손상된 재료의 탄소성변형에 대한 운동학적 해석

Kinematic Description of Damage-Elastoplastic Deformation

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요 지

본 논문에서는 4차 손상유효 tensor를 이용하여 유한 탄소성변형에 대한 운동학적인 손상해석이 소개된다. 이는 뼈대구조에서의 연속체 손상역학의 유효응력개념을 통하여 이루어 진다. 소규모 변형율상태에만 적용할 수 있는 등가변형율 혹은 에너지법과는 달리 제안된 운동학적인 방법은 유효변형율과 유한변형율에 적용할 수 있는 소규모 변형상태에 있어서의 손상탄소성변형율 사이의 관계를 제공한다. 이는 실제 형상과 가상의 유효형상 모두에 대한 변형장의 운동학을 직접 고려하여 수행된다. 이 방법은 등가변형율이나 변형율 에너지 경우처럼 소규모 변형율에 한정되지 않으며, 유한변형율에 대한 에너지등가의 가정과 일치함을 보여준다. 본 논문에서는 손상이 탄소성영역에서 운동학적으로 표현되며, 손상유효 tensor는 2차 손상 tensor를 통해 손상을 운동학적 측정값이 항으로 특징지워 진다.

Abstract

In this paper the kinematics of damage for finite elastoplastic deformations is introduced using the fourth-order damage effect tensor through the concept of the effective stress within the framework of continuum damage mechanics. Unlike the approach of strain equivalence or energy equivalence, which is applicable only to small strains, the proposed kinematic description provides a relation between the effective strain and the damage elastoplastic strain in finite deformation. This is accomplished by directly considering the kinematics of the deformation field both real configuration. The proposed approach shows that it is equivalent to the hypothesis of energy equivalence at finite strains. The damage effect tensor in this work is explicitly characterized in terms of a kinematic measure of damage in the elastoplastic domain through a second-order damage tensor.

Keywords : *damage mechanics, kinematics of damage, finite strain*

1. INTRODUCTION

In 1958, Kachanov[15] introduced the concept of effective stress in damaged materials. This pioneering work started the subject that is now known as continuum damage mechanics.

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Research in this area has steadily grown and reached a stage that warrants its use in today's engineering applications. Continuum damage mechanics is now widely used in different areas including brittle failure (Krajcinovic[16,17], Krajcinovic and Foneska[18], Lubarda *et. al.*[26]), ductile failure (Lemaitre[21,22], Chaboche[5~8], Chow and Wang[9]), composite materials (Allen *et. al.*[1], Boyd et. al.[4], Voyiadjis and Kattan[34], Voyiadjis and Park[35,36]) and fatigue (Chow and Wei[10]). In this theory, a continuous damage variable is defined and used to represent degradation of the material which reflects various types of damage at the micro-scale level like nucleation and growth of voids, cavities, micro-crack, and other microscopic defects.

In continuum damage mechanics, the effective stress tensor is usually not symmetric. This leads to a complicated theory of damage mechanics involving micropolar media and the Cosserat continuum. Therefore, to avoid such a theory, symmetrization of the effective stress tensor is used to formulate a continuum damage theory in the classical sense (Lee *et. al.*[20], Sidoroff[31], Cordebois and Sidoroff[12], Murakami and Ohno[29], Betten[2], and Lu and Chow[24]. A linear transformation tensor, defined as a fourth-order damage effect tensor is used to symmetrize the effective stress tensor.

The kinetics of damage is well defined presently through the effective stress concept.

However, the kinematics of the deformation with damage is only considered indirectly and is only limited to the small strain theory based on the hypothesis of the strain equivalence[23] or energy equivalence[32]. The finite deformation damage models by Ju[13] and Zbib[39] emphasize that "added flexibility" due to the existence of microcracks or microvoids is already embedded in the deformation gradient implicity. Murakami[28] presented the kinematics of damage deformation using the second-order damage tensor. However, the lack of an explicit formulation for the kinematics of finite deformation with damage leads to the failure in obtaining an explicit derivation of the kinematics that directly consider the damage deformation.

The kinematics of damage is described here using the second-order damage tensor. The deformation gradient of damage is defined using the second-order damage tensor. The Green deformation tensor of the elastic damage deformation is also derived.

For a detailed review of the principles of continuum damage machanics as used in this work, the reader is referred to the works of Kachanov[18], Lemaitre[21,22], Krajcinovic[17], Lubarda and Krajcinovic[25], Chaboche[6~8], Murakami[28], Sidoroff[31,32], and Voyiadjis and Kattan[33]

2. THEORETICAL PRELIMINARIES

Referring to Figure 1, the initial undeformed configuration of a body is denoted by C°, while the elastic damage deformed configuration after the body is subjected to a set of external agencies is denoted by C. The body in configuration C° undergoes a sequence of deformations starting with an elastoplastic deformation without damage, followed by a damage deformation. This is indicated by path I in Figure 1. The configuration denoted by C^{ep} implies the elastoplastic deformed configuration. The initial undeformed body may have a pre-existing damage state. A fictitious effective configuration for the body denoted by $\bar{C}$ is assumed to be obtained from C by removing all the damage that the body has undergone. This is the fictitious effective configuration which is based on the effective stress concept. In this configuration, the body has only deformed elastoplastically without damage. $\bar{C}$ is obtained from C by applying a specific stress distribution on the body in order to remove all existing damage in configuration C. The initial undeformed body may have a pre-existing damage state. In addition to the fictitious effective configuration $\bar{C}$, the initial fictitious effective configuration denoted by $\bar{C}^{\circ}$ is defined by removing the initial damage from the initial undeformed configuration of the body by applying a specific state of stress. In the case of no initial damage existing in the undeformed body, the initial fictitious effective configuration is identical to the initial undeformed configuration.

3. DESCRIPTION OF DAMAGE STATE

The damage state can be described using an even order tensor(Leckie[19], Onat[30] and Betten[3]. Ju[14] pointed out that even for isotropic damage one should employ a damage tensor(not a scalar damage variable) to characterize the state of damage in materials. However, the damage generally is anisotropic due to the external agency condition or the material nature itself. Although the fourth-order damage tensor can be used directly as a linear transformation tensor to define the effective stress tensor, it is not easy to characterize physically the fourth-order damage tensor compared to the sceond-order damage tensor. In this work, the damage is considered as a symmetric second-order tensor. The second-order damage tensor is given by Murakami[27] as follows

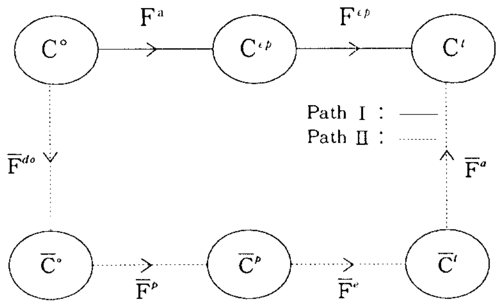

Fig. 1 Schematic repersentation of damage-elastoplastic deformation configurations

$$\phi_{ij} = \sum_{k=1}^{3} \hat{\phi}_k \hat{n}_i^k \hat{n}_j^k \text{ (no sum in k)} \tag{1}$$

or

$$\boldsymbol{\phi} = \boldsymbol{b}^T \hat{\boldsymbol{\phi}} \boldsymbol{b} \tag{2}$$

where hat $\hat{n}^k$ is an eigenvector corresponding to the eigenvalue, hat $\hat{\phi}_k$, of the damage tensor, ϕ. The principal damage tonsor, $\hat{\phi}$, in equation (2) is given by

$$\hat{\phi}_{ij} = \begin{bmatrix} \hat{\phi}_1 & 0 & 0 \\ 0 & \hat{\phi}_2 & 0 \\ 0 & 0 & \hat{\phi}_3 \end{bmatrix} \tag{3}$$

and the second order transformation tensor $\boldsymbol{b}$ is given by

$$b_{ik} = \begin{bmatrix} n_1^1 & n_2^1 & n_3^1 \\ n_1^1 & n_2^2 & n_3^2 \\ n_1^3 & n_2^3 & n_3^3 \end{bmatrix} \tag{4}$$

This proper orthogonal transformation tensor requires that

$$b_{ik} b_{kj} = \delta_{ik} \tag{5}$$

or

$$b\, b^T = I \tag{6}$$

and the determinant of the matrix [b] is given by

$$|[b]| = 1 \tag{7}$$

Voyiadjis and Venson[38] quantified the physical values of the eigenvalues hat $\hat{\phi}_k$(k = 1, 2, 3) and the second-order damage tensor ϕ for the unidirectional fibrous composite by measuring the crack density with the assumption that one of the eigendirections of the damage tensor coincides with the fiber direction provided the load is applied uniformely along the fiber direction. This introduces a distinct kinematic measure of damage which is complimentary to the deformation kineamtic measure of strain. A thermodynamically consistent evolution equation for the damage tensor ϕ together with a generalized thermodynamic force conjugate, Y, to the damage tensor is persented in the paper by Voyiadjis and Park[35, 36].

4. CONCEPT OF EFFECTIVE STRESS

In a general state of deformation and damage, the effective stress tensor $\bar{\sigma}$ is related to the stress tensor σ by the following linear transformation (Murakami and Ohno [29])

$$\bar{\sigma}_{ij} = M_{ikjl}\, \sigma_{kl} \tag{8}$$

or

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{M}\, \boldsymbol{\sigma} \tag{9}$$

where σ is the Cauchy stress tensor and M is a fourth-order linear transformation operator called the damage effect tensor. Depending on the form used forbold $\boldsymbol{M}$, it is very clear from equation (8) that the effective stress tensor $\bar{\sigma}$ is generally not symmetric. Using a nonsymmetric effecitve stress tensor as given by equation (8) to formulate a constitutive model will result in the introduction of the Cosserat and a micropolar continua. However, the use of such complicated mechanics can be easily avoided if the proper fourth-order linear transformation tensor is formulated in order to symmetrize the effective stress tensor. Such a linear transformation tensor called the damage effect tensor is obtained in the literature[20, 31] using symmetrization methods. However, it lacks a systematic and consistent approach.

Recently, Voyiadjis and Park[37] provided a solid basis for such transformation of the second-order stress tensor and its justification for the symmetrization. The effective stress tensor is symmetrized using the following expression by Lee et. al.[20].

$$\bar{\sigma}_{ij} = (\delta_{ik} - \phi_{ik})^{-1/2} \sigma_{kl} (\delta_{jl} - \phi_{jl})^{-1/2} \tag{10}$$

The fourth-order damage effect tensor is defined such as

$$M_{ikjl} = (\delta_{ik} - \phi_{ik})^{-1/2} (\delta_{jl} - \phi_{jl})^{-1/2} \tag{11}$$

However it is extremely difficult to obtain an explicit form of the square root of the second order tensor in equation (11). Another approach is that of the damage effect tensor using the fourth order damage tensor ψ as defined by Chaboche[5]

$$M_{ikjl} = (I_{ikjl} - \psi_{ikjl})^{-1} \tag{12}$$

where * is a fourth order identity tensor and is given by

$$I_{ikl} = \frac{1}{2}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}) \tag{13}$$

However, it is not easy to characterize physically the fourth order damage tensor ψ_{ijkl} as opposed to the second-order damage tensor ϕ_{ij}. For the case of isotropic damage, the fourth-order damage tensor is defined by Ju[14] as follows

$$\psi_{ikl} = d_1\delta_{ik}\delta_{l} + d_2 I_{ikl} \tag{14}$$

where d_1 and d_2 are scalars (dependent or independent) damage variables. Using the second-order anisotropic damage tensor ϕ_{ij} in the damage effect tensors given by equation (11), one may lose the physical sence of the net stress tensor due to the presence of the off diagonal elements in the damage tensor ϕ_{ij}. In order to avoid this problem, the damage tensor in the principal axes is used in conjunction with the damage effect tensor. However, the eigendirections of the damage tensor do not coincide with the eigendirections of the stress tensor but rather with the conjugate force tensor. Since the damage tensor ϕ always has three orthogonal principal directions $\hat{n}^k$(k = 1, 2, 3) and three corresponding principal values ϕ_k(k = 1, 2, 3), equation (10) can be expressed as follows in the coordinate system that coincides with the three orthogonal principal directions of the damage tonsor

$$\hat{\bar{\sigma}}_{mn} = (\delta_{mp} - \hat{\phi}_{mp})^{-1/2}\hat{\sigma}_{pq}(\delta_{nq} - \hat{\phi}_{nq})^{-1/2} \tag{15}$$

The effective stress tensor in the principal damage direction coordinate system is given by

$$\hat{\bar{\sigma}}_{mn} = b_{mi} b_{nj} \bar{\sigma}_{ij} \tag{16}$$

Similarly, the stress tensor in the principal damage direction coordinate system is given by

$$\hat{\sigma}_{pq} = b_{pk} b_{ql} \sigma_{kl} \tag{17}$$

Using the principle damage direction coordinate system, equation (8) is expressed as follows

$$\hat{\bar{\sigma}}_{mn} = \hat{M}_{mpnq}\hat{\sigma}_{pq} \tag{18}$$

The fourth-order damage effect tensor given by equation (11) should now be expressed as follows

$$\hat{M}_{mpnq} = (\delta_{mp} - \hat{\phi}_{mp})^{-1/2}(\delta_{nq} - \hat{\phi}_{nq})^{-1/2} \tag{19}$$

This tensor is termed the principal damage effect tensor.

5. FOURTH-ORDER ANISOTROPIC DAMAGE EFFECT TENSOR

The explicit representation of the fourth-order damage effect tensor M using the second-order damage tensor ϕ is of particular importance in the constitutive modeling of damage mechanics. However, it is impossible to use the damage tensor ϕ instead of the principal damage tensor $\hat{\phi}$ directly in the frmulation. Therefore the damage effect tensor M in equation (8) should be obtained from equation (15) using the coordinate transformation.

Substituting equations (16) and (17) into equation (15), one obtains the following relation

$$\bar{\sigma}_{ij} = b_{mi} b_{nj} b_{pk} b_{ql} \hat{M}_{mpnq} \sigma_{kl} \quad (20)$$

The fourth-order tensor M in equation (8) now reduces to the following expression as follows

$$M_{ikjl} = b_{mi} b_{nj} b_{pk} b_{ql} \hat{M}_{mpnq} \quad (21)$$

It is clear that the fourth-order damage effect tensor presented by equation (11) differs from the damage effect tensor obtained by equation (21). Therefore the fourth-order damage effect tensor pesented by equation (11) should be expressed in the principal damage direction coordinate system using the principal damage tensor $\hat{\phi}$.

One of the explicit expressions for the fourth-order damage effect tensors using the principal damage effect tensor given by equation (19) is presented here. The principal damage effect tensor given by equation (19) can be written as follows

$$\hat{M}_{mpnq} = \hat{a}_{mp} \hat{a}_{nq} \quad (22)$$

where the second-order tensor a is given by

$$\hat{\boldsymbol{a}} = [\boldsymbol{I} - \hat{\phi}]^{-\frac{1}{2}} \quad (23)$$

or

$$\hat{a}_{mp} = [\delta_{mp} - \hat{\phi}_{mp}]^{-\frac{1}{2}}$$

$$\begin{bmatrix} \frac{1}{\sqrt{1-\hat{\phi}_1}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-\hat{\phi}_2}} & 0 \\ 0 & & \frac{1}{\sqrt{1-\hat{\phi}_3}} \end{bmatrix} \quad (24)$$

Substituting equation (22) into equation (21), one obtains the following relation

$$M_{ikjl} = b_{mi} b_{nj} b_{pk} b_{ql} \hat{a}_{mp} \hat{a}_{nq}$$
$$= a_{ik} a_{jl} \quad (25)$$

Using equation (25), a second-order tensor is defined as follows

$$a_{ik} = b_{mi} b_{pk} \hat{a}_{mp} \quad (26)$$

The matrix form of equation (26) is as follows:

$$[a] = [b]^T [\hat{a}][b]$$

6. KINEMATICS OF DAMAGE-ELASTO-PLASTIC DEFORMATION

A schematic drawing representing the kinematics of elastoplastic deformation and damage is shown in Figure 1. In Figure 1, the fictitious effective deformation gradient denoted by $\bar{F}$ referred to the fictitious undeformed configuration, $\bar{C}^{\circ}$, is only elastoplastic since deformation due to damage is fictitiously removed. Thus

$$\bar{F} = \bar{F}^{ep}$$

The deformation gradient referred to the undeformed configuration, $\bar{C}^{\circ}$, denoted by F is polarly decomposed into the elastoplastic deformation gradient denoted by F^{ep} and the damage deformation gradient denoted by F^{d} such that

$$F = F^{ep} F^{d}$$

The fictitious effective Green deformation tensor is given by

$$\bar{G} = \bar{F}^T \bar{F}$$

$$\overline{F}^{ep^T}\overline{F}^{ep}$$

The Green deformation tensor of the damage-elastoplastic deformation can be obtained through either path I or path II as shown in Figure 1. Path I gives the Green deformation tensor as follows:

$$G = F^T F$$
$$= F^{d^T} F^{ep} F^a \quad (31)$$

Considering path II the Green deformation tensor is obtained as follows:

$$G = \overline{F}^{d^T}\overline{F}^{ep^T}\overline{F}^{ep}\overline{F}^{d} - (\overline{F}^{d^T}\overline{F}^{d}\overline{F}^{do^T}\overline{F}^{do}) \quad (32)$$

where $\overline{F}^{do}$ and $\overline{F}^{a}$ are the fictitious effective initial damage deformation gradient and the fictitious effective final damage diformation gradient, respectively. It should be noted that the deformation gradients following paths I and II are not related directly since an additional state of stress needs to be superimposed to go from either configuration C° to $\overline{C}^\circ$ or C to $\overline{C}$. This additional stress state removes all possible damage inherited in the material during the course of loading from configuration C° to C. However, the Green deformation tensors may be obtained following paths I or II. This is clearly indicated in eqation (32) where one needs to remove $(\overline{F}^{d^T}\overline{F}^{a})$ due to the additional state of stress superimposed in the body in order to remove the damage in the material. However, one needs to add the initial fictitiously removed deformation due to damage. Both relate the real deformed configuration to the fictitious undamage deformed configuration. For simplicity, one assumes that no initial damage exsists in the initial undeformed body. Consequently one obtains the following relation such that

$$\overline{F}^{do^T}\overline{F}^{do} = I \quad (33)$$

and

$$\overline{F} = F^{ep}$$
$$= \overline{F}^{ep} \quad (34)$$

Using equations (30), (31), (32), (33) and (34), one obtains the Green deformation tensor such that

$$G = F^{d^T}\overline{G}\overline{F}^{d} \quad (35)$$

or

$$G = \overline{F}^{d^T}\overline{G}\,\overline{F}^{d} - (\overline{F}^{d^T}\overline{F}^{d} - I) \quad (36)$$

Form equation (36), one obtains the effective Green deformation tensor as follows

$$G = \overline{F}^{d^{-T}}[G + (\overline{F}^{d^T}F^{d} - I)]\overline{F}^{d^{-1}}$$
$$= \overline{F}^{d^{-T}}G\,\overline{F}^{d^{-T}} - F^{d^{-T}}\overline{F}^{d^{-1}} + I \quad (37)$$

Equating quations (35) and (36), one obtains the following relationship

$$F^{d^{-T}}\overline{G}F^{d} = \overline{F}^{d^T}\overline{G}\,\overline{F}^{d} - \overline{F}^{d^T}\overline{F}^{d} + I \quad (38)$$

The Green-Saint-Venant strain tensor termed the strain tensor simply in this work is defined as follows

$$\varepsilon = \frac{1}{2}(G - I) \quad (39)$$

The corresponding effective strain tensor is defined such that

$$\bar{\varepsilon} = \frac{1}{2}(\bar{G} - I) \tag{40}$$

Substituting equation (37) into equation (40), one obtains the effective strain in terms of the elastic-damage Green tensor and the fictitious effective damage gradient such that

$$\bar{\varepsilon} = \frac{1}{2}\bar{F}^{d^{-T}}(G - I)\bar{F}^{d^{-1}} \tag{41}$$

Finally one obtains the relation between * and * using equations (39) and (41) such that

$$\bar{\varepsilon} = \bar{F}^{d^{-T}}\varepsilon\bar{F}^{d^{-1}} \tag{42}$$

Alternatively, the strain tensor is given by

$$\varepsilon = \bar{F}^{d^T}\bar{\varepsilon}\bar{F}^{d} \tag{43}$$

The proposed approach provides a relation between the effective strain and the damage -elastoplastic strain applicable to also finite strains and is not confined to small strains as in the case of the strain equivalence or the strain energy equivalence approach. Since the fictitious effective deformed cofiguration denoted by $\bar{C}$, is obtained by removing the damage from the real deformed configuration denoted by C, the fictitious effective deformed volume denoted by $\bar{\Omega}$ is similarly obtained as follows

$$\begin{aligned}\bar{\Omega} &= \Omega - \Omega^d \\ &= \sqrt{(1-\hat{\phi}_1)(1-\hat{\phi}_2)(1-\hat{\phi}_3)}\,\Omega\end{aligned} \tag{44}$$

or

$$\Omega = \bar{J}^{d}\bar{\Omega} \tag{45}$$

where Ω is the deformed volume, Ω^d is the damage volume, and bar $\bar{J}^d$ is the Jacobian of the damage deformation. The Jacobian of the damage deformation is given by

$$\bar{J}^{d} = \frac{1}{\sqrt{(1-\hat{\phi}_1)(1-\hat{\phi}_2)(1-\hat{\phi}_3)}} \tag{46}$$

However, the Jacobian of the damage is defined such that

$$\begin{aligned}\bar{J}^{d} &= \sqrt{|\bar{G}^{d}|} \\ &= \sqrt{|\bar{F}^{d^T}\bar{F}^{d}|} \\ &= \sqrt{|\bar{F}^{d^T}||\bar{F}^{d}|}\end{aligned} \tag{47}$$

The determinant of the matrix [a] in equation (27) is given by

$$\begin{aligned}|[a]| &= |[b]^T|\ |[\hat{a}]|\ |[b]| \\ &= |[\hat{a}]| \\ &= \frac{1}{\sqrt{(1-\hat{\phi}_1)(1-\hat{\phi}_2)(1-\hat{\phi}_3)}}\end{aligned} \tag{48}$$

Thus one assumes the following relation similar to equation (8) without loss of generality

$$\begin{aligned}\hat{\bar{\sigma}}_{ij} &= \hat{M}_{ikjl}\hat{\sigma}_{kl} \\ &= \hat{a}_{ik}\hat{\sigma}_{jl}\hat{\sigma}_{kl} \\ &= \hat{\bar{F}}^{d}_{ik}\hat{\bar{F}}^{d}_{jl}\hat{\sigma}_{kl}\end{aligned} \tag{49}$$

for stresses coinciding with the principal directions of damage. Consequently one obtains

$$\hat{\bar{F}}^{d}_{ij} = \hat{a}_{ij} \tag{50}$$

and

$$\hat{\bar{F}}^{d}_{ij} = a_{ij} \tag{51}$$

Although the identity is established between $\bar{J}^d$ and $|a|$, this is not sufficient to demonstrate the validity of equation (50). Equation (50) is assumed here based on the physics of the geometrically symmetrized effective stress concept[11, 37]. Equation (42) may now be expressed as follows

$$\bar{\varepsilon}_{ij} = a_{ik}^{-1} a_{jl}^{-1} \varepsilon_{kl} \tag{52}$$

or

$$\bar{\boldsymbol{\varepsilon}} = \boldsymbol{a}^{-1} \boldsymbol{\varepsilon} \boldsymbol{a}^{-1^T}$$
$$= \boldsymbol{M}^{-1} \boldsymbol{\varepsilon} \tag{53}$$

Similiarly, equation (43) can be writen as follows

$$\varepsilon_{ij} = a_{ik} a_{jl} \bar{\varepsilon}_{kl}$$
$$= M_{ikjl} \bar{\varepsilon}_{kl} \tag{54}$$

or

$$\boldsymbol{\varepsilon} = \boldsymbol{a} \bar{\boldsymbol{\varepsilon}} \boldsymbol{a}^T$$
$$= \boldsymbol{M} \bar{\boldsymbol{\varepsilon}} \tag{55}$$

The relations combining the strain of the damage-elstoplastic deformation and the effective strain in equations (52) and (54) indicate that these relationships are equivalent to those obtained using the hypothesis of energy equivalence (Cordebois and Sidoroff [12]). The details of the damage elasto-plastic constitutive models using the proposed kinematics, the evolution laws of damage and the numerical implementation using finite element method will be stated in the forthcoming paper.

7. CONCLUSION

The fourth-order anisotropic damage effect tensor, M, expressed by the second-order damage tensor ϕ, is reviewed in the process of the geometrical symmetrization of the effective stress tensor with an introduction of a distinct kinematic measure of damage which is complimentary to the deformation kinematic measure of strain. The explicit representation of the fourth-order damage effect tensor is obtained with reference to the principal damage direction coordinate system.

The damage-elastoplastic kinematics at finite strain allows one to obtain the strain tensor of the elastoplasic damage deformation without the use of either the hypothesis of energy equivalence or strain equivalence. The proposed approach provides a relation between the effective strain and the damage-elastoplastic strain applicable to finite strains, not confined to small strains as in the case of the strain equivalence or strain energy equivalence approaches.

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