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구속된 기계시스템의 운동제어 설계

조 중 선*

Motion Control Design of Constrained Mechanical Systems

Joongseon Joh*

ABSTRACT

본 논문은 구속된 기계 시스템의 운동 제어 설계를 위한 새로운 방법을 제안한다. 구속된 기계 시스템의 운동 제어에는 지금까지 주로 사용되어온 Lagrange의 운동 방정식에 의한 모델링 보다 Udwadia 와 Kalaba에 의해 제안된 운동 방정식에 의한 모델링이 더욱 적합함을 보였으며 이는 Holonomic 및 Nonholonomic 구속 조건을 비롯한 대부분의 구속 조건이 포함된다. 문헌에 잘 알려진 두 시스템을 시뮬레이션을 통하여 비교 함으로써 본 논문에 제안된 방법이 보다 우수한 결과를 보여줌을 확인 할 수 있었다. 또한 지금까지 불가능 하였던 비선형 일반 속도(generalized velocity)를 포함한 구속 조건도 용이하게 제어됨을 보임으로써 광범위한 구속된 기계 시스템의 운동 제어 문제를 통일된 방법으로 접근 할 수 있음을 제시하였다.

Key Words: Udwadia-Kalaba's equation of motion(Udwadia-Kalaba의 운동방정식),
motion control of constrained mechanical systems(구속된 기계시스템의 운동제어),
nonholonomic constraint(비홀로노믹 구속조건),
Moore-Penrose generalized inverse(Moore-Penrose의 일반화된 역행렬)

I. INTRODUCTION

Interests on the control of mechanical systems with kinematic constraints are increasing recently. The constraints in the category encompasses usually holonomic and nonholonomic constraints. Numerous papers have been published on the control of mechanical systems with holonomic constraints[(1)(2)(3)]. On the other hand, control of mechanical systems with nonholonomic constraints is investigated somewhat recently and relatively small number of papers are found in literature[(4)(5)(6)]. Furthermore, they usually deals with some specific examples.

Traditionally, Lagrangian mechanics is adapted for the modeling of mechanical systems with holonimic or nonholonomic constraints. It requires the use of Lagrangian multipliers which in turn

* 창원대학교 제어계측공학과

means force measurement when one needs to feedback the Lagrangian multipliers. It becomes usually a source of uncertainty and is desirable to avoid it in the case of motion control.

Udwadia and Kalaba[7] proposed a new method which deals general equations of motion for constrained discrete dynamic systems. Explicit equations of motion and generalized force of constraint can be obtained for the system with constraints of the form $A(q,\dot{q},t)\ddot{q}=b(q,\dot{q},t)$ where A is a known $m\times n$ matrix, b is a known m vector, and q is n generalized coordinates. It includes, among others, the usual holonomic and nonholonomic constraints.

In this paper, the Udwadia and Kalaba's equation is modified for the motion control design of constrained mechanical systems. In Section II, the Udwadia and Kalaba's equation is summarized briefly. The equation is modified in Section III for the motion control design of constrained mechanical systems. New dynamics of the constrained mechanical system and new input matrix are introduced. Two examples from literature are simulated in Section IV for verification of the use of Udwadia and Kalaba's equation for mechanical systems with nonholonomic constraints. Section V concludes this paper with concluding remarks.

II. BACKGROUND MATERIAL

Udwadia and Kalaba[7] proposed a novel method which yields the explicit general equations of motion for constrained discrete dynamic systems. The method can handle many kinds of constraints including holonomic and nonholonomic constraints. Main results of the method is briefly reviewed in this section for later use.

The equations of motion of an unconstrained discrete dynamic system can be represented as

$$M(q,t)\ddot{q}=Q(q,\dot{q},t) \tag{1}$$

where $q\in R^n$ is the generalized coordinates, $\dot{q}=\frac{d}{dt}q$, $\ddot{q}=\frac{d}{dt}\dot{q}$, the $n\times n$ matrix M is symmetric and positive definite, and n-vector Q is the portion of the system that is not directly related to the acceleration $\ddot{q}$

Suppose that the system is subjected to the following constraints,

$$A(q,\dot{q},t)\ddot{q}=b(q,\dot{q},t) \tag{2}$$

where A is a known $m\times n$ constraint matrix and b is a known m-vector. It can be seen that (2)includes holonomic, nonholonomic, and many other kinds of constraints.

The explicit equations of motion of a discrete dynamic system (1) under (2) can be written as

$$M(q,t)\ddot{q}=Q(q,\dot{q},t)+Q_c(q,\dot{q},t) \tag{3}$$

where n-vector $Q_c(q,\dot{q},t)$ represents the generalized constraint forces.

The first main result of Udwadia and Kalaba[7] is the derivation of the explicit equations of motion under constraints (2) as

$$M\ddot{q}=Q+K(b-AM^{-1}Q) \tag{4}$$

where the $n\times m$ matrix $K(q,\dot{q},t)=M^{1/2}(AM^{-1/2})^{+}$ and where is $M^{1/2}$ a unique positive definite square root of a positive definite matrix M and '+' denotes the Moore-Penrose generalized inverse (see Penrose[8]).

The second main result is the derivation of the explicit equations for the generalized constraint forces as

$$Q_c(q,\dot{q},t)=K(b-AM^{-1}Q). \tag{5}$$

It is worth to note that equations (4) and (5) are derived independent to any specific problem.

III. A NEW MODEL FOR MOTION CONTROL OF CONSTRAINED MECHANICAL SYSTEMS

In this section, a new modeling method for motion control of constrained mechanical systems is derived using the Udwadia-Kalaba approach. The constraints can be holonomic, nonholonomic, and many other forms even nonlinear in $q, \dot{q},$ and t.

Consider a mechanical system to be controlled whose model is represented as

$$M(q,t)\ddot{q} + G(q,\dot{q},t) = \tau \tag{6}$$

where n-vector τ is the external forcing term. Suppose that the system is under the constraint

$$f(q,\dot{q},t) = 0 \tag{7}$$

where $f(\cdot): R^n \times R^n \times R \rightarrow R^m$ is assumed to be C^1 (i.e., differentiable).

The control problem is to find , which may be dependent on $q, \dot{q},$ and t, such that one may achieve a given task while observing the constraint for all $t \geq t_0, t_0$ is the initial time. The constrained mechanical system (6) and (7) can be rewritten like the form of (1) and (2) in order to use the Udwadia-Kalaba approach. First, the unconstrained equations of motion (6) can be rewritten as

$$M(q,t)\ddot{q} = \tau - G(q,\dot{q},t). \tag{8}$$

Comparison of (1) and (8) gives

$$Q(q,\dot{q},t) = \tau - G(q,\dot{q},t). \tag{9}$$

Secondly, the constraint (7) can be represented like the form of (2) by differentiating it with respect to t. Differentiation of $f(q,\dot{q},t)$ in (7) with respect to t yields

$$\frac{df}{dt} = \frac{\partial f}{\partial q}\frac{dq}{dt} + \frac{\partial f}{\partial \dot{q}}\frac{d\dot{q}}{dt} + \frac{\partial f}{\partial t} \tag{10}$$

or

$$\frac{df}{dt} = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial \dot{q}}\ddot{q} + \frac{\partial f}{\partial t} \tag{11}$$

Therefore equations (2), (7), and (11) give

$$A(q,\dot{q},t) = \frac{\partial f}{\partial \dot{q}} \quad \text{and} \quad b(q,\dot{q},t) = \frac{\partial f}{\partial q}\dot{q} - \frac{\partial f}{\partial t}. \tag{12}$$

Note that the derivatives such that $\frac{\partial f}{\partial q}$ are to be interpreted as vector forms since the dimensions of f, q and $\dot{q}$ can be more than one. So, a mechanical system in (6) which is to be controlled under the constraints (7) is represented like equations (1) and (2) in the Udwadia-Kalaba approach through (8) and (12).

Application of (4) to the mechanical system with (8) and (12) yields

$$M\ddot{q} = \tau - G + M^{1/2}C^{+}[b - AM^{-1}(\tau - G)] \tag{13}$$

where $C = AM^{-1/2}$ Of course, the existence of M^{-1} is assumed in the approach.

Equation (13) can be rearranged as

$$\begin{aligned} &M\ddot{q} + G - M^{1/2}C^{+}(b + AM^{-1}G) \\ &\quad = (I - M^{1/2}C^{+}AM^{-1})\tau. \end{aligned} \tag{14}$$

The left-hand-side of (14) is the "new" dynamics of the constrained mechanical system and the right-hand-side is the control with "new" input matrix. Usually, the rank of $I - M^{1/2}C^{+}AM^{-1}$ is lower than that of I and the system is "underactuated". It means that the system can be feedback stabilized to an equilibrium manifold with smooth feedback in this case. It is pointed out in Bloch and McClamroch[4], Campion *et al.*[5], Bloch *et al.*[6], and Su and Stepanenko[9].

It is worth to note that (14) is decoupled with the generalized constraint forces. It means that the motion of a mechanical system under constraints can be controlled using only position and velocity feedback, i.e., it does not need force sensing.

Application of (5) to the mechanical system with (8) and (12) gives

$$Q_c = M^{1/2}C^{+}(b + AM^{-1}G) - M^{1/2}C^{+}AM^{-1}\tau. \tag{15}$$

Equation (15) gives the possibility of the control of the generalized constraint forces if desired.

The new modeling method is suitable for motion control since the constraints are embedded into the "new" dynamic equation. The validity of using the Udwadia-Kalaba approach for motion control purpose is demonstrated in the following section with some simulated examples.

IV. SIMULATED EXAMPLES

Validity and efficiency of using the Udwadia-Kalaba approach to the control of mechanical system with nonholonomic constraint is demonstrated in the example 1.

Example 1. Equations of motion of a wheeled robot moving on a horizontal plane in Su and Stepanenko[9] with $L = P = 1$ are adopted in this example, where P is the radius of the wheels and $2L$ is the length of the axis of the front wheels. It is constituted by a rigid trolley equipped with non-deformable wheels. The physical configuration and derivation of the equations can be found in Campion *et al.*[5] and d'Andrea-Novel *et al.*[10].

The unconstrained equations of motion can be expressed as

$$\begin{aligned} M\ddot{x} &= -(u_1 + u_2)\sin\theta \\ m\ddot{y} &= (u_1 + u_2)\cos\theta \\ I_0\ddot{\theta} &= u_1 - u_2 \end{aligned} \tag{16}$$

where x and y are generalized coordinates which represent the position of a reference point of the robot, θ is a generalized coordinate for orientation of the robot, m is the mass of the robot, I_0 is its inertia with respect to a vertical axis which is passing through the reference point, and u_1 and u_2 are control inputs.

Comparison of (16) with (8) and (9) yields and

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} -(u_1 + u_2)\sin\theta \\ (u_1 + u_2)\cos\theta \\ u_1 - u_2 \end{bmatrix} \tag{17}$$

The nonholonomic constraint is expressed as

$$\dot{x}\cos\theta + \dot{y}\sin\theta = 0. \tag{18}$$

Differentiation of (18) with respect to t gives

$$[\cos\theta \ \ \sin\theta \ \ 0]\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \dot{x}\dot{\theta}\sin\theta - \dot{y}\dot{\theta}\cos\theta. \tag{19}$$

Comparison of (19) with (2) yields

$$\begin{aligned} &A = [\cos\theta \ \ \sin\theta \ \ 0] \text{ and} \\ &b = \dot{x}\dot{\theta}\sin\theta - \dot{y}\dot{\theta}\cos\theta). \end{aligned} \tag{20}$$

Substitution of (17) and (20) into (4) gives

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_0 \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -\sin\theta & -\sin\theta \\ \cos\theta & \cos\theta \\ 1 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + m\begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}(\dot{x}\dot{\theta}\sin\theta - \dot{y}\dot{\theta}\cos\theta). \tag{21}$$

Equation (21) represents a new equations of motion where the nonholonomic constraint (18) is embedded. When the outputs are chosen as $[y \ \theta]^T$ the governing equations of motion become

$$\begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos\theta \\ 1 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + m\begin{bmatrix} \sin\theta \\ 0 \end{bmatrix}(\dot{x}\dot{\theta}\sin\theta - \dot{y}\dot{\theta}\cos\theta). \tag{22}$$

If the control task is to drive $y, \dot{y}, \theta,$ and $\dot{\theta}$ to $y = \dot{y} = \theta = \dot{\theta} = 0$, can choose u_1 and u_2 as

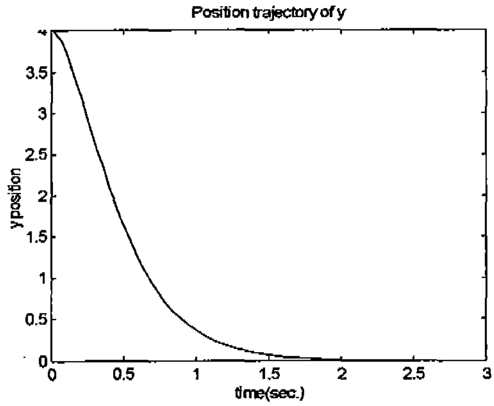
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos\theta \\ 1 & -1 \end{bmatrix}^{-1}\left(-m\begin{bmatrix} \sin\theta \\ 0 \end{bmatrix}(\dot{x}\dot{\theta}\sin\theta - \dot{y}\dot{\theta}\cos\theta) + \begin{bmatrix} -l_1 y - l_2\dot{y} \\ -m_1\theta - m_2\dot{\theta} \end{bmatrix}\right), \ \theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots \tag{23}$$

where l_1, l_2, m_1, and m_2 can be determined to satisfy desired performances. This results in

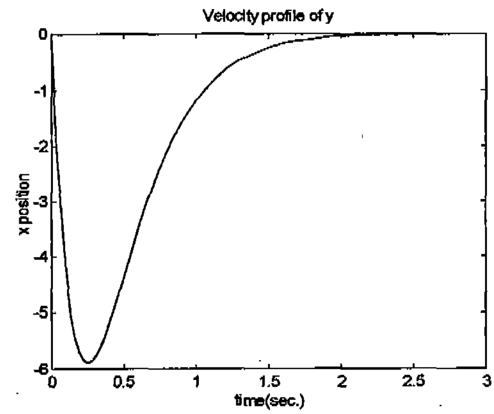
$$\begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -l_1 y & -l_2\dot{y} \\ -m_1\theta & -m_2\dot{\theta} \end{bmatrix}. \tag{24}$$

Equation (24) assures that the control task can be achieved.

A simulation is performed to the system (21) using the control (23). System parameters and initial conditions are the same as in Su and Stepanenko[9] for comparison, i.e., $m = 0.5$, $I_0 = 0.5$, $x(0) = 0$, $\dot{x}(0) = 0$, $y(0) = 4$, $\dot{y}(0) = 0$, $\theta(0) = 45°$ and $\dot{\theta}(0) = 0$. Control parameters l_1, l_2, m_1 and m_2 are chosen to be $l_1 = m_1 = 16$ and $l_2 = m_2 = 8$.



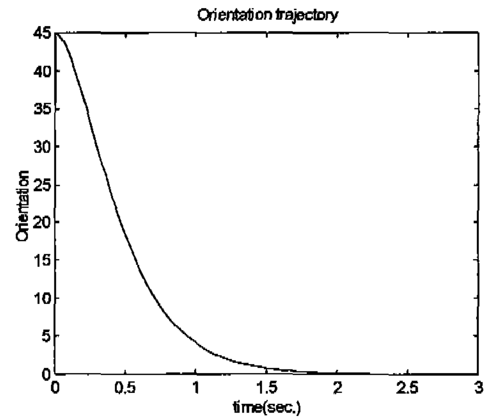
(a) Position trajectory of y



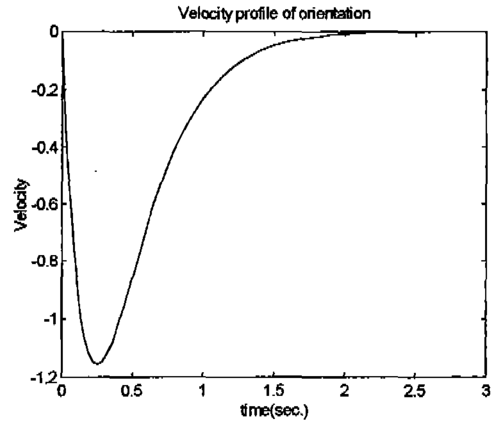
(b) Velocity profile of $\dot{y}$

Fig. 1 Position and velocity evolution of y

Figure 1 and 2 show that the control task is achieved satisfactorily. Much smoother and smaller velocity profiles (in magnitude) than Su and Stepanenko[9] are obtained even though the position and orientation trajectories are similar. Figure 3 shows the control profiles. Much smoother and smaller controls can be observed also.



(a) Orientation trajectory of θ



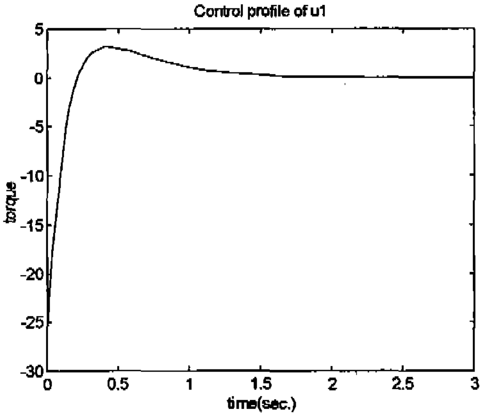
(b) Velocity profile of $\dot{\theta}$

Fig. 2 Orientation and velocity evolution of θ

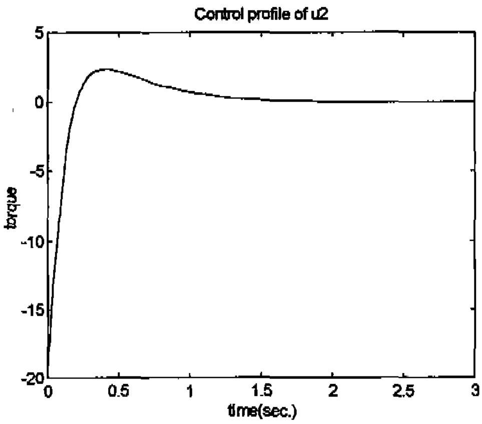
Figure 4 shows that the nonholonomic constraint is satisfied during the motion control.

Another simulated example is investigated for a system with a constraint which is nonlinear in generalized velocities. So far no other modeling technique is able to address such constraint.

Example 2. Consider a simple planar Cartesian manipulator in McClamroch and Wang[1]. The unconstrained equations of motion are expressed as

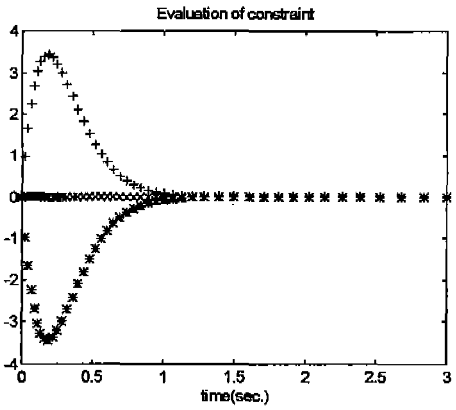


(a) Control profile of u_1



(b) Control profile of u_2

Fig. 3 Control profiles of u_1 and u_2.



(+ + + : $\dot{x}\cos\theta$, * * * : $\dot{y}\sin\theta$, x x x : $\dot{x}\cos\theta+\dot{y}\sin\theta$)

Fig. 4 Evaluation of constraint $\dot{x}\cos\theta+\dot{y}\sin\theta$

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{25}$$

where q_1 and q_2 are generalized coordinates and u_1 and u_2 are control inputs.

Comparison of (25) with (8) and (9) yields

$$M=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \; and \; Q=\begin{bmatrix}u_1\\ u_2\end{bmatrix}. \tag{26}$$

The constraint equation is given by

$$4q_1^2+q_2^2-1=0. \tag{27}$$

The constraint, after taking derivative with respect to t twice, is

$$[8q_1 \;\; q_2]\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=-8\dot{q}_1^2-\dot{q}_2^2. \tag{28}$$

Comparison of (28) with (2) yields

$$A=[8q_1 \;\; q_2] \text{ and } b=-8\dot{q}_1^2-\dot{q}_2^2. \tag{29}$$

Substitution of (26) and (29) into (4) gives

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\frac{1}{64q_1^2+q_2^2}\begin{bmatrix}q_2^2 & -8q_1q_2\\ -8q_1q_2 & 64q_1^2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}+\frac{-(8\dot{q}_1^2+\dot{q}_2^2)}{64q_1^2+q_2^2}\begin{bmatrix}8q_1\\ q_2\end{bmatrix}. \tag{30}$$

Apparently the numerator $64q_1^2+q_2^2$ is nonzero since $4q_1^2+q_2^2-1=0$ or $64q_1^2+16q_2^2-16=0$. Equation (30) represents a new equations of motion where the constraint (27) is embedded. Note that the determinant of the "new" input matrix is zero, i.e., rank=1. Therefore, the two control u_1 and u_2 are dependent. We propose to choose

$$u_1=u_2=u. \tag{31}$$

The system is

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\frac{1}{64q_1^2+q_2^2}\begin{bmatrix}q_2^2 & -8q_1q_2\\ -8q_1q_2 & 64q_1^2\end{bmatrix}u+\frac{-(8\dot{q}_1^2+\dot{q}_2^2)}{64q_1^2+q_2^2}\begin{bmatrix}8q_1\\ q_2\end{bmatrix}. \tag{32}$$

Let the output variables, i.e., the manipulated variable, be q_1 and $\dot{q}_2$. The dynamics governing the output are

$$\begin{bmatrix}\dot{q}_1\\ \ddot{q}_1\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_2\end{bmatrix}+\frac{1}{64q_1^2+q_2^2}\begin{bmatrix}0\\ q_2^2-8q_1q_2\end{bmatrix}u+\begin{bmatrix}0\\ 8q_1\end{bmatrix}\frac{-(8\dot{q}_1^2+\dot{q}_2^2)}{64q_1^2+q_2^2}. \quad (33)$$

Choose the control u to be the following:

$$u=\frac{64q_1^2+q_2^2}{q_2^2-8q_1q_2}\left(-k_1q_1-k_2\dot{q}_1+\frac{8\dot{q}_1^2+\dot{q}_2^2}{8q_1(64q_1^2+q_2^2)}\right). \quad (34)$$

The resulting closed-loop system is

$$\begin{bmatrix}\dot{q}_1\\ \ddot{q}_1\end{bmatrix}=\begin{bmatrix}0 & 1\\ -k_1 & -k_2\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix}. \quad (35)$$

The system can be made asymptotically stable by any choice of $k_1>0$ and $k_2>0$. This renders $q_1\to 0$ and $q_2\to 0$ as $t\to\infty$.
If the task is to drive and then the control law needs to be modified:

$$u=\frac{64q_1^2+q_2^2}{q_2^2-8q_1q_2}\left(-k_1q_1-k_2\dot{q}_1+\frac{8\dot{q}_1^2+\dot{q}_2^2}{8q_1(64q_1^2+q_2^2)}+k_1q_{1d}+k_2\dot{q}_{1d}\right). \quad (36)$$

It is assumed that q_{1d} and $\dot{q}_{1d}$ are constants. It shows that the problem in McClamroch and Wang[1] can be handled easily.

Now, suppose that the constraint is changed to

$$\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right)=1. \quad (37)$$

This implies that the kinetic energy of the system, even under the external control u_1 and u_2, are kept constant. Note that the constraint is *nonlinear* in $\dot{q}_1$ and $\dot{q}_2$

Differentiation of the constraint with respect to t yields

$$\frac{1}{2}(2\dot{q}_1\ddot{q}_1+2\dot{q}_2\ddot{q}_2)=0 \quad (38)$$

or

$$[\dot{q}_1 \quad \dot{q}_2]\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=0. \quad (39)$$

Comparison of (39) with (2) yields

$$A=[\dot{q}_1 \quad \dot{q}_2] \text{ and } b=0. \quad (40)$$

Substitution of (26) and (40) into (4) gives

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}q_2^2 & -\dot{q}_1\dot{q}_2\\ -\dot{q}_1\dot{q}_2 & \dot{q}_1^2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}. \quad (41)$$

Equation (41) represents a new equations of motion where the nonlinear constraint (37) in embedded. The two control u_1 and u_2 are dependent since the input matrix is of rank 1. We propose to choose

$$u_1=u_2=u \quad (42)$$

then (41) becomes

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\dot{q}_2^2 & -\dot{q}_1\dot{q}_2\\ -\dot{q}_1\dot{q}_2 & \dot{q}_1^2\end{bmatrix}\begin{bmatrix}u\\ u\end{bmatrix}. \quad (43)$$

Choose the output to be q_1 and $\dot{q}_1$. The governing dynamics are

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}=\begin{bmatrix}\dot{q}_1\\ \frac{1}{2}(\dot{q}_2^2-\dot{q}_1\dot{q}_2)u\end{bmatrix} \quad (44)$$

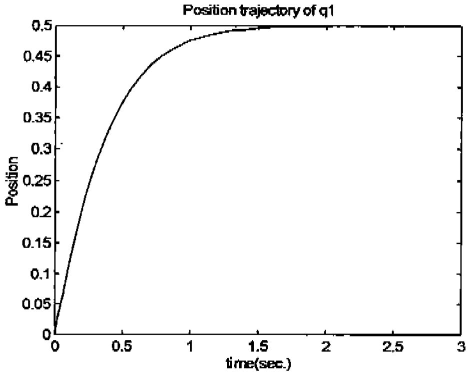
To drive $q_1\to q_{1d}$ and $\dot{q}_1\to\dot{q}_{1d}$ the control law is chosen to be:

$$u=\frac{2}{\dot{q}_2^2-\dot{q}_1\dot{q}_2}\left(-k_1(q_1-q_{1d})-k_2(\dot{q}_1-\dot{q}_{1d})\right) \quad (45)$$

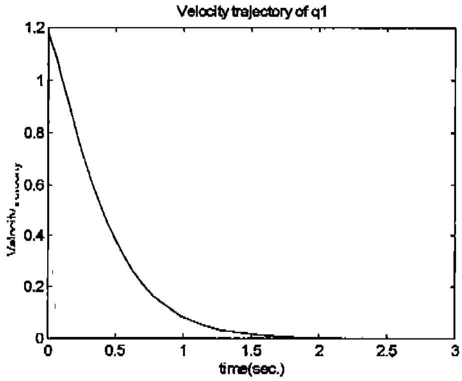
where $k_1>0$ and $k_2>0$ and q_{1d} and $\dot{q}_{1d}$ are constants.

A simulation is performed with initial conditions as $q_1(0)=0, \dot{q}_1(0)=1.1832$, $q_2(0)=0$, and $\dot{q}_2(0)=0.7746$. It means $\dot{q}_1(0)^2=1.4$ and $\dot{q}_2(0)^2=0.6$ in order to satisfy the constraint $\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right)=1$
The desired outputs are $q_{1d}=0.5$ and $\dot{q}_{1d}=0$. Control parameters are chosen to be k_1=16 and k_2=8.

Figure 5 shows that the control task is achieved satisfactorily.



(a) Position trajectory of q_1



(b) Velocity profile of $\dot{q}_1$

Fig. 5 Position and velocity evolution of q_1

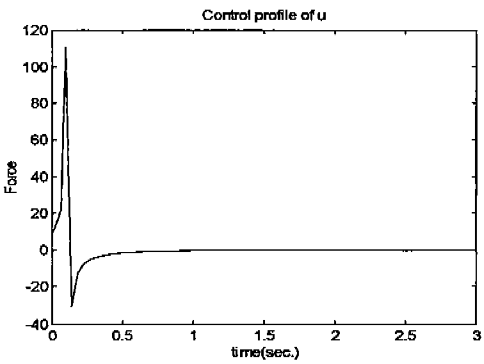
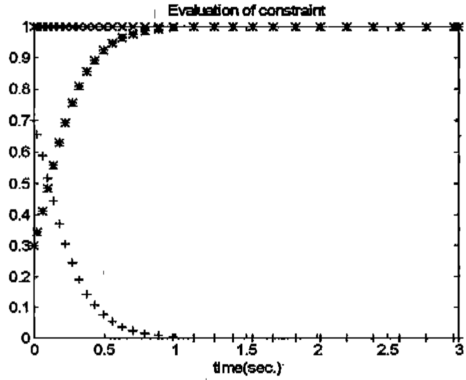


Fig. 6 Control profile of u

Figure 6 shows the control profile of u.

Figure 7 reveals that the constraint which is nonlinear in $\dot{q}_1$ and $\dot{q}_2$ is satisfied during the motion control.



(+++ : $\frac{1}{2}\dot{q}_1^2$, *** : $\frac{1}{2}\dot{q}_2^2$, xxx : $\frac{1}{2}(\dot{q}_1^2+\dot{q}_2^2)$)

Fig. 7 Evaluation of constraint $\frac{1}{2}(\dot{q}_1^2+\dot{q}_2^2)$

V. CONCLUDING REMARKS

It is investigated that the new modeling technique using the Udwadia-Kalaba approach has many advantages for the motion control of constrained mechanical systems. Control of mechanical systems with many constraints including nonholomic constraint can be modeled in a unified way and results a new equations of motion where the constraints are embedded. It applies to the constraints that can be nonlinear in q, $\dot{q}$, and t. So far, no other technique applies to when f is nonlinear in $\dot{q}$.. The resulting control scheme is only position and velocity feedback. It does not need force sensing (i.e., no force feedback). All other technique requires the use of Lagrange multiplier which in turn means force measurement (when one needs to feedback the Lagrange multiplier).

Because of the explicit expression of the equation of motion, one can see explicitly how the input is affected due to the presence of constraint (ref. eq. (14), when the input matrix is changed from I to $I - M^{1/2}C^{+}AM^{-1}$). This shows the effect of constraint on control. Many other work (such as McClamroch and Wang[1] and Su and Stepanenko[9]) treats the Lagrange multiplier as an *external* signal to the system.

Two simulated examples are investigated to show the validity of the method. It should be mentioned that any elaborated control schemes can be developed more easily to achieve more complex control tasks from the proposed modeling technique.

REFERENCES

1. McClamroch, N. H. and Wang, D., "Feedback Stabilization and Tracking of Constrained Robots," *IEEE Trans. on Automatic Control*, Vol. 33, No. 5, pp. 419-426, 1988.
2. Su, C., Leung, T.P., and Zhou, Q.J., "Adaptive control of robot manipulators under constrained motion," *Proc. IEEE Conf. Decision Control*, pp. 2650-2655, 1990.
3. Su, C., Leung, T.P., and Zhou, Q.J., "Force/motion control of constrained robot using sliding mode," *IEEE Trans. Automatic Control*, Vol. 37, pp. 668-672, 1992.
4. Bloch, A. M. and McClamroch, N. M., "Control of Mechanical Systems with Classical Nonholonomic Constraints," *Proc. of the 28th Conf. on Decision and Control*, Tampa, FL, pp. 201-205, 1989.
5. Campion, G., d'Andrea-Novel, B., and Bastin, G., "Controllability and State Feedback Stabilizability oh Nonholonomic Mechanical Systems," *in Advanced Robot Control*, New York: Springer-Verlag, pp. 106-124, 1991.
6. Bloch, A. M., Reyhanoglu, M., and McClamroch, N. H., "Control and Stabilization of Nonholonomic Dynamic Systems," *IEEE Trans. on Automatic Control*, Vol. 37, No. 11, pp. 1746-1757, 1992.
7. Udwadia, F. E. and Kalaba, R. E., "A New Perspective on Constrained Motion," *Proc. of Royal Society of London*, Series A, Vol. 439, pp. 407-410, 1992.
8. Penrose, R., "A Generalized Inverse for Matrices," Proc. of the Cambridge *Philosophical Society*, Vol. 51, Part 3, pp. 406-413, 1955.
9. Su, C. and Stepanenko, Y., "Robust Motion/Force Control of Mechanical Systems with Classical Nonholonomic Constraints," *IEEE Trans. on Automatic Control*, Vol. 39, No. 3, pp. 609-614, 1994.
10. d'Andrea-Novel, B., Bastin, G., and Campion, G., "Modeling and Control of Non Holonomic Wheeled Mobile Robots," *Proc. of the 1991 IEEE Int. Conf. on Robotics and Automation*, Sacramento, CA, pp. 1130-1135, 1991.