929	1.5660	2.3657	3.4003	3.7963	4.1991	4.7473
2	400	1.1220	1.2574	1.3929	1.5660	2.3657	3.4003	3.7937	4.1846	4.7229
4	40	1.1723	1.3139	1.4454	1.6123	2.4592	3.7846	4.2715	4.7269	5.2746
4	60	1.2008	1.3246	1.4392	1.5854	2.2815	3.3892	3.8192	4.2300	4.7354
4	80	1.2054	1.3262	1.4385	1.5823	2.1908	3.1739	3.5500	3.9169	4.3739
4	100	1.2062	1.3262	1.4385	1.5815	2.1954	3.0554	3.3892	3.7185	4.1315
4	200	1.2069	1.3262	1.4385	1.5815	2.1831	2.9200	3.1708	3.4123	3.7062
4	400	1.2069	1.3262	1.4385	1.5815	2.1831	2.9131	3.1539	3.3785	3.6408
4	800	1.2069	1.3262	1.4385	1.5815	2.1831	2.9131	3.1539	3.3785	3.6392
12	120	1.5923	1.7115	1.8169	1.9531	2.5139	3.2539	3.5046	3.7377	4.0031
12	180	1.5539	1.6539	1.7469	1.8608	2.3323	2.9508	3.1615	3.3600	3.5908
12	240	1.5477	1.6415	1.7308	1.8385	2.2685	2.8139	2.9954	3.1669	3.3715
12	300	1.5462	1.6392	1.7277	1.8323	2.2454	2.7523	2.9154	3.0685	3.2554
12	600	1.5462	1.6385	1.7262	1.8308	2.2331	2.7023	2.8423	2.9700	3.1300
12	1200	1.5462	1.6385	1.7262	1.8308	2.2331	2.7008	2.8400	2.9662	3.1246
12	2400	1.5462	1.6385	1.7262	1.8308	2.2331	2.7008	2.8400	2.9662	3.1246

Table 2 Percentiles of $\Phi^3_{2,n-2d-3}$ Statistic for the Seasonal Trends Model

		Probability of a Smaller Value								
d	$n=md$	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
2	20	0.6553	0.9133	1.1687	1.5100	3.2680	5.8993	6.9513	9.1480	10.767
2	30	0.7980	1.0840	1.3607	1.7160	3.3007	5.8653	6.8927	8.1960	9.711
2	40	0.8820	1.1753	1.4553	1.8073	3.3040	5.8313	6.8340	7.5787	8.968
2	50	0.9313	1.2240	1.5027	1.8473	3.3040	5.7980	6.7767	7.1780	8.444
2	100	0.9960	1.2780	1.5473	1.8793	3.3040	5.6373	6.5053	6.5247	7.413
2	200	1.0007	1.2807	1.5487	1.8800	3.3040	5.3433	6.0280	6.4413	7.204
2	400	1.0007	1.2807	1.5487	1.8800	3.3040	4.8487	5.2913	6.4407	7.197
4	40	1.3987	1.7640	2.5433	2.6347	4.8860	8.2960	9.5620	10.771	12.361
4	60	1.6507	2.0447	2.5433	2.9267	4.9553	7.6813	8.7107	9.723	11.085
4	80	1.7827	2.1813	2.5433	3.0413	4.9640	7.4587	8.3440	9.221	10.405
4	100	1.8520	2.2473	2.5433	3.0867	4.9647	7.3773	8.1860	8.981	10.043
4	200	1.9253	2.3080	2.5433	3.1153	4.9653	7.3320	8.0687	8.765	9.649
4	400	1.9287	2.3100	2.5433	3.1160	4.9653	7.3313	8.0667	8.760	9.631
4	800	1.9287	2.3100	2.5433	3.1160	4.9653	7.3313	8.0667	8.760	9.631
12	120	6.114	6.8533	7.5193	8.3313	11.549	15.920	17.432	18.981	20.803
12	180	6.114	6.8533	7.5193	8.3313	11.549	15.377	16.641	17.866	19.307
12	240	6.114	6.8533	7.5193	8.3313	11.549	15.253	16.439	17.527	18.778
12	300	6.114	6.8533	7.5193	8.3313	11.549	15.220	16.387	17.425	18.591
12	600	6.114	6.8533	7.5193	8.3313	11.549	15.221	16.369	17.380	18.489
12	1200	6.114	6.8533	7.5193	8.3293	11.549	15.221	16.369	17.380	18.488
12	2400	6.114	6.8533	7.5193	8.3313	11.549	15.221	16.369	17.380	18.488

Table 3 Empirical Power of Size 0.05 Test for $\Phi_{1,n-2d-3}^{2d+3}$

$\rho \setminus \rho_d$	0.8	0.9	0.95	0.98	0.99	0.995	1.0
0.8	.8633	.9656	.9960	1.0	1.0	1.0	1.0
0.9	.1627	.1731	.3659	.6250	.7039	.7492	.7890
0.95	.3008	.1486	.0942	.1262	.1494	.1622	.1712
0.98	.2854	.1277	.0663	.0613	.0536	.0616	.0539
0.99	.2132	.1179	.0745	.0584	.0535	.0543	.0519
0.995	.1648	.1190	.0769	.0634	.0569	.0559	.0500
1.0	.1321	.1173	.0851	.0603	.0528	.0520	.0486

Table 4 Empirical Power of Size 0.05 Test for $\Phi_{2,n-2d-3}^{3}$

$\rho \setminus \rho_d$	0.8	0.9	0.95	0.98	0.99	0.995	1.0
0.8	.6969	.6046	.5780	.4622	.3442	.2671	.1860
0.9	.1848	.1649	.1530	.1567	.1404	.1324	.1232
0.95	.2266	.1582	.1071	.0810	.0697	.0721	.0625
0.98	.1926	.1057	.0655	.0603	.0515	.0558	.0502
0.99	.1338	.0794	.0538	.0502	.0533	.0537	.0450
0.995	.0984	.0624	.0568	.0535	.0503	.0540	.0468
1.0	.0727	.0597	.0536	.0513	.0499	.0513	.0503

Table 5 Empirical Power of Size 0.05 Test for LM

$\rho \setminus \rho_d$	0.8	0.9	0.95	0.98	0.99	0.995	1.0
0.8	.4641	.5691	.6834	.7838	.8267	.8397	.8666
0.9	.3013	.3618	.4272	.4961	.5401	.5574	.5841
0.95	.2203	.2175	.2374	.2605	.2725	.2795	.2945
0.98	.1755	.1515	.1224	.1016	.0928	.0892	.0891
0.99	.1676	.1158	.0782	.0619	.0566	.0545	.0575
0.995	.1614	.1005	.0679	.0518	.0479	.0473	.0452
1.0	.1671	.0875	.0523	.0506	.0505	.0425	.0448

5. Numerical Example

The proposed test of multiple unit roots is applied to the quarterly Korean GNP series over the period 1970 through 1991, whose plot showes a seasonal pattern with period 4, Figure 1.

The plot shows the series may contain both the stochastic and deterministic trends. Hence for this quarterly data we fit model (3.5) with $p=4$ and include at least four lags. The values of the test statistics Φ_1 and Φ_2 together with the estimation results for ϕ_i's, $\widetilde{\alpha}_j$'s, $\widetilde{\beta}_j$'s and θ_k's for $i=1,2,3$, $j=1,2,3,4$ and $k=1,2,3,4$ are summarized in Table 6.

From Table 6 we observe that H_{01} is rejected while H_{02} is not rejected. The acceptance of H_{02} together with the significant α_j's imply that the rejection of H_{01} may be due to nonzero nuisance parameters. Since the seasonal deterministic trends are not significant we consider the reduced model without these terms. Test statistics for the reduced model can be easily obtained following Corollary 4.1 of Hasza and Fuller (1982). The test statistics for the reduced model in Table 7 are denoted by $R\Phi_1^{d+3}$ and $R\Phi_2^3$ for convenience and the critical values are read from Table 5.1 of Hasza and Fuller (1982). From Table 7 we observe that the null hypothesis of simultaneous unit roots together with zero seasonal deterministic levels is again rejected while H_{02} is not rejected. The estimation results lead us to conclude that the Korean GNP series contains both the regular and seasonal unit roots together with the nonzero deterministic seasonal levels. As a conclusion if we do not have a priori information about the existence of nuisance parameters and our interest is only to determine whether the series contains simultaneous unit roots it is safe to use the test statistic (2) of Theorem 2.

Table 6 Estimation results of the Korean GNP data in model (3.5)}

ϕ_i	0.978 (0.056)	-0.208 (0.149)	0.427 (0.122)	
$\widetilde{\alpha}_j$	-1770.013 (539.288)	863.074 (459.829)	218.337 (454.918)	2666.989 (531.461)
$\widetilde{\beta}_j$	-64.248 (83.960)	75.333 (71.482)	84.548 (75.878)	64.713 (82.236)
$\widetilde{\theta}_k$	-0.025 (0.177)	0.092 (0.116)	0.124 (0.116)	-0.025 (0.111)
Φ_i	$\Phi_1^{2d+3}=4.404$	$\Phi_2^3=6.202$		
c-value	4.119	8.281		

Standard errors of estimators are in parentheses and 'c-value' means the critical value at significance level 0.05.

Table 7 Estimation results of the Korean GNP data without the seasonal deterministictrends

ϕ_i	1.000 (0.014)	-0.302 (0.121)	0.649 (0.082)	
$\tilde{\alpha}_j$	-1838.395 (544.692)	1041.139 (310.480)	680,262 (299.738)	2133.783 (438.273)
$\tilde{\theta}_k$	0.017 (0.111)	0.145 (0.110)	0.163 (0.109)	-0.100 (0.103)
Φ_i	$R\Phi_1^{2d+3}=5.259$	$R\Phi_2^3=5.671$		
c-value	3.93	8.96		

Standard errors of estimators are in parentheses and 'c-value' means the critical value at significance level 0.05.

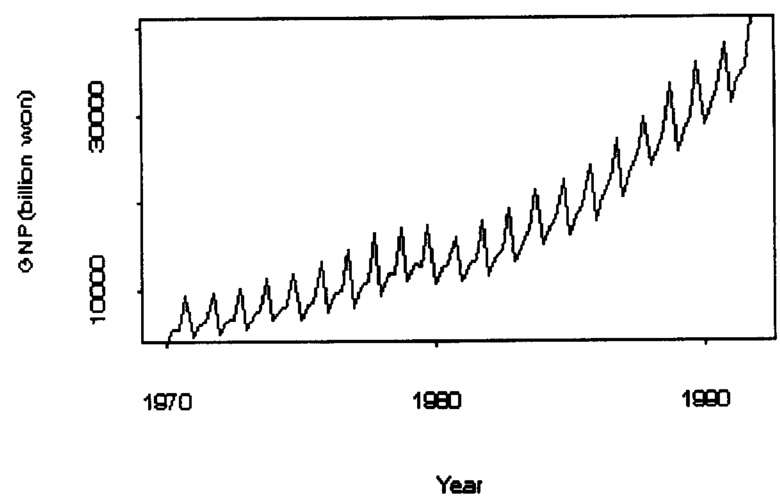

Figure 1 Time Series Plot of the Korean G.N.P. Series

Acknowledgement

We would like to thank to Professor Sung K. Ahn and anonymous refree for helpful comments which lead the significant improvements over the earlier version of the draft.

References

[1] Ahn, S.K. and Cho, S. (1993). A Note on Tests for Seasonal Unit Roots in the Presence of Deterministic Trends, *Journal of the Korean Statistical Society*, Vol. 22, 1, 113-124.

[2] Chan, N.H. and Wei, C.Z. (1988). Limiting Distributions of Least Squares Estimates of Unstable Autoregressive Processes. *The Annals of Statistics*, Vol. 16, 367-401.

[3] Dickey, D.A. and Fuller, W.A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *The Journal of American Statistical Association*, Vol. 74, 427-431.

[4] Dickey, D.A. and Fuller, W.A. (1981). Likelihood Ratio Test Statistics for Autoregressive Time Series with a Unit Root, *Econometrica*, Vol. 49, 1057-1072.

[5] Hasza, D.P. and Fuller, W.A. (1979). Estimation for Autoregressive Processes with Unit Roots, *The Annals of Statistics*, Vol. 7, 1106-1120.

[6] Hasza, D.P. and Fuller, W.A. (1982). Testing for Nonstationary Parameter Specifications in Seasonal Time Series Models, *The Annals of Statistics*, Vol. 10, 1209-1216.

[7] Li, W.K. (1991). Some Lagrange Multiplier Tests for Seasonal Differencing, *Biometrika*, Vol. 78, 49-56.

[8] Park Y.J. and Cho S. (1995). Lagrange Multiplier Test for both Regular and Seasonal Unit Roots, *The Korean Communications in Statistics*, Vol. 2, 101-114.