Application of Fuzzy Logic to Sliding Mode Control for Robot Manipulators

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Abstract

In this paper, a new fuzzy sliding mode control algorithm is presented for trajectory control of robot manipulators. A fuzzy logic is applied to a sliding mode control algorithm to have the sliding mode gain adjusted continuously through fuzzy logic rules. With this scheme, the stability and the robustness of the proposed fuzzy logic control algorithm are proved and ensured by the sliding mode control law. The fuzzy logic controller requires only a few tuning parameters to adjust. Computer simulation results are given to show that the proposed algorithm can handle uncertain systems with large parameter uncertainties and external disturbances.

I. Introduction

Since Mamdani[6] and Mamdani and Assilian[7] firstly applied fuzzy theory to control a laboratory model of a steam engine, fuzzy control has been used successfully in a variety of applications. However, it appears that one of the most difficult theoretical issues in applying fuzzy logic to control system is that the lack of stability analysis, and tuning a fuzzy logic controller involves adjusting many parameters.

Recently, methodology, which uses in its idealized form piecewise continuous feedback control laws, resulting in the state trajectory 'sliding' along a discontinuity or sliding surface in the state space, known as *sliding mode control*, has been researched. The concept of sliding mode control has been studied in detail by Utkin[10, 11], where it has been used to stabilize a class of non-linear systems. Sliding mode control is effectively used in the trajectory control of robot arms and has been studied by many researchers in recent years[3, 8, 12].

For faster manipulator dynamics in the presence of model uncertainties such as parameter perturbations, unknown joint frictions and inertias, and external disturbances, fixed controllers cannot be implemented accurately. Fuzzy logic manipulator control laws may alter the control signal to account for changes in robot dynamics and disturbances in the environment.

The fuzzy logic theory in sliding mode control area is used to improve the overall performance of the conventional sliding mode control algorithms in recent years, for example[1, 4, 5]. The algorithm in[1] uses a fuzzy rule base for tuning the thickness of a boundary layer for sliding mode controller. With this scheme, the tracking performance can be improved and good for rejection of control chattering phenomenon. However, large parameter variation and disturbances cannot be handled. Other schemes, introducing fuzzy inference into sliding mode control are presented in[4, 5]. However, these schemes require multiple fuzzy logic processing, thus the computation load required is very high, for multi-input-multi-output plants such as multi-link robots.

In this paper, a fuzzy logic is applied to a sliding mode control algorithm to have the sliding mode gain adjusted continuously through fuzzy logic rules. With this scheme, the stability and the robustness of the proposed fuzzy logic control algorithm are proved and ensured by the sliding mode control law. The algorithm presented in this paper requires only two input value fuzzy regions and one output value fuzzy regions with nine fuzzy control rules even for multi-link robot manipulators. Therefore, the computation required to formulate the driving torque for multi-link robots is reduced greatly. The presented algorithm has other advantages that: fairly large parameter variation and disturbances can be handled; only position and velocity terms need to be measured to implement, hence, it can also be applicable to most electromechanical (mechatronic) systems.

Computer simulation results are given to show that the proposed algorithm can handle uncertain systems with large parameter uncertainties and external disturbances.

The organization of this paper is as follows: section 2 presents some mathematical issues of the control problem; section 3 presents the fuzzy sliding mode control algorithms; in section 4, as an application example, a two link robot manipulator is chosen to show that the proposed control scheme possesses good pro-

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perties under large parameter uncertainties and disturbances; section 5 concludes the paper.

II. Problem Formulation

Consider the rigid body dynamic n-link manipulator derived via the Euler-Lagrange equations(see [2], and Figure 4):

$$\begin{aligned} M(q)\ddot{q}+F(x) &= \tau \\ F(x) &= V(q,\dot{q})\dot{q}+D\dot{q}+G(q)+\tau_d \end{aligned} \tag{1}$$

where $q\in R^n$ and $\dot{q}$ are joint angle and angular velocity, respectively. $M(q)\in R^{n\times n}$ is the inertia matrix, which is symmetric, positive definite. $V(q,\dot{q})\in R^{n\times n}$ contains centrifugal and Coriolis terms. $D\dot{q}\in R^n$ and $G(q)\in R^n$ describe viscous friction and gravity, respectively. $\tau_d\in R^n$ represents the unknown disturbances, such as static friction or Coulomb friction. For simplicity, these terms are combined and expressed as $F(x)$ in (1), where $x=[q,\ \dot{q}]^T$ is the state vector. Note that $F(x)$ is the nonlinear function which is not exactly known but the extent of the imprecision on $F(x)$ is upper bounded by a known continuous function of x. $\tau\in R^n$ is the vector of input torques.

The control problem is to synthesize a control law for u such that the state x traces the desired trajectory, $x_d=[q_d\ \ \dot{q}_d]^T$, with a certain precision defined by

$$\|q_d-q\|\le\gamma_1,\ \ \|\dot{q}_d-\dot{q}\|\le\gamma_2,\ \ \gamma_1>0,\ \ \gamma_2>0$$

It is assumed that $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are well defined and bounded for all operational time t.

Define

$$e=q_d-q,\quad z=\dot{e}+\Lambda e \tag{2}$$

with $\Lambda=diag(\Lambda_1,\Lambda_2,\cdots,\Lambda_n),\ \Lambda_i>0$.

Then, from (1) and (2),

$$M\dot{z}=M(\Lambda\dot{e}+\ddot{q}_d)+F-u \tag{3}$$

Eq.(3) cannot be given exactly due to the disturbances, modelling uncertainties and unknown parameters, but upper bounds for the norms of (3) can be estimated as follows.

Lemma 1

(a) There exists bounded differentiable functions $\theta(t)\in R^n$ *and bounded nonlinear functions* $l_1(t),l_2(t)\in R^n$ *such that*

$$\begin{aligned} &z^T[M(\Lambda\dot{e}+\ddot{q}_d)+F]+\frac{1}{2}z^T\dot{M}z \\ &\quad=\theta\|z\|+(l_1+l_2\|z\|)\|z\|^2,\quad \forall q,\dot{q} \end{aligned} \tag{4}$$

(b) There exists constant $\eta>0$ *such that*

$$\begin{aligned} &z^T[M(\Lambda\dot{e}+\ddot{q}_d)+F]+\frac{1}{2}z^T\dot{M}z \\ &\quad\le\phi\eta\|z\|,\quad \forall q,\dot{q} \end{aligned} \tag{5}$$

with

$$\phi=1+\|z\|+\|z\|^2 \tag{6}$$

Proof : Noting that q_d and $\dot{q}_d$ are bounded, $F(x)$ is a bounded function at most quadratic in $\dot{q}$, we have that q and $\dot{q}$ are bounded on $\|e\|$ and $\|\dot{e}\|$ respectively. From (2), $\dot{e}=z-\Lambda e$ and thus $\dot{e}(s)=[I-T(s)]z(s)$ with $T(s)=\Lambda(sI+\Lambda)^{-1}$. Denote the H_∞ norm of a stable transfer function by $\|\cdot\|_\infty$. Then, it follows that $\|\dot{e}\|\le\|z\|+\|T\|_\infty\|z\|$. Since $T(s)$ is stable, $\|T\|_\infty$ is bounded and thus $\|\dot{e}\|$ is bounded on $\|z\|$. Now, it is clear that there exist bounded nonlinear functions $\theta(t),r_1(t),r_2(t)\in R^n$ such that

$$M(\Lambda\dot{e}+\ddot{q}_d)+F=\theta+r_1\|z\|+r_2\|z\|^2 \tag{7}$$

Moreover, without loss of generality, θ can be chosen as differentiable functions. Since M is bounded differentiable function of q is bounded on $\dot{q}$. Thus, there exist bounded nonlinear functions $l_1(t),l_2(t)\in R^n$ such that (a) holds. Let η be the largest value of $\sup_t\|\theta(t)\|$, $\sup_t\|l_1(t)\|$ and $\sup_t\|l_2(t)\|$. Then, clearly (b) holds. △△△

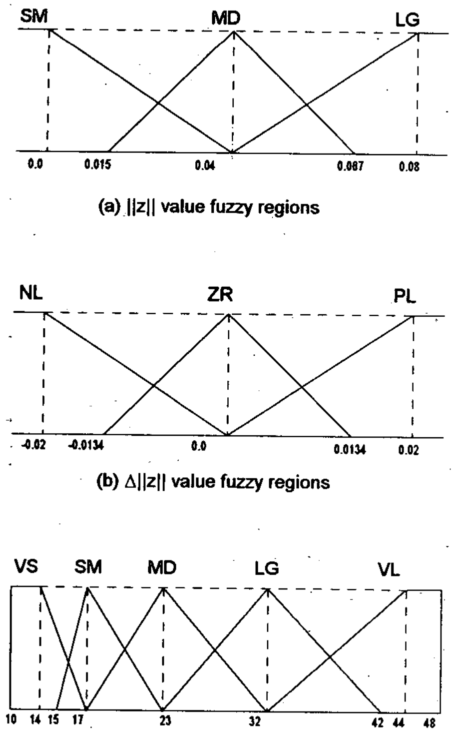
Note that $z(s)=(sI+\Lambda)e$, and $(sI+\Lambda)^{-1}$ is a strictly proper and stable transfer function. Thus $z(t)=0,\forall t>t_0$, $e(t)\to0$ and $\dot{e}(t)\to0$. Therefore, the control problem becomes how to choose the control law, such that $\|z\|\le\varepsilon$, where $\varepsilon>0$ is a predefined tracking error precision (see [9]).

III. Configurations of Fuzzy Sliding Mode Controllers

1. Control Law

The fuzzy sliding mode control law to compute the control input for the uncertain system (1) is designed as

$$u=\begin{cases}\phi\Psi\dfrac{z}{\|z\|}, & \text{if}\|z\|>\varepsilon \\ & \quad ;\ \ \begin{array}{l}\Psi=\text{output of FLC}\\ \text{to satisfy } \Psi>\eta,\\ 0<\varepsilon<1\end{array} \\ \phi\Psi\dfrac{z}{\varepsilon}, & \text{otherwise}\end{cases} \tag{8}$$



(a) ||z|| value fuzzy regions

(b) Δ||z|| value fuzzy regions

(c) output value(ψ) fuzzy regions

Fig. 1. Fuzzy regions and boundary values of fuzzy subsets after tuning.

Table 1. Fuzzy control rules after tuning for simulation of two-link manipulator.

	‖Z‖ value fuzzy regions			
		SM	MD	LG
Δ‖Z‖ value fuzzy regions	SM	VS	SM	MD
	ZR	VS	SM	LG
	PL	VS	MD	VL

The meaning of this table is presented as If ‖Z‖ =SM and Δ‖Z‖ =NL then make Ψ=VS, etc.

with ϕ defined by (6). The structure of (8) is shown in Figure 4.

2. *Fuzzy Logic Control (FLC)*

The FLC input signals are $\|z\|$ and change in the $\|z\|$, i.e. $\Delta\|z\|$. The FLC output signal is change in the sliding mode control gain Ψ. It is generally known that the use of more membership functions allows one to construct more nonlinear relations. In this paper, three membership functions associated with each input variable and five membership functions for the output variable. Therefore, the maximum number of fuzzy control rules is 9. The membership functions associated with $\|z\|$, $\Delta\|z\|$, and Ψ are given in Figure 1. The relationship between the two input fuzzy variables is expressed using nine IF...THEN... fuzzy rules, which is summarized in Table 1. Then, we have the following result.

Theorem 1 : Consider the system (1) with the control law (8). Let η be given by Lemma 1, and Ψ is the output of FLC to satisfy $\Psi > \eta$. Then the closed-loop system is globally stable in the sense that the tracking error z is globally bounded by

$$\|z\| \le \frac{\varepsilon\eta}{\Psi} < \varepsilon \tag{9}$$

Proof : Choose a Lyapunov function

$$v = \frac{1}{2} z^T M z \tag{10}$$

Differentiating (10) with respect to t and substituting (3) gives

$$\dot{v} = \frac{1}{2} z^T \dot{M} z + z^T [M(\Lambda \dot{e} + \ddot{q}_d) + F - u] \tag{11}$$

When $\|z\| > \varepsilon$, from Lemma 1(b) and (8), $\dot{v}$ can be expressed as

$$\dot{v} \le \phi(\eta - \Psi)\|z\| \tag{12}$$

Note that Ψ is obtained from the fuzzy logic rule to satisfy $\Psi > \eta$. Hence, $\dot{v} < 0$ for any z with $\|z\| > \varepsilon$. This implies that z is bounded by $\|z\| \le \varepsilon$.

Thus after the transient response, $\|z\| \le \varepsilon < 1$. In this case, $u = \phi \Psi z / \varepsilon$. Then, $\dot{v}$ can be expressed as $\dot{v} \le \phi\eta\|z\| - \phi\Psi\|z\|^2/\varepsilon$, or

$$\dot{v} \le \phi(\eta - \Psi\|z\|/\varepsilon)\|z\| \tag{13}$$

In (13), we see that $\dot{v} < 0$ for all $\|z\| > \varepsilon\eta/\Psi$. This implies that z is bounded by $\|z\| \le \varepsilon\eta/\Psi$. Note that $\Psi > \eta$, thus we see that $\varepsilon\,\eta/\Psi < \varepsilon$. Therefore, we can conclude that $\|z\|$ is globally bounded by (9). △△△

3. *Fuzzy Rule Inference and Defuzzification*

Once the relationship between the two input variables (in this paper, $\|z\|$ and $\Delta\|z\|$) is expressed, it is needed to evaluate those relations using fuzzy inference. Fuzzy inference, or rule evaluation, employs the composition rule called as min-max inference of fuzzy relations to calculate numerical conclusions to linguistic rules based on system input values. Even though the output of the algorithm is a fuzzy set, the output of the fuzzy controller must be a single crisp value which will serve as input to the controlled process (in this paper, the output of the fuzzy controller will serve as the gain of the sliding mode controller). Therefore, the algebraic manipulation procedure is required to compute the degree of applicability of each rule and the final crisp defuzzified control action at the given moment. Several

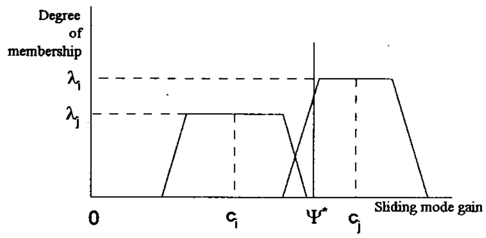


Fig. 2. Defuzzification-Height method.

defuzzification methods have been developed such as COA(center of area), MIM(mean of maxima), Height Method, etc. In this paper, the Height Method is used. The expression to compute the Height Method is in the following.

$$\Psi^* = \frac{\lambda_i c_i + \lambda_j c_j}{\lambda_i + \lambda_j} \tag{14}$$

where Ψ is the final output of fuzzy logic controller (FLC), and the meaning of $\lambda_i, \lambda_j, c_i, c_j$ are degree of each membership and centre of each region respectively as shown in Figure 2.

Note that the final output of FLC is the sliding mode gain Ψ. Then the gain Ψ is used to calculate the sliding model control output u in (8). We see from theorem 1 that if parameters of the FLC are adjusted so that the FLC output Ψ is obtained as $\Psi > \eta$ for all operation time t, the closed-loop system is guaranteed to be stable in the sense that the tracking error is bounded by (9).

Note that Eq.(8) is a sliding mode control algorithm with the sliding mode gain Ψ is defined by fuzzy logic rule, and with $\Omega = \{z | z(t) = 0\}$ as the sliding surface. While the sliding mode control has good robustness, the control law has to be discontinuous across Ω. So, $\varepsilon > 0$ is chosen in algorithm (8) to eliminate the chattering of the control law. With these control algorithms, small tracking errors can be achieved by choosing a small ε. However, small ε with big Ψ will usually result in undesirable vibration on the control signal. Therefore each fuzzy region boundary values are required to be tuned to get a reasonably good result. The boundary value also need to be subjectively chosen so that each fuzzy subset could overlap with its neighbours. This overlap can give a fuzzy controller smooth and stable surface.

Ⅳ. An Application Example

As an application example, a two-link robot manipulator has been simulated, controlled by the fuzzy sliding mode controller, presented in this paper. The manipulator was modelled as a set of nonlinear coupled differential equations (see [2], and Figure 4)

$$\begin{aligned} \tau &= M(q)\ddot{q} + F(\boldsymbol{x}) \\ &= M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + \tau_d \end{aligned} \tag{15}$$

with $F(\boldsymbol{x}) = V(q, \dot{q})\dot{q} + G(q) + \tau_d$, $\tau \in \boldsymbol{R}^2$, $M \in \boldsymbol{R}^{2\times 2}$, $V \in \boldsymbol{R}^{2\times 2}$, $G \in \boldsymbol{R}^2$, and $\tau_d \in \boldsymbol{R}^2$ where

$$M = \begin{bmatrix} m_1 l_1^2 + m_2[l_1^2 + l_2^2 + 2 l_1 l_2 \cos(q_2)] & m_2 l_1 l_2 \cos(q_2) + m_2 l_2^2 \\ m_2 l_1 l_2 \cos(q_2) + m_2 l_2^2 & m_2 l_2^2 \end{bmatrix}$$
$$V = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 & -m_2 l_1 l_2 \sin(q_2)\dot{q}_1 - m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$
$$G = \begin{bmatrix} m_1 l_1 g\cos(q_1) + m_2 g[l_2 \cos(q_1+q_2) + l_1 \cos(q_1)] \\ m_2 l_2 g\cos(q_1+q_2) \end{bmatrix}$$
$$\tau_d = \begin{bmatrix} k_1 sgn(\dot{q}_1) \\ k_2 sgn(\dot{q}_2) \end{bmatrix}$$

with m_n, l_n, q_n are mass, length, joint angle of link n respectively, g is gravity and $c_1 := \cos(q_1)$, $s_{12} := \sin(q_1+q_2)$ *etc.*.

The desired trajectory is supposed to be

$$\begin{aligned} q_{d1} &= 1 + 0.2\sin(\pi t) \\ q_{d2} &= 1 - 0.2\cos(\pi t) \end{aligned}$$

for $t \in [0, 8]$, and the disturbance τ_d was added in the form of

$$\tau_d = \begin{bmatrix} 5\sin(4\pi t) \\ 5\sin(4\pi t) \end{bmatrix} \tag{16}$$

Parameters used in the simulation were

$$l_1 = l_2 = 1\,m, \quad m_1 = m_2 = 1kg$$

While the manipulator was being operated, m_2 was changed from $1kg$ to $3kg$ at t = 2sec and $3kg$ to $1kg$ at t = 4sec. The sampling time was set to be 10^{-3}sec and the controller parameters were chosen to be

$$\Lambda = 6I, \quad \varepsilon = .04$$

Each fuzzy region boundary values after tuning are shown in Figure 1, and the overall system structure is shown in Figure 4.

Figure 3 shows the simulation results under presented control law Eq.(8): (a) position errors; (b) the sliding mode control gain Ψ, *i.e.* fuzzy controller outputs, for $t \in [0, 8]$; (c) control torques for each link of the manipulator.

As shown in Figure 3, tracking errors(position errors) are less than 0.0058rad for no load change. Between t = 2sec and t = 4sec, the tracking errors are increased due to the additive load (m_2) change, and Ψ were adjusted by the fuzzy logic controller to reduce the tracking errors. We see that the tracking performance is very good using the control law, presented in this paper.

Note that the simulation results achieved in the presence of the disturbance of (16) over the interval $t \in [0, 8]$, indicate that the proposed algorithm works effectively for both the given parameter uncertainties and the disturbances.

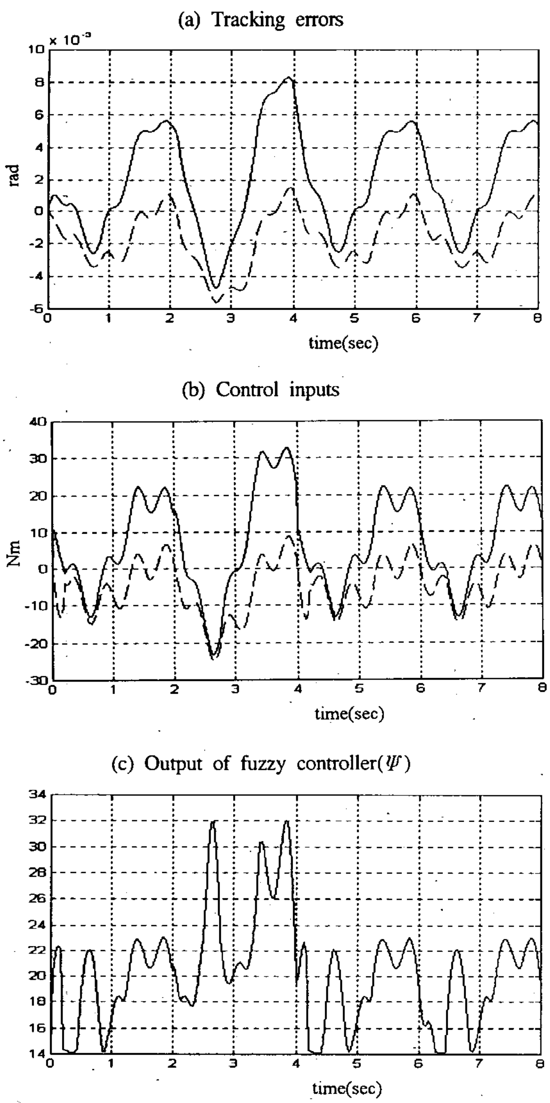


Fig. 3. Simulation results under proposed control algorithm.

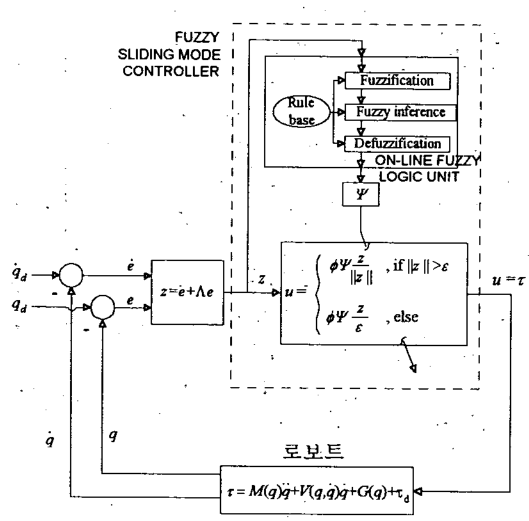


Fig. 4. Structure of proposed control system.

V. Conclusions

In this paper, a sliding mode control algorithm is presented, and a fuzzy logic is applied to the sliding mode control to perform the sliding mode gain is adjusted continuously through fuzzy logic rules for trajectory control of robot manipulators. The stability of the proposed fuzzy logic controllers is proved and the robustness is ensured by the sliding mode control law. From the experimental computer simulation results, it can be seen that the proposed algorithm possesses good properties under large parameter uncertainties and disturbances, and achieved a high performance control.

For the implementation of presented algorithm, only z_i (i.e e and $\dot{e}$) are required. Therefore, the presented algorithm is model free, and can be applicable to most electromechanical (mechatronic) systems.

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