Impulse Response Filtration Technique for the Determination of Phase Velocities from SASW Measurements

SASW 시험에 의한 위상속도 결정을 위한 임펄스 응답필터 기법

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요 지

표면파를 이용하여 지반의 강성을 추정하는 기법인 SASW 시험에서 위상속도(phase velocity)를 결정하기 위해서는 위상각(phase angle)의 전개(unwrapping)가 필수적이다. 포장 구조에서처럼 깊이에 따라 강성의 차이가 현저한 경우는 기존의 위상각 전개방식으로는 정확한 위상속도를 결정하기가 용이하지 않다. 이는 기존의 위상각 전개방식은 주 위상각(principal phase angle)에 2π의 정수배를 더하는 것인데, 위상각 스펙트럼(phase spectrum)에서 정수배를 결정하는 데에 어려움이 있기 때문이다.

본 연구에서는 이러한 문제점을 해결하기 위해서, 임펄스 응답 필터 기법(Impulse Response Filtation Technique), 또는 IRF기법이라고 하는 새로운 위상각 분석 기법을 제안하였다. IRF 기법의 원리는 임펄스 응답을 필터 처리함으로써 파군(wave group)을 분리하는 것인데, 파군의 분리는 임펄스 응답에 대한 Gabor spectrogram을 분석한 정보를 근거로 한다. Gabor spectrogram은 전파되는 파의 에너지를 주파수-시간 공간에서 나타내는 contour 그림으로서, 파군의 전파 상황을 시각적으로 표현하는 수단이다. 이렇게 필터 처리된 임펄스 응답을 이용하면, 위상각 스펙트럼의 분석을 정확하게 할 수 있으며, 위상각의 전개에 있어서 난해함을 제거할 수 있다.

끝으로, 전형적인 포장 구조에 대하여 이론적으로 SASW 시험을 모사하였으며, 그 결과를 이용하여 IRF 기법의 효용성을 입증하였다.

Abstract

The calculation of phase velocities in Spectral-Analysis-of-Surface-Waves (SASW) measurements requires unwrapping phase angles. In case of layered systems with strong stiffness contrast like a pavement system, conventional phase unwrapping algorithm to add in-

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teger multiples of 2π to the principal value of a phase angle may lead to wrong phase velocities. This is because there is difficulty in counting the number of jumps in the phase spectrum especially at the receiver spacing where the measurements are in the transition zone of defferent modes. A new phase interpretation scheme, called "Impulse Response Filtration (IRF) Technique," is proposed, which is based on the separation of wave groups by the filtration of the impulse response determinded between two receivers. The separation of a wave group is based on the impulse response filtered by using information from Gabor spectrogram, which visualizes the propagation of wave groups at the frequency-time space. The filtered impulse response leads to clear interpretation of phase spectrum, which eliminates difficulty in counting number of jumps in the phase spectrum. Verification of the IRF technique was performed by theoretical simulation of the SASW measurement on a pavement system which complicates wave propagation.

Keywords : SASW method, Dispersion curve, Phase unwrapping, Phase spectrum, Phase velocity

1. INTRODUCTION

Dispersion characteristics of surface waves have been widely used to investigate stiffness profiles of subsurface materials. The Spectral-Analysis-of-Surface-Waves(SASW) measurement is one approach to evaluate stiffness profiles. The SASW measurement involves using a seismic source and two receivers in a line with the source. SASW is basically a process of retrieving phase velocities for a band of frequencies. For the calculation of phase velocities, there are several methods available such as phase difference, cross correlogram, and a variation of multiple-filter technique. The phase difference method is the one adopted for SASW measurements.

The phase difference method involves unwrapping of principal values of the phase. Conventional phase unwrapping has been done by adding integer multiples of 2π to the principal value of a phase angle at each frequency. The selection of these integer multiples in unwrapping the phase is done by using engineering judgment based on the *a priori* shear velocities of the materials or the phase velocities from other SASW measurements using a different receiver spacing. Interpretation of the jumps in the phase spectrum is not always straightforward. Interpretation is especially difficult in layered systems with strong stiffness contrast. Difficulties in the interpretation of the phase spectrum are encountered at some receiver spacings where transition between different modes is occuring.

The algorithm proposed here is based on the filtration of the impulse response between two receivers. The filtered impulse response leads to a backbone phase spectrum without interference of later arrivals and background noise. The backbone phase spectrum provides information for determining the jumps of the saw-tooth pattern in the original wrapped phase spectrum to be used in unwrapping the phase spectrum. Since this backbone phase spectrum incorporates the backbone phase spectrum the new algorithm is called the impulse response filtration (IRF) technique. This new algorithm can also be used to enhance the phase spectrum. This can be a valuable tool to extract usable information when multiple coherent signal averaging is not feasible nor economical.

2. IMPULSE RESPONSE FILTRATION TECHNIQUE

Investigation of the stiffness profile of subsurface materials incorporates the measurement of stress waves. Stress waves generated by an impact source or a vibrator are propagated through a layered media, and the motion is recorded by receivers placed at a certain distance from the source. The wave motions recorded by a pair of receivers, which are located in a line with a source, are interpreted to retrieve the dispersion characteristics of the material.

In the case of layered systems with strong stiffness contrast, like pavement systems, the propagation of more than one wave group is observed due to the refraction and reflection of stress waves at the interface between layers. Different wave groups which travel with different velocities interfere with each other. As a result, the measured phase difference spectra between two receivers are complicated due to the interference of different wave groups. This may lead to the wrong phase velocity dispersion curves. Thus, it is important to investigate wave groups in recorded wave motions to retrieve a correct phase velocity disperson curve. In terms of investigation of wave groups it is better to use the impulse response rather than individual time signals, because we can concentrate on only the response of the materials between receivers.

The impulse response can be calculated by deconvolution of the time signal for the second receiver out of the time signal for the first receiver, or by the inverse Fourier transform of a transfer function. Of these two approaches, the second is more practical and less time consuming in the engineering applications. The impulse response by the inverse Four-

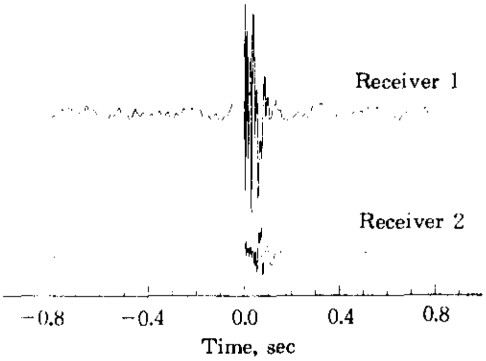


Fig. 1 Time history of acceleration recorded at two receivers at the runway of JFK airport

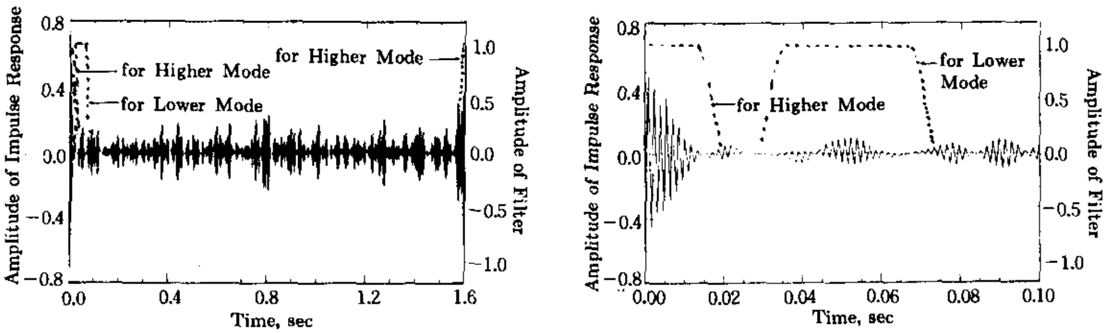
ier transform of a transfer function, $h(t)$, is defined by

$$h(t) = \int H_{YX}(f) e^{i2\pi ft} dx \quad \cdots\cdots (1)$$

in which H_{YX} = transfer function of signals for two receivers. The transfer function, H_{YX} is the spectral ratio of the system response to the excitation, and expressed by

$$H_{YX} = \frac{F_Y}{F_X}$$
$$= \frac{A_Y}{A_X}\{\cos(\theta_Y - \theta_X) + i\sin(\theta_Y - \theta_X)\} \quad \cdots\cdots (2)$$

in which F_X, F_Y are Fourier transforms of time signals, A_X, A_Y are spectral amplitudes, and θ_X, θ_Y are phase angles of time signals for two receivers, respectively. The transfer function is the output/input amplitude ratio and phase shift over a band of frequency, and it can be interpreted as a response spectrum to a reference spectrum. It implies that the impulse response, the inverse Fourier transform of the transfer function, is the time signal captured at the second receiver when a unit impulse source is applied at the first receiver position. In Fig. 1, a pair of time signals measured at the runway of JFK airport are shown. The impulse response for these time singnals are calculated and shown in Fig. 2. The Gabor spectrogram (Dziewonski *at el*., 1969), which represents the original signal as a linear combination of time-and-frequency shifted Gaussian functions, is a good tool in the investigation of wave groups. The Gabor spectrogram for the impulse response of Fig. 2 is shown in Fig. 3. Two distinct wave groups are observed in the Gabor spectrogram. One is an earlier arrival which appears in the higher frequency range above about 105 Hz. The other is a later arrival which appears in the lower frequency range below about 105 Hz. The former represents a low velocity wave group as compared to the latter. Throughout this work, the later arrival is called a lower mode, and the earlier arrival is called a higher mode. The frequency which is a boundary of two different modes is called mode conversion frequency. Due to the existence of two separate wave groups, the phase spectrum is governed by the lower mode in the frequency range lower than the mode conversion frequency, and is gov-



(a) Impulse Response

(b) Expanded Impulse Response

Fig. 2 Impulse response for a pair of receivers and the filter applied to generate backbone impulse response erned by the higher mode in the frequency range higher than the mode conversion frequency. Therefore, just unwrapping the phase, without the knowledge of the complication of the wave groups, may lead to erroneous phase velocities. For the correct phase unwrapping of the complicated wave data, the separation of the two wave groups is required. The separation of wave groups can be done by extracting either the lower mode or the higher mode. For the extraction of each mode, the impulse response should be appropriately filtered to give only the signals corresponding to each mode. In the case of the impulse response of Fig. 2, the lower mode can be seen in the range from 0.03 sec to 0.07 sec, and the higher mode can be seen in the range from 0 sec to 0.02 sec, as shown in the Gabor spectrogram of Fig. 3.

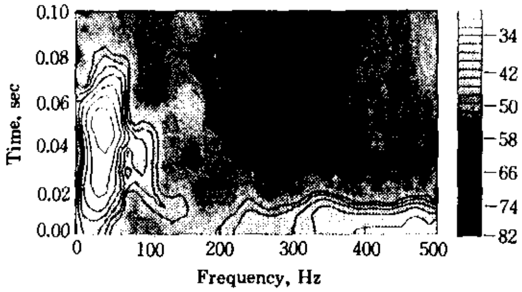
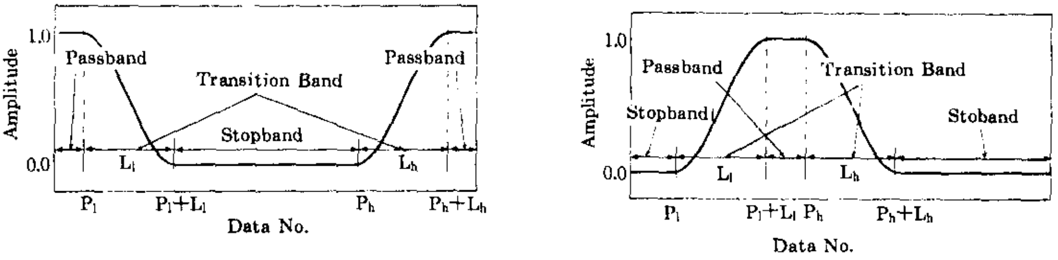


Fig. 3 Gabor spectrogram for the original impulse response of Fig.2

The schematic design of the filters to extract the lower and higher modes is shown in Fig. 4. The proposed filter consists of three bands, which are passband, transition band and stopband. The passband, which is flat and provides good spectral resolution as a boxcar filter does, passes the major part of the impulse response. The stopband is flat and zero-values, designed to eliminate later arrivals. The parameters for passband, transition band and stopband are determined by the shape of the impulse response.

In the frequency domain, the multiplication of the time signal with the filter is equivalent to the convolution of the filter spectrum with the Fourier transform of the time signal. Convolution is a moving average process with the filter spectrum as a weighting factor. Therefore, the sharp features of the spectrum are smeared out by the convolution, and a phase shift of the signal results.

The transition band of the proposed filter is cosine-tapered. This smooth transition gives less leakage of the spectral energy and provides better phase behavior. A phase shift through digital filtering of the time signal is inevitable, but it can be minimized by altering the shape of the filter. The wider the transition band, the less phase shift is caused. Therefore, it is a good strategy to have the transition band as wide as possible. In addition, the filtered time signal is used only as a basis to unwrap the original time signal, and is not directly used for the calculation of phase velocities. By this rationale, the phase shift caused



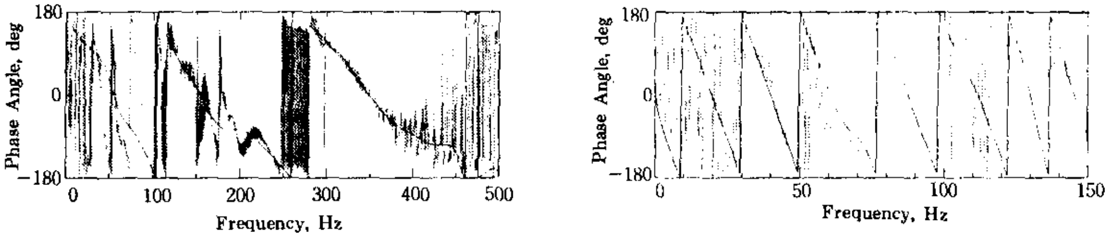
(a) Filter for the Higher Mode

(b) Filter for the Lower Mode.

Fig. 4 Filter for the Higher and lower modes

by the proposed filter is not a problem in unwrapping the original time signal.

The filters for lower and higher modes applied to this impulse response are shown together with the impulse response in Fig. 2. The filtered impulse response is called a backbone impulse response, and the backbone phase spectrum is generated, based on the Fourier transform of the backbone impulse response. The backbone phase spectra for higher and lower modes are shown in Figs 5. (a) and (b), respectively. The backbone phase spectra have clear saw-tooth patterns because backbone curves mainly carry one wave group at a given frequency.



(a) Higher Mode

(b) Lower Mode

Fig. 5 Comparison of backbone phase spectrum with original phase spectrum

The jumps in the backbone phase spectrum are used as bases to unwrap the original phase spectrum. The final unwrapped phase spectrum is the combination of the phase spectra for the lower mode and the higher mode. For the frequencies below 105 Hz at which the mode is changed, the phase spectrum of the lower mode should be used to determine the final backbone phase spectrum and, above 105 Hz, the phase spectrum of the higher mode should be used. In Fig. 6 the final unwrapped backbone phase specrum determined from the higher and lower modes is shown with the original phase spectrum. By this approach the difficulty in interpreting the jumps in the complicated original phase spectrum can be eliminated.

In summary, the procedure for impulse response filtration technique is as follows:

1. Calculate impulse response either from the time signals recorded at two receivers or from the measured transfer function.
2. Generate Gabor spectrogram for the impulse response determined at the previous step.
3. Apply a time window to the impulse response. The time window for the higher mode can be written as in Eq. 3.

$$f(n)=\begin{cases} 1, & 1\leq n<p_l \\ \frac{1}{2}\left(\cos\frac{n-p_l}{L_l}\pi+1\right), & p_l\leq n<p_l+L_l \\ 0, & p_l+L_l<n<p_h \\ \frac{1}{2}\left(\cos\frac{p_h+L_h-n}{L_h}\pi+1\right), & p_h\leq n<p_h+L_h \\ 1, & p_h+L_h<n\leq N \end{cases} \qquad \cdots\cdots (3)$$

The one for the lower mode can be written as in Eq. 4.

$$f(n)=\begin{cases} 0, & 1\leq n<p_l \\ \frac{1}{2}\left(\cos\frac{p_l+L_l-n}{L_l}\pi+1\right), & p_l\leq n<p_l+L_l \\ 1, & p_l+L_l<n<p_h \\ \frac{1}{2}\left(\cos\frac{n-p_h}{L_h}\pi+1\right), & p_h\leq n<p_h+L_h \\ 0, & p_h+L_h<n\leq N \end{cases} \qquad \cdots\cdots (4)$$

In Eqs. 3 and 4, p_l, p_h are the initial points of the transition band, L_l, L_h are the lengths of the cosine-tapered window, and N is the total number of points in the impulse response. For the higher mode, the filter should be symmetric about the center. Therefore, L_l is equal to L_h and p_h is equal to $N-(p_l+L_l)$. Parameters for the time window should be determined based on the Gabor spectrogram.

4. Calculate the modified transfer functions of the filtered impulse responses, which are the Fourier transforms of the modified impulse response, for the lower and higher modes, respectively. The phase spectrum of the modified transfer function is called a backbone phase spectrum.
5. Unwrap the phase spectra of the modified transfer functions for the lower and higher modes, respectively. Unwrapping of the phase spectra can be done easily by using a threshold like 300° because these phase spectra have clear saw-tooth patterns. These two unwrapped phase spectra are combined to form the final backbone unwrapped phase spectrum. Below the mode conversion frequency, the phase spectrum of lower mode is used,

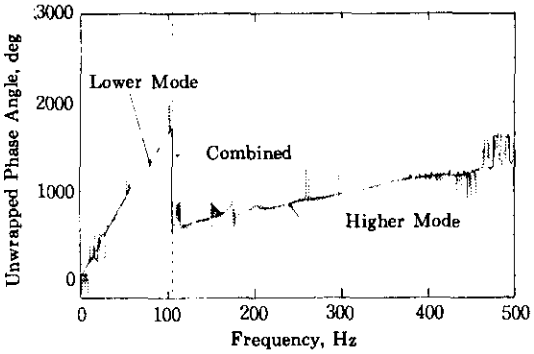


Fig. 6 Comparison of unwrapped backbone phase spectrum and unwrapped original based spectrum based on IRF technique

and above the mode conversion frequency, the phase spectrum of higher mode is used.

6. Unwrap the phase spectrum of the original transfer function based on the jumps in the backbone phase spectrum. The number of jumps used in unwrapping should be inherited from each backbone phase spectrum of lower and higher modes.

3. VERIFICATION OF IMPULSE RESPONSE TECHNIQUE

To verify the impulse response filtration technique for the interpretion of the phase spectrum, an SASW measurement was theoretically simulated by using the experimental setup in Fig. 7. The modeling algorithm for the theoretical simulation is based on the dynamic stiffness matrix method (Kausel and Peek, 1982).

For the given experimental setup on a pavement system, a pair of time signals at the two receiver positions and a phase velocity dispersion curve were generated. The theoretically determined time signals are shown in Fig. 8, and a portion of the impulse response calculated from these time signals is shown in Fig. 9.

In order to look for mode conversion, a Gabor spectrogram was generated for the original impulse response. The contour plot of the Gabor spectrogram is shown in Fig. 10. As in the Gabor spectrogram for the real measurements on the runway of JFK airport which is shown in Fig. 3, two different wave groups are clearly observed. They are clearly separated with the boundary of about 114 Hz. The lower mode appears at the frequency range below about 114 Hz, and the higher mode appears at the frequency range.above about 114 Hz.

Based on the information of the modes of wave groups, the impulse response filters were constructed appropriately for the lower and the highter modes. The filters for IRF are shown in Fug. 9 together with the applied filters to generate backbone phase spectra. The phase unwrapping of original phase spectrum was performed in two parts. The one is for the frequencies below the mode conversion frequency and the other is for the frequencies above the mode conversion frequency.

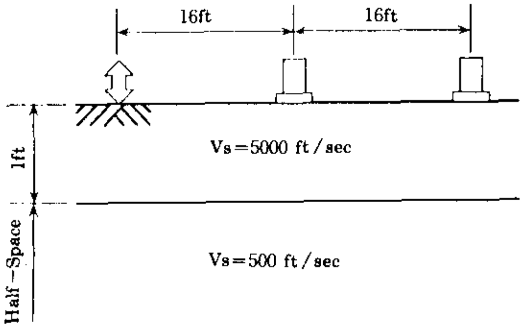


Fig. 7 Experimental setup used for theoretical experiment

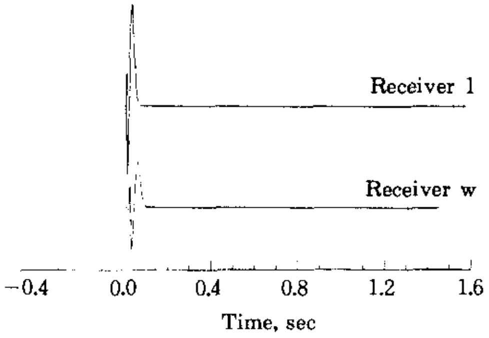


Fig. 8 Time history of displacements theoretically determined at two receivers in a line with source

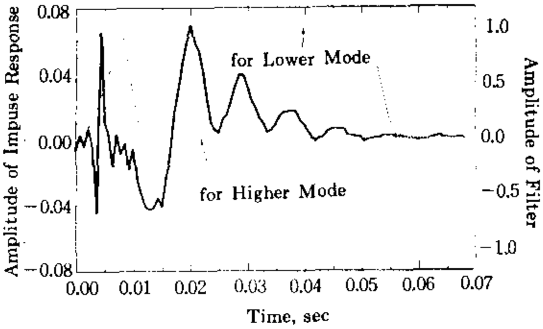


Fig. 9 Original impulse response, backbone impulse response and the filters applied to generate backbone impulse responses

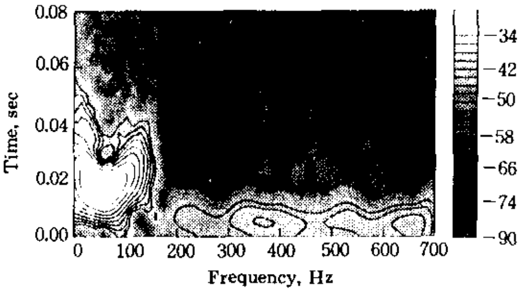


Fig. 10 Gabor spectrogram for the impulse response in Fig.9

Also, the theoretical unwrapped phase spectrum was backcalculated by using the relationships between the unwrapped phase angle, theoretical phase velocities shown in Fig. 12 and the receiver spacing. The relationships can be written as :

$$\theta=\frac{d}{\lambda}\cdot 360° \quad \cdots\cdots (6)$$

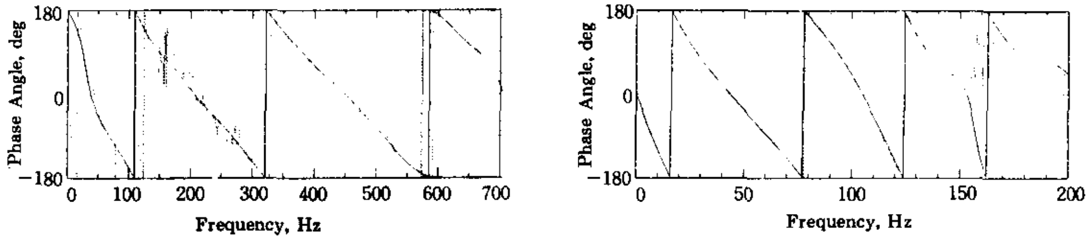
where θ=unwrapped phase angle, d=receiver spacing, and λ=wavelength.

The theoretical phase spectrum is compared with the unwrapped phase spectrum determined by the impulse response filtration technique in Fig. 13. This comparison shows that the original phase spectra interpreted by IRF technique is in perfect agreement with the theoretically determined unwrapped phase spectrum and the interpretation of the phase spectrum by the impulse response filtration technique is correct.

4. CONCLUSIONS

A new algorithm called impulse response filtration(IRF) technique has been developed, which enables unwrapping of the phase spectrum determined by surface wave measurements specifically on the pavement systems. This new method significantly outperforms the conventional phase unwrapping scheme even in the case of complicated wave propagation which carries several wave groups and uncorrelated signals.

Specially at the pavement system where the wave propagation is composed of two distinct wave groups, the theoretical simulation of an SASW measurement revealed that phase unwrapping without the knowledge of mode conversion leads to wrong phase velocities and that mode conversion should be realized to retrieve correct phase velocities from the surface wave measurements.



(a) Higher Mode

(b) Lower Mode

Fig. 11 Comparison of Backgone Phase Spectra with Original Phase Spectrum

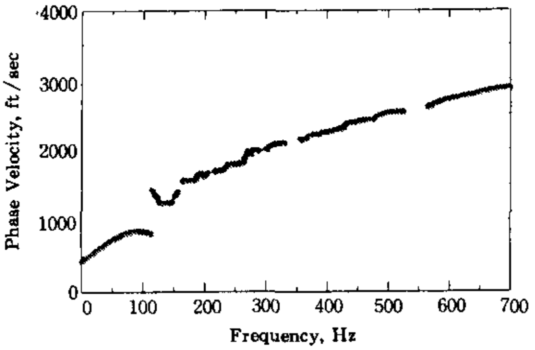


Fig. 12 Theoretical phase velocity dispersion curve for an experimental setup of Fig.7

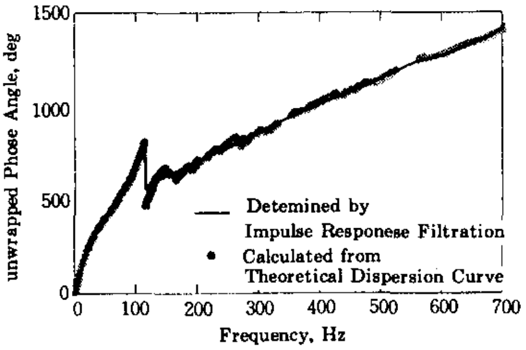


Fig. 13 Comparison of the unwrapped phase spectrum calculated from the theoretical dispersion curve with the phase spectrum determined by IRF technique

5. ACKNOWLEDGMENT

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NOTATION

The following symbols are used in this paper.

A_X, A_Y = spectral amplitudes of time signals for a frequency
d = receiver spacing
F_X, F_Y = Fourier transforms of time signals
$f(n)$ = filter function
H_{YX} = transfer function
$b(t)$ = impulse response
L = the length of the transition band in the proposed filter
l = wavelength
N = the length of the time signal
p_l = the length of the passband
q = the unwrapped phase angle
θ_X, θ_Y = the phase shifts of time signals for a frequency

REFERENCES

1. Dziewonski, A., Bloch, S., and Landisman, M.(1969), "A Technique for the Analysis of Transient Seismic Signals", Bull. Seismol. Soc. Am. Vol. 59, No. 1, pp.427-444
2. Joh, S.-H.(1992), A Users's Guide to WinSASW: Data Reduction and Analysis Program for SASW Measurement, The University of Texas at Austin.
3. Joh, S.-H.(1996), Advances in Interpretation and Analysis Techniques for Spectral-Analysis-of-Surface-Waves (SASW) Measurements, Ph.D. Dissertation, The University of Texas at Austin
4. Karl, J.H.(1989), An Introduction to Digital Signal Processing, Academic Press, Inc.
5. Kausel, Eduardo, and Peek, Ralf(1982), "Dynamic Loads in the Interior of a Layered Stratum: An Explicit Solution", Bull. Seismol. Soc. Am. Vol. 72, No. 5, pp.1459-1508

(접수일자. 1996. 10. 5)