Analytical Studies for SASW Measurements Underwater

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요 지

주파수영역 표면파기법 (SASW)을 수중지반탐사에 적용하기 위한 일련의 해석적 연구가 실시되었다. SASW실험으로 지반강도분포를 정밀히 예측하기 위해서는 실험조건, 고차모드의 영향, 응답파의 정리 기법 등, 실험결과에 영향을 미치는 모든 제반 요소들을 고려한 정확하고도 경제적인 이론분산곡선을 결정해야만 한다. 따라서, 본 논문에서는 수중 표면파의 이론분산곡선을 구하기 위한 해석기법들을 개발하고 그 이론적 배경을 설명하였다. 또한, 개발된 해석기법들을 수중지반에서 실제로 실시된 SASW 실험결과의 분석에 적용하여 수중지반의 강도특성을 예측하는 예를 보였다. 이 결과로부터 개발된 해석기법들과 더불어서 수중지반탐사를 위한 SASW 실험방법의 적용성이 검토되었다.

Abstract

Analytical studies were conducted to develop the Spectral-Analysis-of-Surface-Waves (SASW) method for underwater use. For the precise estimation of the in-situ soil stiffness profile from SASW measurements, it is essential to determine economical and reasonable theoretical dispersion curves reflecting various experimental conditions. In this paper, therefore, analytical methods are mainly discussed, which were developed to determine theoretical dispersion curves of surface waves propagated along the soil-water interface. Application of the analytical methods is then illustrated by an example involving estimation of a stiffness profile through a forward modeling process of SASW measurements.

Applicabilities of the SASW method as well as the developed analytical methods are evaluated, respectively, from the example.

Keywords : SASW, Underwater, Surface wave, Theoretical dispersion curve, Soil stiffness

1. Introduction

A seismic technique called the Spectral-Analysis-of-Surface-Waves(SASW) method has been developed to profile stiffness properties near the surface of layered soil media (Nazarian and Stokoe, 1986). In the SASW method, seismic surface waves propagating along the ground surface are measured by two vertically-oriented receivers positioned along

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a line at different distances from the source and analyzed by using a spectral analysis technique. An experimental dispersion curve for surface waves, characterized by the variation of the phase velocity with frequency, is determined from the phase difference between the signals at the two receivers. A geotechnical site is characterized by a profile of shear wave velocity with depth that provides a theoretical dispersion curve consistent with the experimental one.

The SASW method has been successfully applied to explore various geotechnical sites (Stokoe et al., 1989). Recently, experimental studies have been performed to extend the SASW method to layered soil systems overlaid by water(Luke et al., 1993). Analytical studies have also been performed to support the interpretation of experimental data obtained from the SASW method underwater(Wright et al., 1994: Lee, 1996).

In the analytical studies, it is one of important objectives to develop analytical methods to determine economical and reasonable theoretical dispersion curves of surface waves. In this paper, therefore, development of these analytical methods is mainly discussed. An example is also shown, in which theoretical dispersion curves determined from the analytical methods were applied to a forward modeling of SASW measurements underwater. Applicabilities of the SASW method as well as the developed analytical methods for underwater use are examined on the basis of the results obtained from the example.

2. Analytical Methods for Determining Dispersion Curves

To determine theoretical dispersion curves for layered soil deposits overlaid by water, Lee(1996) suggested two analytical methods: "the normal mode, so-called the 2 dimensional(2D) solution": and "the complete solution, so-called the 3 dimensional(3D) solution". In both analytical methods, dispersion curves were determined by solving a wave propagation problem for homogeneous, isotropic, and horizontally layered elastic media.

2.1 The Dynamic Stiffness Matrix Approach

To obtain general solutions for a wave propagation problem, the stiffness matrix approach established by Kausel and Roesset(1981) was used. In this approach, the forces at the interfaces between layers are related directly to displacements at the same locations by a dynamic stiffness matrix. Thus, displacements developed at any of the interfaces of either an isolated soil or water layer to an external load or seismic excitation can be directly determined by using this approach. The stiffness matrices for the isolated soil and water layers can be found in Lee(1996).

To determine dynamic response for an entire layered system as shown in Fig.1, a global stiffness matrix, $[\mathbf{K}]$, needs to be assembled as seen in Fig. 2 for the complete system considering the layer stiffness at each "node" (interface) of the system. The global load vectors and displacements in Fig. 2 correspond to external forces and dynamic responses at the interfaces, respectively, and are referred to as the state vectors. The state vectors for a layered system are shown in Fig. 1.

2.2 An Analytical method: the 2 Dimensional Solution

According to the vibration theory, dispersion curves can be determined on the basis of the normal modes(roots) of vibration of a layered system. Normal modes are obtained

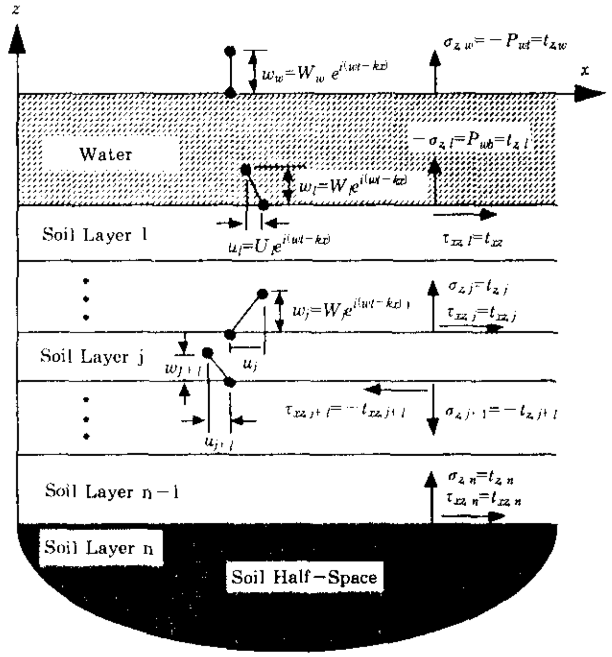

Fig.1 Displacements, stresses and external load vectors at the interfaces of a layered soil system overlaid by water on a $x-z$ plane of a Cartesian coordinate system.

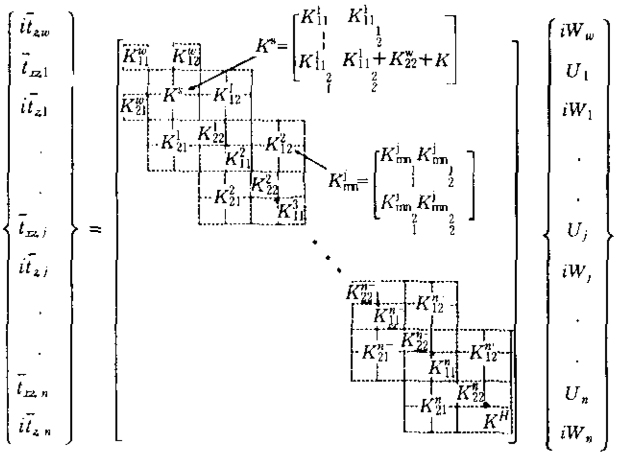

Fig.2 Interrelationship between the load and displacement state vectors through the global stiffness matrix for a system composed of layers of soil overlying a soil half-space and overlaid by a water layer.

from the characteristic equation formulated with the stiffness matrix approach.

The normal modes of the system can be obtained from the equation given in Fig. 2. providing no external loads exist. In this case, the equation is expressed as

$$[\mathbf{K}]\{\mathrm{U}\}=0 \tag{1}$$

where $[\mathbf{K}]$ is the global stiffness matrix of the system, and $\{\mathrm{U}\}$ is the vector for displacements at the layer interfaces in the system. To obtain the non-trivial solution for displacements from Eq. 1, the determinant of the stiffness matrix should be zero:

$$[\mathbf{K}]=0 \tag{2}$$

This equation is called the "characteristic" equation ("period" equation, "secular" equation, or "dispersion" function). The roots of the characteristic equation are the normal modes and are called here the 2-D solution. For practical convenience in this study, the roots of the characteristic equation are expressed in terms of phase velocities rather than wave numbers corresponding to prescribed wavelengths or frequencies. The interrelations of phase velocities with wavelengths or frequencies represent the dispersion curve.

2.3 An Analytical method: the 3 Dimensional Solution

As a more sophisticated and a more realistic solution than the 2D solution, the 3D solution is also developed. The 3-D solution is the harmonic displacements due to dynamic loads in the frequency-spatial domain in a cylindrical coordinate system from which dispersion curves can be generated. In a cylindrical coordinate system shown in Fig. 3, the harmonic displacements in the spatial domain are expressed by "inverse Hankel transform integrals" of the harmonic displacements in the wave number domain, and can be solved by evaluating the integrals. This technique is called the "integral transform technique"(Ewing et al., 1957)

For a specified loading condition, a particular solution for the displacement in the frequency-spatial domain can be obtained by using the integral transform technique. For a present problem, the loading condition considered consists of a harmonic vertical disk load applied in the frequency-spatial domain at the soil-water interface. This assumes that a

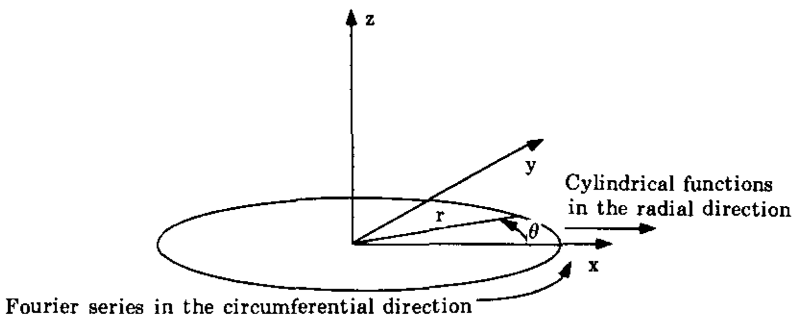

Fig.3 Illustration of the decomposition of the displacements and stresses in the circumferential and radial directions in a cylindrical coordinate system.

uniform harmonic vertical load is applied over a circular area with the radius of the disk. This representation of the loading is frequently used to represent the source for seismic tests.

A particular solution for the vertical displacement corresponding to the specified load was obtained as

$$u_z(r, w)=qR\int_0^\infty \bar{u}_z(k, w)\ J_1(kR)J_0(kr)dk \tag{3}$$

where k is wave number; q is the amplitude of the disk load; r is the distance from the center of the disk load; R is the radius of the disk; $u_z(r, w)$ and $\bar{u}_z(k, w)$ are the vertical displacements in spatial and wave number domain, respectively, and the bar denotes the frequency-wave number domain; and $J_0(kr)$ and $J_1(kr)$ are the Bessel functions of the first kind, and order zero and one, respectively.

The displacements expressed by Eq. 3 can be calculated by: (1) assembling the stiffness matrix for the coupled soil-water system; (2) solving the displacements corresponding to the unit harmonic vertical load for each of the wave numbers by using the relationship between displacements and loads interrelated with the stiffness matrix ; (3) evaluating the integrals in Eq. 3 with the displacements obtained in the step 3; (4) repeating the procedure from step 1 through step 3 for various frequencies.

The complex vertical displacements calculated from Eq. 3 provide phase information at various locations in the spatial domain for the calculation of phase velocities of the surface wave. The phase velocity of the surface wave for each frequency is calculated from the phase difference between the displacements at two distances from the source. Dispersion curves are then determined from the relationship between the phase velocities and the frequencies.

An analytical difficulty remains in determining the displacements by using Eq. 3. Analytical evaluation of these integrals is limited when there is a large number of layers, and closed-form evaluation of the integrals can only be obtained for very simple cases. Thus, the integrals need to be evaluated numerically.

Among the various numerical methods to evaluate the Hankel transform integrals given by Eq. 3, a numerical integration method called the "fast field" technique(Schmidt, 1985) was found to be useful for dealing with underwater problems and was chosen for this study. In the fast field technique, the Bessel functions in the Hankel transform integrals are separated into incoming and outgoing parts by expressing them in terms of Hankel functions. The Hankel functions are then replaced by the asymptotic expressions for large arguments. The Hankel transform integrals then become similar to the Fourier transform and are evaluated easily by means of the fast Fourier transform technique.

3. Application of the Developed Analytical Methods

The 2D and 3D solutions presented in this paper were applied to estimate soil stiffness profiles from experimental dispersion curves. To estimate an in situ soil stiffness profile theoretical dispersion curves are computed by using assumed stiffness profiles, and compared to experimental dispersion curves. The assumed stiffness profile is adjusted repeatedly until the theoretical dispersion curve matches the experimental dispersion curve within acceptable limits.

3.1 Experimental Dispersion Curve

The case considered here consists of soft material at the surface that increases in stiffness with depth. The test site is located in the Gulf of Mexico. The seafloor of the test site is essentially flat, and the water is about 27m deep. Luke(1994) determined the experimental dispersion curves for the test site as shown in Fig. 4. Surface wave velocities for wavelength ranging from approximately 0.6m to 100 m were determined, and, thus, soil profiles approximately as deep as 30 to 50m could be investigated from this result. Further details of the experimental set up, the testing technique, and equipment can be obtained from Luke(1994).

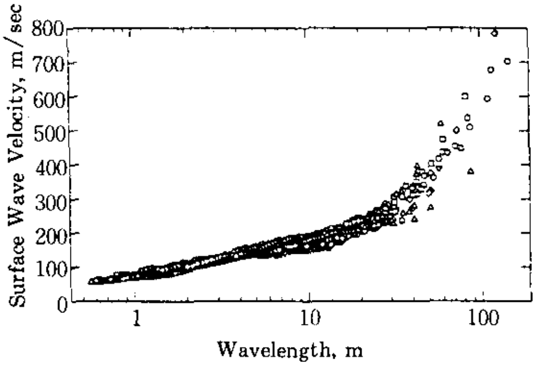

Fig.4 Composite dispersion curves determined from the cross power spectrum obtained by applying the SASW method to the time records of every pair of geophones(After Luke, 1994).

3.2 Estimation of the in situ Soil Stiffness Profile

Theoretical dispersion curves were calculated from the 2-D and the 3-D solutions and compared to the experimental dispersion curves. The experimental dispersion curve for only one location at the 68m offset from the source was selected because the dispersion curves for all the locations had a similar trend of dispersion.

The final soil stiffness profile determined after iterative forward modeling is shown in Fig. 5. The stiffness profile for depths to 30m from the soil-water interface was assumed. The soil below the depth of 30m was assumed to behave as a half-space. The water depth of 27m was assumed on the basis of the actual water depth. To simulate the experimental set up in the 3-D solution, the receiver spacing was assumed to be the same as the one used in the experiments by using "the alternative method" described by Lee(1996). The receiver spacing was 5m and the locations of the receivers were 68 and 73m from the source.

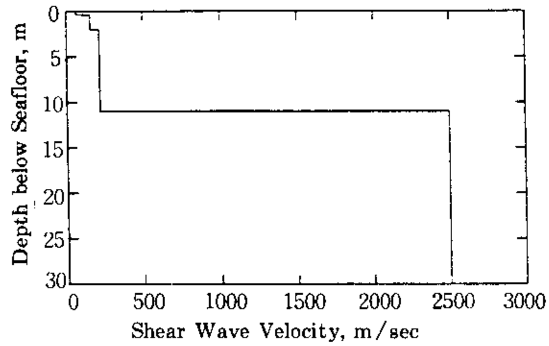

Fig.5 An estimated soil stiffness profile with depth at the 68m offset from the source for the test site in the Gulf of Mexico(Water depth=27m).

The theoretical dispersion curves computed from both the 2-D and 3-D solutions are compared to the experimental dispersion curve in Fig. 6. The theoretical dispersion curves from both the solutions agree well with the experimental dispersion curve over the wavelength range shown in this figure. Thus, the contribution of higher modes appears to be negligible for this wavelength range although the soil stiffness abruptly increases in a large amount at a depth of 11m (from 215 to 2500 m/sec) as seen in Fig. 5.

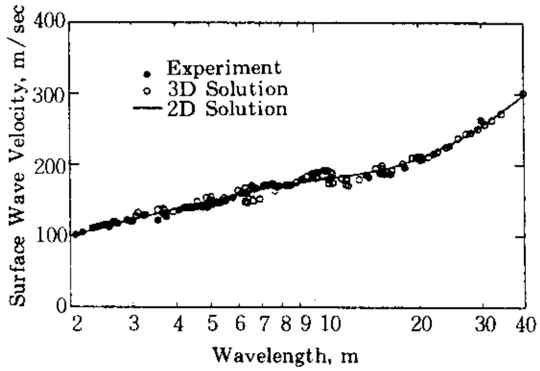

Fig.6 Comparison of experimental and theoretical dispersion curves at the offset of 68m for the Gulf of Mexico data.

4. Comments in Application of the Analytical Methods

At the outset of this study, the 2D solution was preferred to the 3D solution for several reasons: (1) the dispersion curves determined by the 2D solution (the first of the normal modes) were expected to be acceptable in many underwater cases because experience acquired from many on land cases showed that surface wave propagation was dominated by the first mode; (2) the effort to obtain a 2D solution was less than for a 3D solution; and (3) the basic theory of the 2D solution was easily understandable.

In some cases, however, the dispersion curves determined from the 2D and 3D solutions were often significantly different in the range of the frequencies of interest. The major cause of the difference was revealed to be the contributions of higher modes. These cases are often encountered where stiffness decrease in some depth interval or stiffness contrast between layers is significant. For these cases, it was believed that the contributions of higher modes to the dispersion curves were thought to be significant. When the contributions of higher modes are significant, the dispersion curves determined from the 2D solution are not realistic for a significant range of frequencies because the 2D solution does not identify the higher mode contributions. Realistic dispersion curves can only be determined by the 3D solution. In addition, for such cases, it was shown that the dispersion curves vary with the receiver locations. Therefore, it is necessary to consider the receiver spacing for the 3D solution for the determination of dispersion curves matching those from experiments.

Accordingly, it can be suggested that reliable theoretical dispersion curves can be determined by both the 2D and 3D solutions if the contributions of higher modes are negligible. However, it is almost impossible to suggest a certain criterion determining for all cases whether the contributions of higher modes are negligible or significant. Thus, except for cases where the stiffness increases only gradually with depth, it is recommended that dispersion curves determined from the 2D solution be verified by using the 3D solution. If the contributions of higher modes are very significant, the 3D solution should be used and consideration should be given to the receiver spacing.

5. Conclusions

Two analytical methods to determine theoretical dispersion curves were developed in this study. The 2D solution characterizes the disturbance assuming plane wave propagations in Cartesian coordinates. The 3D solution is expressed in terms of motions by solving propagation of waves with a curved wave front in cylindrical coordinates resulting from a vertical disk load on the soil-water interface. Though determining dispersion curves from the 3D solution is a more sophisticated and more realistic approach, this method is more computationally intensive. Thus, the 2D solution is preferred in many cases where it provides an adequate solution, i.e., where contributions of higher modes are negligible.

Theoretical dispersion curves from the 2-D and 3-D solutions were compared with the experimental dispersion curve for the site where the stiffness increased with depth(Gulf of Mexico). For this case, theoretical dispersion curves from both the 2-D and 3-D solutions agreed well with the experimental dispersion curves. Thus, the 2-D solution was found to be adequate for this example. Accordingly, it could be concluded that contribution of higher modes was apparently negligible for the wavelength range where the dispersion curves reflected the soil stiffness increasing gradually with depth.

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