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The Minimum Dwell Time Algorithm for the Poisson Distribution and the Poisson-power Function Distribution[1)]

Joo-Hwan Kim[2)]

Abstract

We consider discrimination curve and minimum dwell time for Poisson distribution and Poisson-power function distribution. Let the random variable X has Poisson distribution with mean λ. For the hypothesis testing $H_0: \lambda = t$ vs. $H_1: \lambda = d$ $(d < t)$, the optimal decision rule is; reject H_0 if $X \leq c$. Since a critical value c can not be determined to satisfy both types of errors α and β, we considered discrimination curve that gives the maximum d such that it can be discriminated from t for a given α and β. We also considered an algorithm to compute the minimum dwell time which is needed to discriminate at the given α and β for the Poisson counts and proved its convergence property.

For the Poisson-power function distribution, we reject H_0 if $X \leq c'$. Since a critical value c' can not be determined to satisfy both α and β, similar to the Poisson case we considered discrimination curve and computation algorithm to find the minimum dwell time for the Poisson-power function distribution. We present this algorithm and an example of computation. It is found that the minimun dwell time algorithm fails for the Poisson-power function distribution if the aiming error variance σ_2^2 is too large relative to the variance σ_1^2 of the Gaussian distribution of intensity. In other words, if ℓ is too small, we can not find the minimum dwell time for a given α and β.

1. Introduction

Let the random variable X be the number of counts for a fixed time interval, and suppose it has a Poisson distribution with mean λ. Consider a hypothesis testing $H_0: \lambda = t$ versus

1) This research was supported by Korea Science and Engineering Foundation 1996, 961-0105-032-1.
2) Assistant Professor, Department of Computer Science & Statistics, Dongguk University, Kyong-Ju, 780-714, Korea

$H_1: \lambda = d$ $(d < t)$. The most powerful test with given probability of Type I error α is to be a left-tail test : Reject H_0, if $X \leq c$, where the integer critical value c is chosen to satisfy α. For the present test, the probability of Type II error is $\beta = P(\text{accept } H_0 \mid H_1 \text{ is true})$. Note that we intend that our errors for testing are low.

In radar applications, the quantity α is the probability of leakage (or leakage rate). This is the most serious risk and should be made as small as possible. The quantity β is the probability of false alarm (or false alarm rate).

Since we assume the counts X has a Poisson distribution, the two types of errors are

$$\alpha = \sum_{x=0}^{c} \frac{e^{-t} t^x}{x!} \tag{1.1}$$

and

$$\beta = 1 - \sum_{x=0}^{c} \frac{e^{-d} d^x}{x!} . \tag{1.2}$$

One can select c as the largest integer so that $P[X \leq c] \leq \alpha$. With c now specified, there is a unique d that matches the given β. Note that, for a fixed t, a small d gives a smaller β. However, in general, a critical value c can not be determined to satisfy both α and β that are specified. To describe the operating characteristics of this test, instead of minimizing β, we determine the largest d that can be discriminated from the given t with given α and β.

Since c is an integer, there is an interval of t's that, for the given α, yields the same c and hence d. Hence, plotting this maximum d versus t for the given α and β gives a step function increasing in t with small steps parameterized with the test of hypothesis critical values $c = 0, 1, 2, \cdots$. The plot of d versus t is called the *discrimination curve* (Beyer et al. (1987)), where d is the maximum mean counts such that d can be discriminated from t for a given α and β. The minimum dwell time τ^* is obtained from the intersection of the dwell time line with the discrimination possible region in the discrimination curve.

In section 2, we study the discrimination curve for Poisson distribution and present some results as figures. In section 3, we present discrimination curve and the method of its computation for the Poisson-power function distribution. In section 4, we investigate the minimum dwell time algorithm for the Poisson distribution and we show that this algorithm process must terminate. In section 5, we consider an algorithm for the minimum dwell time and actual errors with minimum dwell time for the Poisson-power function distribution. We present an example of computation for the minimum dwell time algorithm.

2. Discrimination Curve for Poisson

The discrimination curve for the Poisson distribution comes from the following well-known probability theory. Consider a Poisson stochastic process $(X(\tau) | 0 \le \tau < \infty)$, where each $X(\tau)$ has a Poisson distribution with parameter $\lambda\tau$, $\lambda > 0$. For a definition and statement of properties of a Poisson stochastic process, see Cox and Isham (1980). It is not necessary to know that the process of generating counts is a Poisson process; it only necessary to know that the random counts X has a Poisson distribution.

Let W_i be the waiting time between the $(i-1)^{st}$ count and the i^{th} count. Then the waiting time W_i is independent and exponentially distributed with mean $1/\lambda$. Now

$$P[X(\tau) \le c] = P\{W_1 + W_2 + \cdots + W_{c+1} > \tau\}. \tag{2.1}$$

Also the quantities $2\lambda W_i$ have a χ^2 distribution with 2 d.f., so that $2\lambda \sum_{i=1}^{c+1} W_i \sim \chi^2_{2(c+1)}$. Then equation (2.1) becomes

$$P[X(\tau) \le c] = P\{\chi^2_{2(c+1)} > 2\lambda\tau\}. \tag{2.2}$$

Under H_0, let $X = X(\tau)$ with $\lambda\tau = t$. Then

$$P[X(\tau) \le c] = P\{\chi^2_{2(c+1)} > 2t\} = 1 - F_{2(c+1)}(2t), \tag{2.3}$$

where $F_{2(c+1)}$ is the continuous c.d.f. for a χ^2 distributed random variable with 2(c+1) d.f. Therefore, from (2.3), for a given α

$$t = \frac{1}{2} F^{-1}_{2(c+1)}(1-\alpha). \tag{2.4}$$

Similarly, for a given β,

$$d = \frac{1}{2} F^{-1}_{2(c+1)}(\beta). \tag{2.5}$$

The plot of d versus t is called the discrimination curve, where d is the maximum mean counts such that d can be discriminated from t for a given α and β. The values of (t, d) computed is called an operating point. If the operating point (t, d) is below the discrimination curve then t and d can be discriminated for a given α and β; and if not, then they can not be statistically discriminated. We say the regions as "discrimination possible region" and "discrimination impossible region", respectively.

We can now obtain curves of operating points. Note that the bottom left-most point $c=0$ has the coordinates $(t, d) = (-\log\alpha, -\log(1-\beta))$. We make this step function will also joint the bottom left-most point to the horizontal axis.

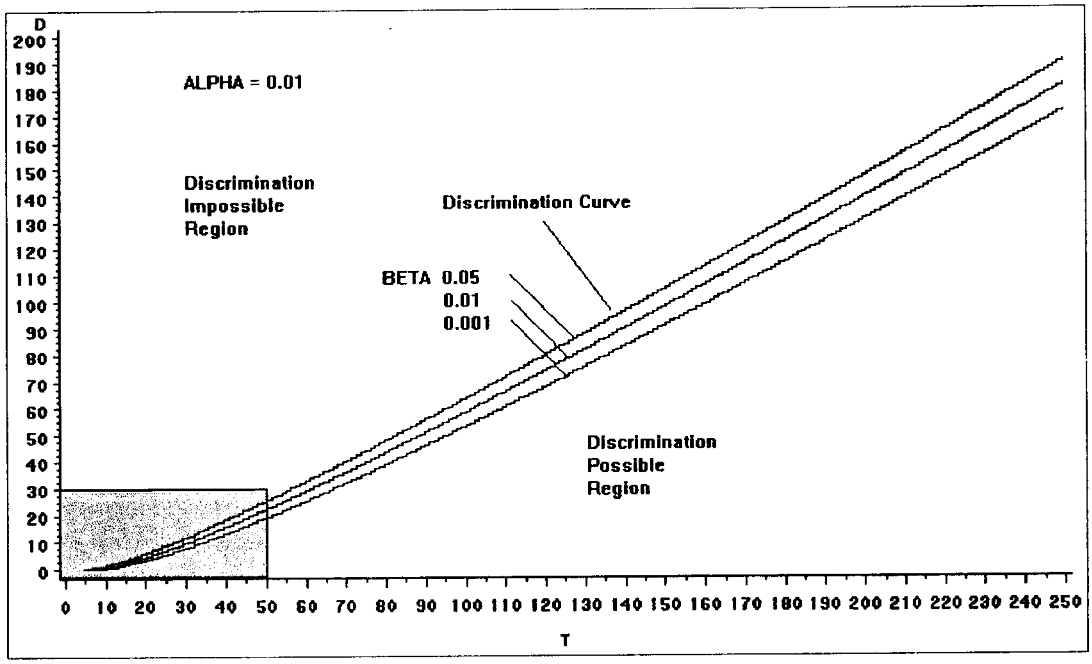

Figure 1A. Discrimination Curve for a Given α and β

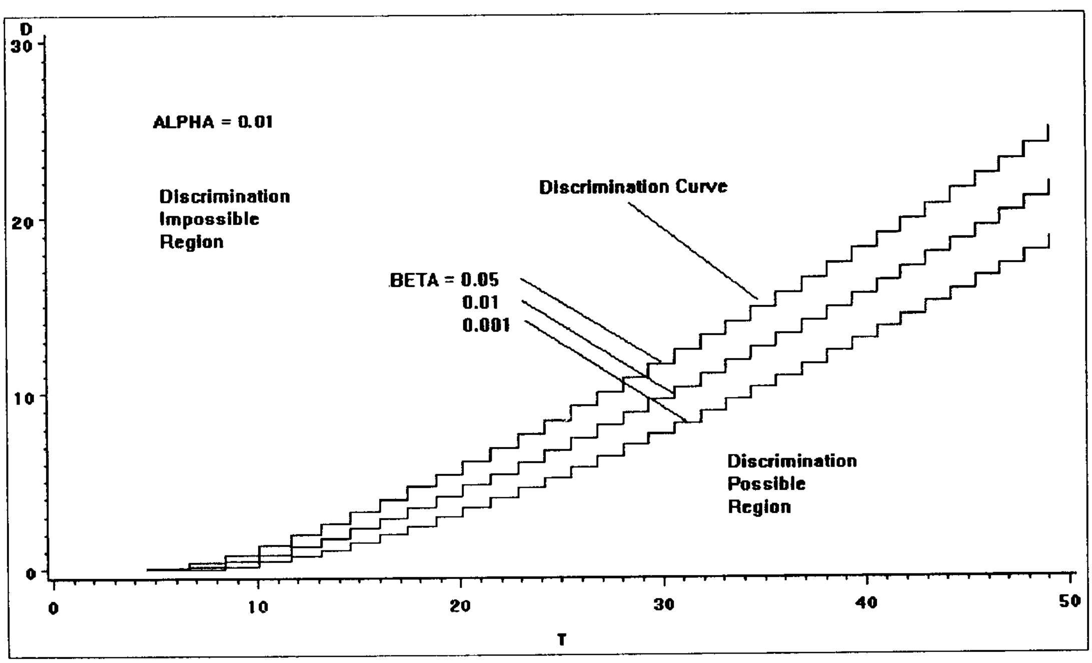

Figure 1B. Detailed Discrimination Curve for a Given α and β

Figure 1A shows graphs of discrimination curve for $\alpha = 0.01$ and various values of $\beta = (0.05, 0.01, 0.001)$. Figure 1B is the detailed graph in the box in Figure 1A. Note that we can compute (2.4) and (2.5) via $d =$ GAMINV$(\beta, c+1)$ and $t =$ GAMINV$(1-\alpha, c+1)$, respectively as a function of c, where GAMINV is a SAS quantile function (SAS 1991). Note also that we can compute d and t using CINV which is a chi-square inverse function: $d = 0.5*$CINV$(\beta, 2(c+1))$ and $t = 0.5*$CINV$(1-\alpha, 2(c+1))$.

3. Poisson-power Function Distribution and Discrimination Curve

In an application of radar discrimination problem, Kim(1991) considered aiming errors of the beam which are deviations between the center of the beam and the center of the object for an object interrogation and make the following two assumptions about aiming errors:

(i) The beam has a circular Gaussian distribution of intensity with standard deviation σ_1. This distribution is on a plane perpendicular to the beam axis.

(ii) Aiming errors yield a circular Gaussian distribution of the beam axis relative to the object center. The standard deviation of the distribution is σ_2.

Kim(1994) derived the exact probability distribution of the counts in presence of aiming errors. We shall first review the distribution briefly. Beckman and Johnson(1987) give evidence from an experiment that the beam has a Pearson Type VII distribution instead of a circular Gaussian distribution in assumption (i). This distribution is much heavier in the tails than is the Gaussian. Kim(1994) compared a circular Gaussian distribution with a Pearson Type VII distribution for scattering distribution of the neutron counts.

Under the assumption of a Poisson distribution of counts and aiming errors, the probability of exactly x neutron counts, $x = 0, 1, 2, \cdots$ being counted is

$$P(x|\lambda) = \frac{1}{x!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\lambda}\lambda^x e^{-(\omega_1^2+\omega_2^2)/(2\sigma_2^2)} \frac{d\omega_1 d\omega_2}{2\pi\sigma_2^2} \qquad (3.1)$$

where λ is defined by

$$\lambda = k \cdot e^{-(\omega_1^2+\omega_2^2)/(2\sigma_1^2)} \qquad (3.2)$$

where k represents the mean return neutron counts without aiming errors. In (3.2) σ_1 is a standard deviation of the circular Gaussian intensity distribution of the beam at the object, and (ω_1, ω_2) are coordinates of points on beam cross section. σ_2 in (3.1) is a standard deviation of the circular Gaussian aiming error distribution of the beam relative to the object.

Using the polar coordinates transformation, and letting

$$\ell = (\sigma_1/\sigma_2)^2 \qquad (3.3)$$

we obtain

$$P(x|\lambda) = \frac{\ell}{k^{\ell} x!} \gamma(x+\ell\,; k), \tag{3.4}$$

where

$$\gamma(\nu; k) = \int_0^k t^{\nu-1} e^{-t}\, dt \tag{3.5}$$

is the incomplete gamma function.

We have defined in (3.2) that k be the mean number of return neutron signals counted with the assumption that no aiming errors are made in the measurement of the parameters and that the beam is perfectly centered on the object. In this case, $\lambda = k$.

The probability distribution in (3.4) can be written as followings:

$$\begin{aligned} P(x; k, \ell) &= \frac{\ell}{k^{\ell} x!} \int_0^k e^{-\omega} \omega^{x+\ell-1}\, d\omega \\ &= \frac{1}{x!} E_{\omega}(e^{-\omega} \omega^{x}) \end{aligned}$$

where E_{ω} represents expected value of ω, and ω has a probability distribution

$$f(\omega) = \ell k^{-\ell} \omega^{\ell-1}, \qquad \ell > 1,\ 0 \le \omega \le k \tag{3.6}$$

The distribution in (3.6) is called the power-function distribution. From the above expression, the distribution in (3.4) is a special case of a compound Poisson distribution where ω has a power-function distribution, and ω is a mean of the Poisson distribution. Thus the probability distribution represented by (3.4) may be reasonably called a Poisson-power function distribution. See Johnson and Kotz(1970) for the definition of compound Poisson distribution. Kim(1995) proved some properties such as unimodality, stochastical ordering, computational recursion formula,, monotone likelihood ratio property, of the distribution.

For the Poisson-power function distribution, the definition of the discrimination curve is the same to the Poisson distribution in section 2. Two types of error rates for the hypothesis testing to the Poisson-power function distribution are

$$\alpha = \sum_{x=0}^{c} P(x: t, \ell) = \sum_{x=0}^{c} \frac{\ell}{t^{\ell} x!} \gamma(x+\ell\,; t) = \sum_{x=0}^{c} \frac{\ell\, \Gamma(x+\ell)}{t^{\ell} \Gamma(x+1)} F(t; x+\ell) \tag{3.7}$$

and

$$\beta = 1 - \sum_{x=0}^{c} \mathrm{P}(x; d, \ell) = 1 - \sum_{x=0}^{c} \frac{\ell\, \Gamma(x+\ell)}{d^{\ell} \Gamma(x+1)} F(d; x+\ell) \tag{3.8}$$

where $F(t; x+\ell)$ is the gamma c.d.f. with shape parameter $x+\ell$, and Γ is the complete gamma function. Let

$$C(k) \equiv \sum_{x=0}^{c} \frac{\ell\, \Gamma(x+\ell)}{k^{\ell} \Gamma(x+1)} F(k; x+\ell) \tag{3.9}$$

Then

$$\alpha = C(t) \tag{3.10}$$

and

$$C(d) = 1 - \beta \tag{3.11}$$

The operating points (t, d) that determine the discrimination curve are obtained for each $c = 0, 1, 2, \cdots$ by solving for t in (3.10) and for d in (3.11). It is not possible to give closed form formulae for t and d similar to (2.4) and (2.5). Because Poisson-power function distribution has stochastical ordering property (See Kim (1995)), $C(k)$ is a continuous and decreasing function of k. We solve (3.10) by an iterative secant method;

$$t_{i+1} = t_i - \frac{\alpha - C(t)}{C'(t)} = t_i - \frac{\Delta t_i(\alpha - C(t_i))}{\Delta C_i}, \tag{3.12}$$

where $\Delta t_i = t_i - t_{i-1}$ and $\Delta C_i = C(t_i) - C(t_{i-1})$. Similarly

$$d_{i+1} = d_i - \frac{\Delta d_i(1 - \beta - C(d_i))}{\Delta C_i} \tag{3.13}$$

where $\Delta d_i = d_i - d_{i-1}$ and $\Delta C_i = C(d_i) - C(d_{i-1})$.

Initial values are chosen to be $d_0 = -\log(1-\beta)$, $d_1 = d_0 + 1$, and $t_0 = -\log\alpha$ and $t_1 = t_0/2$ for $c = 0$ and all ℓ. Initial values for $c+1$ were chosen to be the solution t and d for the previous c. For $\alpha = 0.05$, the convergence criterion was $|\Delta t_i| < .01$ and $|\alpha - P(t_i)| < 10^{-4}$. No convergence difficulties occurred; convergence almost always was obtained in less than 8 iterations of (3.12) and (3.13).

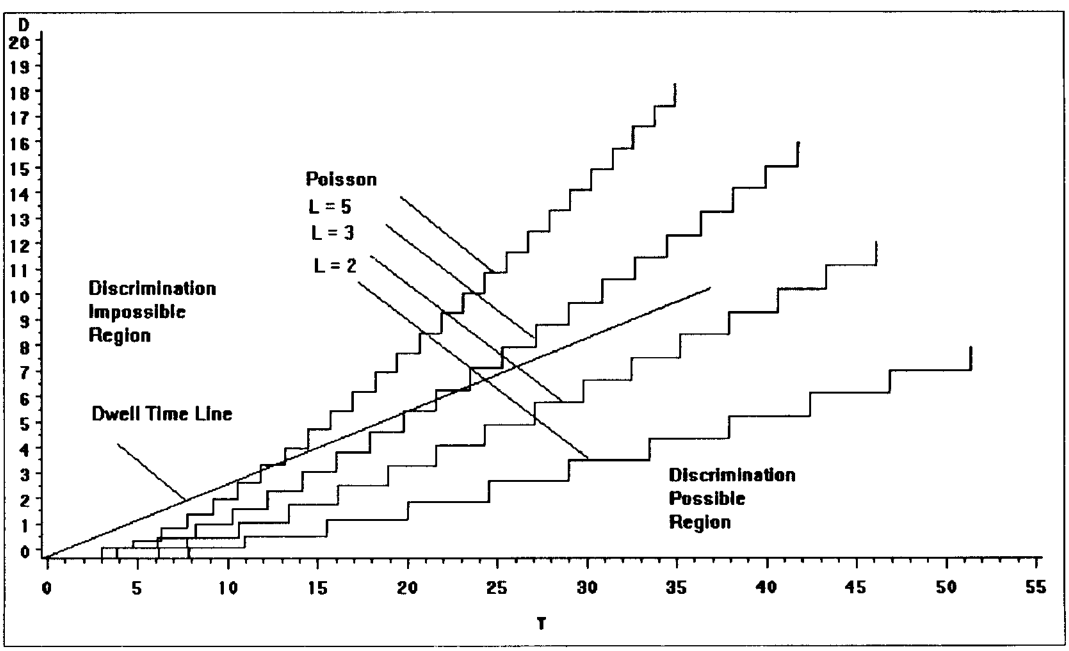

Figure 2. Discrimination Curve for Poisson-power function distribution ($\alpha = \beta = 0.05$)

Figure 2 compares the discrimination curves for choices of $\ell = 5, 3,$ and 2 to the Poisson-power function distribution with that of the Poisson distribution. Note that Kim(1995) proved the fact that these curves converge to the curve in the Poisson distribution as $\ell \to \infty$. Note also that smaller values of ℓ yield smaller discrimination possible regions.

4. Minimum Dwell Time Algorithm for Poisson

Discrimination curve introduced in section 2 can be used to compute the minimum dwell time needed to discriminate at the given α and β. We note the formulae for the mean t and d in terms of the dwell time τ and the mean counts rates $t^{(r)}$ and $d^{(r)}$:

$$t = \tau t^{(r)} \quad \text{and} \quad d = \tau d^{(r)}.$$

The plot of this curve in the (t, d) plane is a straight line L through the origin with slope $d^{(r)}/t^{(r)}$. The minimum dwell time τ^* is obtained from the intersection of L with the discrimination possible region. That is, at the point where the line first enters the discrimination possible region. See Figure 2. The minimum dwell time τ^* is an increasing piecewise continuous function of the slope $d^{(r)}/t^{(r)}$. Consequently, as expected, τ^* decreased as the $d^{(r)}$ decreases or $t^{(r)}$ increases.

In this section, we consider an algorithm to compute the minimum dwell time needed to discriminate at the given α and β and actual errors with the minimum dwell time. Schultz and Vairin (1987) study the simulation for discrimination algorithm design and evaluation.

Let α and β be given. Let $t^{(r)}$ be the mean rate under H_0 and let $d^{(r)}$ be the mean rate under H_1. Since we assume the counts X has a Poisson distribution, we have

$$P(x \mid H_0) = \frac{e^{-\tau t^{(r)}} (\tau t^{(r)})^x}{x!}, \quad x = 0, 1, 2, \cdots \tag{4.1}$$

and

$$P(x \mid H_1) = \frac{e^{-\tau d^{(r)}} (\tau d^{(r)})^x}{x!}, \quad x = 0, 1, 2, \cdots \tag{4.2}$$

Choose the dwell time τ as the smallest dwell time such that there exists an integer c such that if

$$\alpha_\tau^{(c)} = \sum_{x=0}^{c} P(x \mid H_0) \tag{4.3}$$

and

$$\beta_\tau^{(c)} = 1 - \sum_{x=0}^{c} P(x \mid H_1) \tag{4.4}$$

then

$$\alpha_\tau^{(c)} \le \alpha \quad \text{and} \quad \beta_\tau^{(c)} \le \beta. \tag{4.5}$$

The resulting $\alpha_{\tau}^{(c)}$ and $\beta_{\tau}^{(c)}$ are the actual Type I error rate and the Type II error rate with the minimum dwell time, respectively. We shall repeat the computation algorithm in more detail. The SAS program of the algorithm is available from the author.

<Algorithm>

1) Let α and β be given.

2) Let $\varepsilon > 0$ be a small number. Put $\tau = \varepsilon$.

3) Let c take on in succession on the values $0, 1, 2, \cdots$.

4) Calculate $\alpha_{\tau}^{(c)}$ and $\beta_{\tau}^{(c)}$. Note that $\alpha_{\tau}^{(c)}$ is an increasing function of c and $\beta_{\tau}^{(c)}$ is a decreasing function of c.

5) If $\alpha_{\tau}^{(c)} \leq \alpha$ then determine if $\beta_{\tau}^{(c)} \leq \beta$.

6) If both conditions hold, we have the minimum dwell time and stop.

7) If the first condition holds, but the second condition does not, we increase c and repeat.

8) If both conditions fail, we replace τ by 2ε and repeat the computations until we find a τ such that (4.5) is satisfied.

<Example 1> We choose $\alpha = 0.05$, $t^{(r)} = 1.0$, and $d^{(r)} = 0.1$. We obtain the results in table 1. Note that TALPHA represents actual type I error rate and TBETA represents actual type II error rate, and TAU represents minimum dwell time for a given ALPHA and BETA.

OBS	BETA	C	TAU	TALPHA	TBETA
1	0.000	30	40.70	0.049853	0.000000
2	0.005	4	9.20	0.048580	0.002574
3	0.010	3	7.80	0.048477	0.008334
4	0.015	3	7.80	0.048477	0.008334
5	0.020	3	7.80	0.048477	0.008334
6	0.025	3	7.80	0.048477	0.008334
7	0.030	2	6.30	0.049846	0.026183
8	0.035	2	6.30	0.049846	0.026183
9	0.040	2	6.30	0.049846	0.026183
10	0.045	2	6.30	0.049846	0.026183
11	0.050	2	6.30	0.049846	0.026183
12	0.055	2	6.30	0.049846	0.026183
13	0.060	2	6.30	0.049846	0.026183
14	0.065	2	6.30	0.049846	0.026183
15	0.070	2	6.30	0.049846	0.026183
16	0.075	2	6.30	0.049846	0.026183
17	0.080	2	6.30	0.049846	0.026183
18	0.085	1	4.75	0.049747	0.082720
19	0.090	1	4.75	0.049747	0.082720
20	0.095	1	4.75	0.049747	0.082720
21	0.100	1	4.75	0.049747	0.082720

<table 1> Results for the Algorithm and $\alpha = .05$, $t^{(r)} = 1$, and $d^{(r)} = .1$.

We shall show that this process of the algorithm must terminate. Let α and β be given. Assume that the independent Poisson random variable X has mean λ. Now

$$\alpha = P(X \le c \mid \lambda = t) = P(\chi^2_{2(c+1)} > 2t) \tag{4.6}$$

and

$$\beta = P(X > c \mid \lambda = d) = P(\chi^2_{2(c+1)} \le 2d) \tag{4.7}$$

where $\chi^2_{2(c+1)}$ is a Chi-square distributed random variable with $2(c+1)$ d.f. By the Central Limit Theorem,

$$\alpha = 1 - \Phi\left(\frac{2t-2(c+1)}{2\sqrt{c+1}} \right) - \varepsilon_1(c) \tag{4.8}$$

and

$$\beta = \Phi\left(\frac{2d-2(c+1)}{2\sqrt{c+1}} \right) + \varepsilon_2(c) \tag{4.9}$$

where $\varepsilon_1(c) \to 0$ and $\varepsilon_2(c) \to 0$ as $c \to \infty$ and Φ is the c.d.f. of the standard normal random variable. Hence

$$\frac{t-(c+1)}{\sqrt{c+1}} = \Phi^{-1}(1-\alpha-\varepsilon_1(c)) \tag{4.10}$$

and

$$\frac{d-(c+1)}{\sqrt{c+1}} = \Phi^{-1}(\beta-\varepsilon_2(c)) \tag{4.11}$$

where $\Phi^{-1}(x)$ is the inverse function of the cumulative standard normal distribution function. Let $(t(c), d(c))$ be the discrimination curve defined by (2.4) and (2.5) in section 2. Therefore,

$$t(c) = (c+1) + \sqrt{c+1} \cdot \Phi^{-1}(1-\alpha-\varepsilon_1(c)) \tag{4.12}$$

Similarly,

$$d(c) = (c+1) + \sqrt{c+1} \cdot \Phi^{-1}(\beta-\varepsilon_2(c)) \tag{4.13}$$

where $\varepsilon_i(c) \to 0$ as $c \to \infty$.

The above two formulae (4.12) and (4.13) shows that $d(c)/t(c) \to 1$ as $c \to \infty$. Therefore, any half-line starting at the origin with positive slope less than 1 must eventually intersect the discrimination curve and will eventually remain completely under the curve. This proves that for the Poisson counts the minimum dwell time problem has a solution. Further, by taking the step sizes small enough, the algorithm will find the least solution in τ with arbitrary accuracy.

5. Minimum Dwell Time for the Poisson-power Function Distribution

Similar to the Poisson case, the discrimination curve explained in section 3 can be used to compute the minimum dwell time for the Poisson-power function distribution needed to discriminate at the given α and β.

In this Section, we consider an algorithm to compute the minimum dwell time for the Poisson-power function distribution and actual errors with the minimum dwell time. Schultz and Vairin(1987) studied the simulation for discrimination algorithm design and evaluation. The minimum dwell time τ^* for the Poisson-power function distribution is also an increasing piecewise continuous function of the slope $d^{(r)}/t^{(r)}$ and, as expected, τ^* decreased as the $d^{(r)}$ decreases or $t^{(r)}$ increases. Note that a path of (t, d) operating points representing increasing dwell times is a line segment beginning at the origin and that it may not intersect a discrimination curve for some $\ell < \infty$. This situation is described by an example.

Let α, β and ℓ be given. Let $t^{(r)}$ be the mean rate under H_0 and let $d^{(r)}$ be the mean rate under H_1. For the Poisson-power function distribution, we have

$$P(x|H_0) = \frac{\ell}{\tau k^{(r)\ell} x!} \gamma(x+\ell\,; \tau k^{(r)}), \quad x=0,1,2,\cdots \tag{5.1}$$

and

$$P(x|H_1) = \frac{\ell}{\tau d^{(r)\ell} x!} \gamma(x+\ell\,; \tau d^{(r)}), \quad x=0,1,2,\cdots \tag{5.2}$$

where $\gamma(\nu; k)$ is the incomplete gamma function defined in (3.5).

Since we assume $d^{(r)} < t^{(r)}$, put $d^{(r)} = \rho t^{(r)}$ where $\rho = t^{(r)}/d^{(r)} < 1$. Let $t = \tau t^{(r)}$ and $d = \rho t$. To find the minimum dwell time, choose the dwell time τ as the smallest dwell time such that there exists an integer c such that if

$$\alpha^* = \frac{\ell}{t^\ell} \sum_{x=0}^{c} \frac{\gamma(x+\ell\,;t)}{\Gamma(x+1)} \le \alpha \tag{5.3}$$

an

$$\beta^* = 1 - \frac{\ell}{(\rho t)^\ell} \sum_{x=0}^{c} \frac{\gamma(x+\ell\,;\rho t)}{\Gamma(x+1)} \le \beta \tag{5.4}$$

The resulting α^* and β^* in (5.3) and (5.4) are actual Type I error rate and actual Type II error rate, respectively. The algorithm for minimum dwell time and actual errors for the Poisson-power function distribution is almost the same to the Poisson case in section 4. The SAS program of the algorithm is also available from the author.

It is found that the algorithm fails if the aiming error variance σ_2^2 is too large relative to the variance σ_1^2 of the Gaussian distribution of intensity across the beam.

<Example 2> Consider the hypothesis of $H_0: k = t$ vs. $H_1: k = \rho t$, where $\rho = d/t < 1$. In section 3, we showed that the Neyman-Pearson test for the Poisson-power function distribution is a left-tail test. For this reason, for a given α and β in the interval $(0, 1)$, we choose the smallest t that satisfies (5.3) and (5.4), where for each t, the c is the largest integer critical value.

By a different calculation using an integration by parts, the quantities in (5.3) and (5.4) can be written as

$$\alpha^* = 1 - \frac{\gamma(c+1;t) - t^{-\ell}\,\gamma(\ell + c+1; t)}{\Gamma(c+1)} \tag{5.5}$$

and

$$\beta^* = \frac{\gamma(c+1;\rho t) - (\rho t)^{-\ell}\,\gamma(\ell + c+1; \rho t)}{\Gamma(c+1)} \tag{5.6}$$

For an example of the results for algorithm, we choose the following parameter value:

$$t=1 \quad \rho=0.1 \quad \alpha=\beta=0.05$$

We obtain the results in table 2. Note that CALPHA represents actual type I error rate and CBETA represents actual type II error rate, and T represents minimum dwell time for a given ALPHA and BETA.

Note that for $\ell \leq 1.2$, there is no solution to the problem attempting to solve. This is the situation when the aiming error variance is too large relative to the variance of the Gaussian distribution of intensity across the beam. In other words, if the ratio of two variances in ℓ is too small, we can not find the minimum dwell time for a given α and β.

ALPHA	BETA	L	H0	C	CALPHA	CBETA
0.05	0.05		6.3	2	0.049846	0.026183
0.05	0.05	10.0	8	2	0.027984	0.038040
0.05	0.05	7.0	8	2	0.038271	0.035024
0.05	0.05	6.0	8	2	0.045089	0.033540
0.05	0.05	5.0	9	2	0.035086	0.042299
0.05	0.05	4.0	9	2	0.049756	0.039043
0.05	0.05	3.0	14	3	0.043584	0.026510
0.05	0.05	2.8	15	3	0.042096	0.031563
0.05	0.05	2.6	16	3	0.042498	0.036801
0.05	0.05	2.4	17	3	0.044711	0.042082
0.05	0.05	2.2	18	3	0.048883	0.047235
0.05	0.05	2.0	25	4	0.048000	0.040723
0.05	0.05	1.8	34	5	0.049281	0.042484
0.05	0.05	1.6	54	7	0.049893	0.049358
0.05	0.05	1.4	181	20	0.049667	0.048507
0.05	0.05	1.2	2065	170	.	.
0.05	0.05	1.0	3401	170	.	.

<Table 2> Results of the algorithm computation

6. Conclusion

Discrimination curve plots the maximum d that can be discriminated from t for a given α and β. It is a step function increasing in t with small steps parameterized with the critical values $c = 0, 1, 2, \cdots$. Discrimination curve can be used to compute the minimum dwell time needed to discriminate at the given α and β. The minimum dwell time τ^* is obtained from the intersection of the dwell time line L with the discrimination possible region. We showed that $L = d(c)/t(c) \to 1$ as $c \to \infty$ for the Poisson distribution. That is, any half-line starting at the origin with positive slope less than 1 must eventually intersect the discrimination curve and will eventually remain completely under the curve. Therefore, the minimum dwell time problem has a solution for the Poisson counts.

For the Poisson-power function distribution, the definition of discrimination curve is similar to Poisson case. It is found that the minimum dwell time algorithm for the Poisson-power function distribution fails if the aiming error variance σ_2^2 is too large relative to the variance σ_1^2 of the Gaussian distribution of intensity across the beam. In other words, if the ratio of two variances ℓ is too small, we can not find the minimum dwell time for a given α and β.

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