Robust Stability Analysis for a Fuzzy Feedback Linearization Method using a Takagi-Sugeno Fuzzy Model

Hyung-Jin Kang, Cheol Kwon, Hee-Jin Lee, and Mignon Park

Abstract

In this paper, robust stability analysis for the fuzzy feedback linearization regulator is presented. Well-known Takagi-Sugeno fuzzy model is used as the MISO nonlinear plant model. Uncertainty and disturbances are assumed to be included in the model structure with known bounds. For these structured uncertainty and disturbances, robust stability of the closed system is analyzed in both input-output sense and Lyapunov sense. The robust stability conditions are proposed by using multivariable circle criterion and the relationship between input-output stability and Lyapunov stability. The proposed stability analysis is illustrated by a simple example.

I. Introduction

By the feedback linearization method, a class of nonlinear plant can be transformed into a linear system model. Since the transformed linear system model can be easily controlled by well-known and powerful linear control methods, feedback linearization has been widely used in nonlinear control theory[1, 2]. Also, similar design concept has been applied to fuzzy control theory using a fuzzy model[3-5].

A fuzzy model has excellent capability in nonlinear system description and is particularly suitable for the complex and uncertain system. Specially, Takagi-Sugeno fuzzy model can represent a highly nonlinear dynamic system using a small number of rules[6]. Based on Takagi-Sugeno fuzzy model, Sugeno[3] proposed a basic scheme for the fuzzy feedback linearization regulator. He assumed that perfect linearization could be obtained and he analyzed the stability of the linearized system by linear control theory. But, in practical situations, modeling uncertainty and external disturbances are inevitably produced and perfect linearization can not be achieved. Therefore, robust stability analysis is needed to deal with uncertainty and disturbances.

In some previous literatures[7-9], circle criterion and modified circle criterion were applied to analyze the nominal stability of the fuzzy control systems. Also, Tanaka[10-12] suggested various stability and robust stability analysis methods for the fuzzy control systems based on Takagi-Sugeno fuzzy model. In this paper, we have improved these previous researches and proposed robust stability analysis for the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model. To analyze robust stability, we assume that uncertainty and disturbances with known bounds are included in the model structure. For these structured uncertainty and disturbances, the robust stability of the closed system is analyzed in input-output sense and Lyapunov sense by applying multivariable circle criterion.

This paper is organized as follows : In section Ⅱ, problem formulation and a control scheme of the fuzzy feedback linearization regulator are offered. Robust stability analysis for the fuzzy feedback linearization regulator is proposed in section Ⅲ. In section Ⅳ, a simple example is presented to illustrates the proposed analysis method. Finally, the conclusion of the paper is presented in section Ⅴ.

Ⅱ. The Fuzzy Feedback Linearization Regulator Based on T-S Fuzzy Model

Consider the regulation problem of the following n-th order nonlinear SISO system

$$x^{(n)} = f(x) + g(x)u + d \tag{1}$$

where f and g are unknown (uncertain) but bounded continuous nonlinear functions and d denotes the external disturbance which is unknown but bounded. The external disturbances are due to system load, external noise, etc. Let $\mathbf{x}=[x, \dot{x}, \cdots, x^{(n-1)}]^T \in R^n$ be the state vector of the system which is assumed to be available.

In this paper, well-known Takagi-Sugeno fuzzy model is used to identify the unknown nonlinear system of Eqn. (1). Takagi-

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Sugeno fuzzy model is available in IF-THEN form (2) or Input-Output form (3).

· **IF-THEN form**

plant rule i :

$$\begin{aligned} &IF\ x\ is\ M_{i1}\ \text{and}\ \dot{x}\ is\ M_{i2}\ \text{and}\ \cdots\ \text{and}\ x^{(n-1)}\ is\ M_{in} \\ &THEN\ x^{(n)} = (\boldsymbol{a}_i + \Delta\boldsymbol{a}_i(t))^T \cdot \boldsymbol{x} + (b_i + \Delta b_i(t))u + d, \quad (2) \\ &i = 1, 2, \cdots, r \\ &\text{where}\ \boldsymbol{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T,\ \boldsymbol{a}_i,\ \Delta\boldsymbol{a}_i(t) \in R^{1\times n}, \\ &b_i,\ \Delta b_i(t) \in R \end{aligned}$$

In the expression of (2), M_{ij} is the fuzzy set and r is the number of rules. Also, $\Delta\boldsymbol{a}_i(t)$ and $\Delta b_i(t)$ denotes the norm-bounded time-varying modeling uncertainty.

· **Input-Output form**

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(\boldsymbol{x})\{(\boldsymbol{a}_i + \Delta\boldsymbol{a}_i(t))^T \cdot \boldsymbol{x} + (b_i + \Delta b_i(t))u\}}{\sum_{i=1}^{r} w_i(\boldsymbol{x})} + d$$

$$= \sum_{i=1}^{r} h_i(\boldsymbol{x})\{(\boldsymbol{a}_i + \Delta\boldsymbol{a}_i(t))^T \cdot \boldsymbol{x} + (b_i + \Delta b_i(t))u\} + d \quad (3)$$

$$\text{where}\ w_i(\boldsymbol{x}) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)}),\quad h_i(\boldsymbol{x}) = \frac{w_i(\boldsymbol{x})}{\sum_{i=1}^{r} w_i(\boldsymbol{x})}$$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij}. It is assumed in this paper that

$$w_i(\boldsymbol{x}) \geq 0,\quad i = 1, 2, \cdots, r$$

$$\sum_{i=1}^{r} w_i(\boldsymbol{x}) > 0$$

Therefore,

$$h_i(\boldsymbol{x}) \geq 0,\quad i = 1, 2, \cdots, r$$

$$\sum_{i=1}^{r} h_i(\boldsymbol{x}) = 1$$

For Eqn. (3) to be controllable, $\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i \neq 0$ for $\boldsymbol{x}$ in certain controllability region $U_c \subset R^n$ is required. If this controllability requirement is satisfied and there is no uncertainty in Eqn. (3) ($\Delta\boldsymbol{a}_i = 0$, $\Delta b_i = 0$, $d = 0$), the following fuzzy feedback linearization regulator of Eqn. (4) can cancel the nonlinearity of Eqn. (3) and achieve perfect linearization of Eqn. (5).

$$u = \frac{\hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} - \sum_{i=1}^{r} h_i(\boldsymbol{x})\ \boldsymbol{a}_i^T \cdot \boldsymbol{x}}{\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i}$$

$$= \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}}^T - \boldsymbol{a}_i^T) \cdot \boldsymbol{x}}{\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i} \quad (4)$$

where we use the same $\boldsymbol{a}_i$, b_i and $h_i(\boldsymbol{x})$ with the fuzzy model of Eqn. (3) for all i and $\hat{\boldsymbol{a}} \in R^{1\times n}$ is the linear state feedback gain vector. The perfectly linearized system can be written as Eqn. (5).

$$x^{(n)} = \hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} \quad (5)$$

In practical application, however, uncertainty and disturbances are inevitable. Therefore, perfect linearization can not be achieved. By substituting Eqn. (4) into Eqn. (3), the imperfectly linearized system can be written as Eqn. (6). In the next section, robust stability analysis for Eqn. (6) is presented.

$$\begin{aligned} x^{(n)} &= \hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} + \sum_{i=1}^{r} h_i(\boldsymbol{x})\ \Delta\boldsymbol{a}_i(t)^T \cdot \boldsymbol{x} \\ &\quad + \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x})\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i}\{\sum_{i=1}^{r} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}} - \boldsymbol{a}_i) \cdot \boldsymbol{x}\} + d \\ &= \boldsymbol{a}_L^T \cdot \boldsymbol{x} + (\hat{\boldsymbol{a}} - \boldsymbol{a}_L)^T \cdot \boldsymbol{x} + \sum_{i=1}^{r} h_i(\boldsymbol{x})\ \Delta\boldsymbol{a}_i(t)^T \cdot \boldsymbol{x} \\ &\quad + \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x})\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i}\{\sum_{i=1}^{r} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}} - \boldsymbol{a}_i)^T \cdot \boldsymbol{x}\} + d \\ &= \boldsymbol{a}_L^T \cdot \boldsymbol{x} + \boldsymbol{a}_N(t)^T \cdot \boldsymbol{x} + d \quad (6) \end{aligned}$$

$$\begin{aligned} \text{where,}\ \boldsymbol{a}_N(t) &= (\hat{\boldsymbol{a}} - \boldsymbol{a}_L)^T \cdot \boldsymbol{x} + \sum_{i=1}^{r} h_i(\boldsymbol{x})\ \Delta\boldsymbol{a}_i(t)^T \cdot \boldsymbol{x} \\ &\quad + \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x})\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x})b_i}\{\sum_{i=1}^{r} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}} - \boldsymbol{a}_i)^T \cdot \boldsymbol{x}\} \end{aligned}$$

Remark : $\boldsymbol{a}_L$ in Eqn. (6) denotes the reference vector for robust stability analysis and $\boldsymbol{a}_L$ should be chosen so as to satisfy the following two conditions.

i) the asymptotic stability of $x^{(n)} = \boldsymbol{a}_L^T \cdot \boldsymbol{x}$.
ii) the basic assumption of Theorem 1 in the next section.

To implement the fuzzy feedback linearization regulator of Eqn. (4) in the fuzzy rule-based form, we propose the control structure shown in Fig. 1. We divide the fuzzy feedback linearization regulator into two blocks, a fuzzy rule-based controller block and a simple nonlinear function block.

In the proposed structure, the fuzzy controller block shares the same fuzzy sets M_{ij} and the parameters $\boldsymbol{a}_i$ and b_i with the fuzzy model in the premise parts for all i and j. Therefore, $\hat{\boldsymbol{a}}$ is the only design parameter of the fuzzy feedback linearization

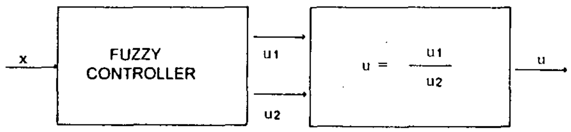

Fig. 1. The control structure of the fuzzy feedback linearization regulator.

regulator of Eqn. (4). The nonlinear function block simply divides u_1 by u_2 to produce the same u as in Eqn. (4). i-th rule of the fuzzy controller block can be represented by the following expression of (7).

$$IF\ x\ is\ M_{i1}\ \text{and}\ \dot{x}\ is\ M_{i2}\ \text{and} \cdots \text{and}\ x^{(n-1)}\ is\ M_{in}$$

$$THEN \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} (\hat{a} - a_i)^T \cdot x \\ b_i \end{bmatrix} \quad i = 1, 2, \cdots, r \tag{7}$$

Ⅲ. Robust Stability Analysis for the Fuzzy Feedback Linearization Regulator

To analyze the robust stability of Eqn. (6), consider two different cases, i) $d \neq 0$ ii) $d = 0$. In case of i) $d \neq 0$, the input-output stability should be guaranteed so as to bound the norm of the state vector x (output) with respect to the norm-bounded disturbance d (input). In our analysis, well-known multivariable circle criterion is used to analyze the input-output robust stability of Eqn. (6). Multivariable circle criterion is L_2 input-output stability analysis tool for the linear system with sector bounded nonlinearities [13-15]. Since $a_{Nj}(t)$ in Eqn. (6) is bounded by the maximum and the minimum obtained in Appendix A for all j and t, it can be treated as time-varying sector bounded nonlinearity. Therefore, multivariable circle criterion can be applied to analyze L_2 robust stability of Eqn. (6). To apply multivariable circle criterion, the closed system of Eqn. (6) should be transformed into the basic configuration of multivariable circle criterion as in Fig. 2. In this basic configuration, the transfer function matrix $G(s)$ can be computed from Eqn. (20) in Appendix B. Applying multivariable circle criterion to the transformed basic configuration, we have proposed a sufficient condition for L_2 robust stability of the fuzzy feedback linearization regulator in Theorem 1.

In case of ii) $d = 0$, Lyapunov stability of the equilibrium $x = 0$ of Eqn. (6) is required with respect to the initial state x_0. From the nonlinear control theory, Lyapunov stability can be related to the input-output stability as in Theorem 2. Therefore, we can derive Lyapunov stability condition for $d = 0$ using Theorem 1 and the relationship between the input-output stability and Lyapunov stability. In Theorem 3, Lyapunov stability condition for $d = 0$ is proposed.

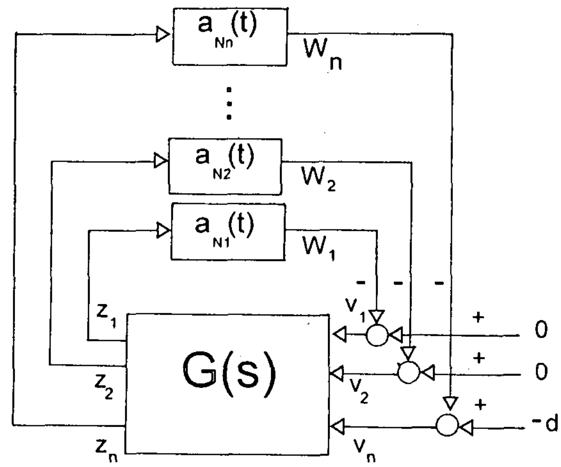

Fig. 2. Basic configuration for multivariable circle criterion.

Since the stability conditions for L_2 stability and Lyapunov stability are identical, the following analysis procedure can be commonly applied to analyze both L_2 stability and Lyapunov stability.

· Procedure for robust stability analysis

Step 1. Select the linear stable reference a_L so as to satisfy the basic assumption of Theorem 1 and compute the transfer function matrix $G(s)$ from Eqn. (20) in Appendix B.

Step 2. From Eqn. (13) and Eqn. (14) in Appendix A, find the maximum and minimum sector bounds of $a_{Nj}(t)$ for all j.

Step 3. Plot Gershigorin bands for all j using the transfer function matrix $G(s)$ and the sector bounds.

Step 4. Check if the sufficient condition of Theorem 1 or Theorem 3 is met.

Theorem 1. L_2 robust stability condition for the fuzzy feedback linearization regulator.

basic assumption : $\max\{a_{Nj}(t)\} \geq \min\{a_{Nj}(t)\} \geq 0, \ \forall j$

L_2 robust stability of the overall system is guaranteed if
$|G_{jj}(j\omega) + g_{cj}| - r_j(j\omega) > r_{cj}, \quad \forall j$
or none of Gershigorin bands enter and encircle the disc centered at $-g_{cj}$ with radius r_{cj} (Fig. 3).

where $r_j(j\omega) = \sum_{k=1, k\neq j}^{n} |G_{jk}(j\omega)|$ or $\sum_{k=1, k\neq j}^{n} |G_{kj}(j\omega)|$

$$g_{cj} = 0.5\left(\frac{1}{\min(a_{Nj}(t))} + \frac{1}{\max(a_{Nj}(t))}\right)$$

$$r_{cj} = 0.5\left(\frac{1}{\min(a_{Nj}(t))} - \frac{1}{\max(a_{Nj}(t))}\right)$$

Proof of the above theorem is the same as the proofs of references [13-15] and hence omitted here.

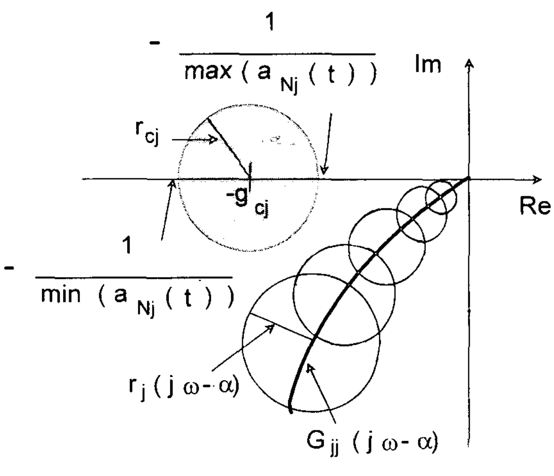

Fig. 3. Graphical analysis of multivariable circle criterion.

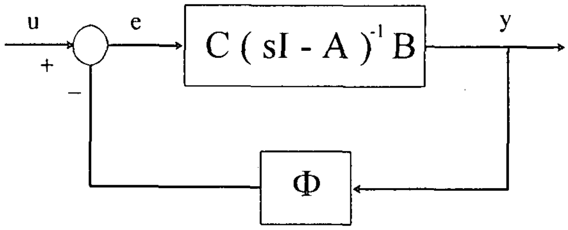

Fig. 4. Relationships between input-output and Lyapunov stability.

Theorem 2. Relationships between input-output and Lyapunov stability.

Consider the following system of Eqn. (8)

$$\dot{x}(t)=Ax(t)+Be(t),\ \ y(t)=Cx(t),\ \ e(t)=u(t)-\Phi[t,y(t)] \quad (8)$$

where $x(t)\in R^n$, $u(t)\in R^m$, $y(t)\in R^l$, and A, B, C are matrices of compatible dimensions and Φ: $R_+\times R^l\rightarrow R^m$ satisfies $\Phi(t,0)=0$, $\forall t\geq 0$ (Fig. 4).

if the following three conditions i), ii) and iii) are satisfied, then $x=0$ is a globally attractive equilibrium of the unforced system.

i) Φ is globally Lipschitz continuous; i.e., there exists a finite constant μ such that
$\|\Phi(t,y_1)-\Phi(t,y_2)\|\leq\mu\|y_1-y_2\|,\ \forall t\geq 0,\ \forall y_1, y_2\in R^l$

ii) the pair (A, B) is controllable, and the pair (C, A) is observable.

iii) the forced system is L_2 stable.

Proof of this theorem can be found in reference [16] (Theorem (46))

Theorem 3. Lyapunov robust stability condition for the fuzzy feedback linearization regulator.

$x=0$ is a globally attractive equilibrium of the unforced system of Eqn. (6) (i.e. $d=0$) if

$|G_{jj}(j\omega)+g_{cj}|-r_j(j\omega)>r_{cj}$, $\forall j$

or none of Gershigorin bands enter and encircle the disc centered at $-g_{cj}$ with radius r_{cj} (Fig. 3).

where $r_j(j\omega)=\sum_{k=1,k\neq j}^{n}|G_{jk}(j\omega)|$ or $\sum_{k=1,k\neq j}^{n}|G_{kj}(j\omega)|$

$$g_{cj}=0.5\left(\frac{1}{\min(a_{Nj}(t))}+\frac{1}{\max(a_{Nj}(t))}\right)$$

$$r_{cj}=0.5\left(\frac{1}{\min(a_{Nj}(t))}-\frac{1}{\max(a_{Nj}(t))}\right)$$

proof : To prove Theorem 3, first, we express the system of Eqn. (6) in the form of Eqn. (8) as in Eqn. (9).

$$\dot{x}(t)=Ax(t)+Be(t),\ \ y(t)=Cx(t),\ \ e(t)=u(t)-\Phi[t,y(t)] \quad (9)$$

where A, B, C are A_L, B_L, C_L in Appendix B, respectively. and

$$u=\begin{bmatrix}0\\0\\0\\\cdots\\-d\end{bmatrix},\ \Phi=A_N(t)=\begin{bmatrix}a_{N1}(t)&0&0&\cdots&0\\0&a_{N2}(t)&0&\cdots&0\\0&0&a_{N3}(t)&\cdots&0\\&&\vdots&&\\0&0&0&\cdots&a_{Nn}(t)\end{bmatrix}$$

Then, for the system of Eqn. (9), we examine three sufficient conditions of Theorem 2.

a) Since $a_{Nj}(t)$ is bounded for all j and t, we can assume that $\|A_N(t)\|\leq\mu$ for all t
where μ is a finite constant.
With this assumption and the property of the induced matrix norm, the following inequality holds for all $t\geq 0$ and for all y_1, y_2.

$$\|A_N(t)y_1-A_N(t)y_2\|=\|A_N(t)(y_1-y_2)\|\leq\|A_N(t)\|\ \|y_1-y_2\|\leq\mu\|y_1-y_2\|$$

Therefore, the system of Eqn. (9) is globally Lipschitz continuous.

b) The controllability and observability test shows that the pair (A, B) is controllable and the pair(C, A) is observable, independent of a_L.

c) If the sufficient condition of Theorem 1 is met, the forced system (i.e. $d\neq 0$) is L_2 stable.

a) and b) show that the system of Eqn. (9) always satisfies the sufficient conditions i) and ii) of Theorem 2. Therefore, according to c), if the sufficient condition of Theorem 1 is met, $x=0$ is a globally attractive equilibrium of the unforced system of Eqn. (6) (i.e. $d=0$).

IV. Example

Consider the stability of the following Takagi-Sugeno fuzzy model with two rules. Membership functions of this fuzzy model are shown in Fig. 5.

Rule 1 : *IF* x *is about* 0

THEN $\dot{x}=(\,a_1+\ \Delta a_1(t))^T\cdot x+(b_1+\Delta b_1(t))u+d$

Rule 2 : *IF* x *is about* $\pm\frac{\pi}{2}$ $(\,|x|<\frac{\pi}{2})$

THEN $\dot{x}=(\,a_2+\ \Delta a_2(t))^T\cdot x+(b_2+\Delta b_2(t))u+d$ **(10)**

or

$$\dot{x}=\frac{\sum_{i=1}^{2} w_i(x)\{(\,a_i+\ \Delta a_i(t))^T\cdot x+(b_i+\Delta b_i(t))u\}}{\sum_{i=1}^{2} w_i(x)}+d$$

$$=\sum_{i=1}^{2} h_i(x)\{(\,a_i+\ \Delta a_i(t))^T\cdot x+(b_i+\Delta b_i(t))u\}+d \quad \textbf{(11)}$$

where $w_i(x)=\prod_{j=1}^{2} M_{ij}(x^{(j-1)})$, $h_i(x)=\frac{w_i(x)}{\sum_{i=1}^{2} w_i(x)}$

where $x=[x,\dot{x}\,]^T$, $a_1=$ [20, 0], $a_2=$[10, 0], $b_1=$ -2, $b_2=$ -1 we assume that $\Delta a_1(t)$, $\Delta a_2(t)$, $\Delta b_1(t)$, $\Delta b_2(t)$ are unknown but bounded as follows.

$-2\le\Delta a_{11}(t)\le 2$, $-1\le\Delta a_{12}(t)\le 1$, $-2\le\Delta a_{21}(t)\le 2$,
$-1\le\Delta a_{22}(t)\le 1$, $-0.01\le\Delta b_1(t)\le 0.01$, $-0.01\le\Delta b_2(t)\le 0.01$

In this example, the following fuzzy feedback linearization regulator of Eqn. (12) is used to stabilize the system of Eqn. (10) or Eqn. (11).

$$u=\frac{\hat{a}^T\cdot x-\sum_{i=1}^{2} h_i(x)\ a_i^T\cdot x}{\sum_{i=1}^{2} h_i(x)b_i}=\frac{\sum_{i=1}^{2} h_i(x)(\,\hat{a}^T-\ a_i^T)\cdot x}{\sum_{i=1}^{2} h_i(x)b_i} \quad \textbf{(12)}$$

where $\hat{a}$ = [-50 ,-30]

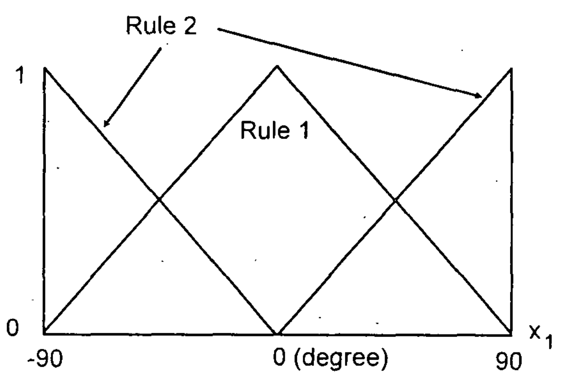

Fig. 5. Membership functions.

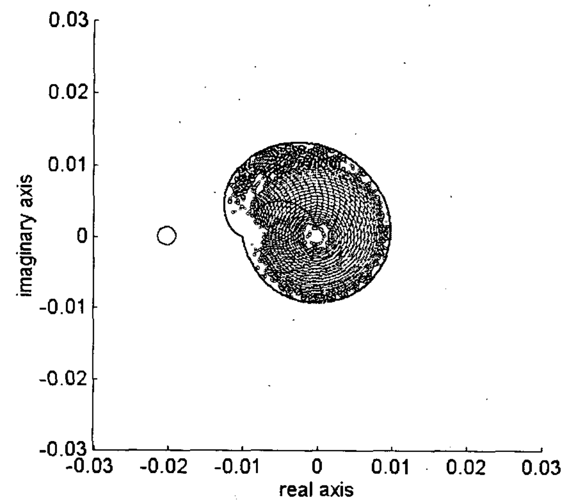

Fig. 6. Gershigorin band and disk (j =1).

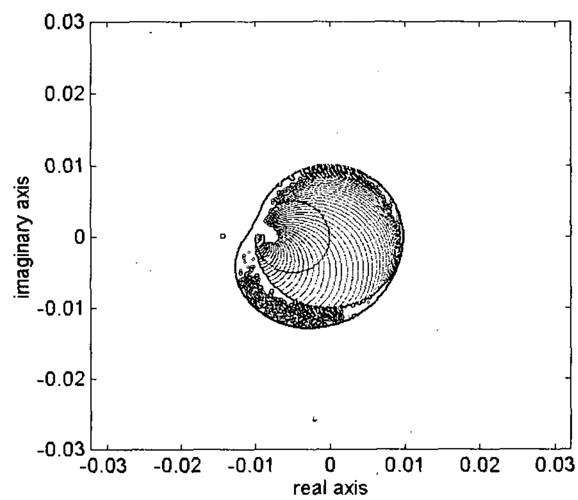

Fig. 7. Gershigorin band and disk (j =2).

According to the analysis procedure, L_2 stability (in case of $d\neq 0$) and Lyapunov stability (in case of $d=0$) of the closed system can be analyzed in the following steps.

Step 1. Select a_L = [-100, -100]. We can easily verify that this a_L satisfies two conditions in Remark.

Then, compute the $G(s)$ as

$$G(s)=\frac{-1}{S^2+100S+100}\begin{bmatrix}1 & 1\\ s & s\end{bmatrix}$$

Step 2. The maximum and minimum sector bounds of $a_{Nj}(t)$ for j=1, 2. can be found as follows.

$$\max_t\ a_{N1}(t)=53.04,\ \min_t\ a_{N1}(t)=46.96$$

$$\max_t\ a_{N2}(t)=71.62,\ \min_t\ a_{N2}(t)=68.38$$

Step 3. Gershigorin bands and the sector disks are given in Fig. 6 and Fig. 7.

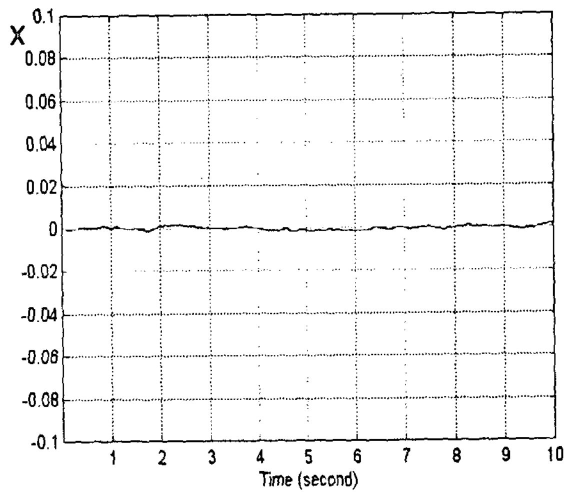

Fig. 8. State variable x (case $d \neq 0$).

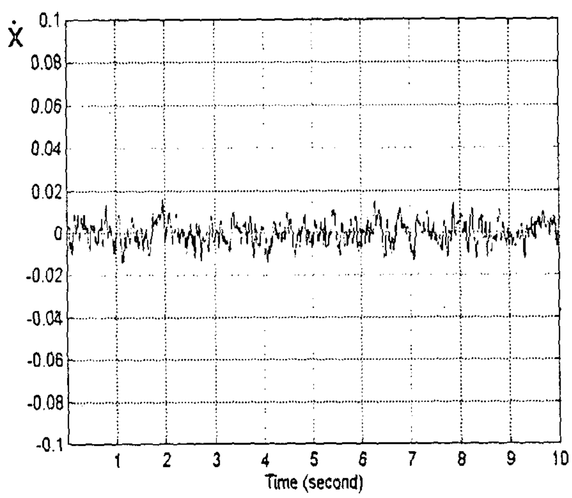

Fig. 9. State variable $\dot{x}$ (case $d \neq 0$).

Step 4. Since the sufficient condition of Theorem 1 and Theorem 3 are met, the fuzzy feedback linearization regulator of Eqn. (11) can stabilize the system of Eqn. (9) or Eqn. (10) in L_2 sense (in case of $d \neq 0$) and the Lyapunov sense (in case of $d=0$).

To verify the stability analysis, the computer simulation is performed for the case $d \neq 0$ and case $d=0$.

i) case $d \neq 0$

In this simulation, random signal d (mean(d)=0, $|d| \leq 1$) is used as external disturbance and initial conditions are assumed to be zero($x_0 = 0$). Fig. 8. and Fig. 9. illustrate the simulation results.

ii) case $d=0$

$d=0$ and initial condition x_0=[1, 0] are used for simulation.

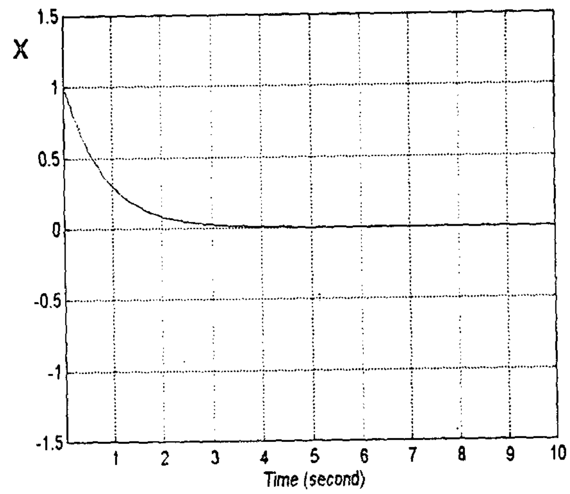

Fig. 10. State variable x (case $d=0$).

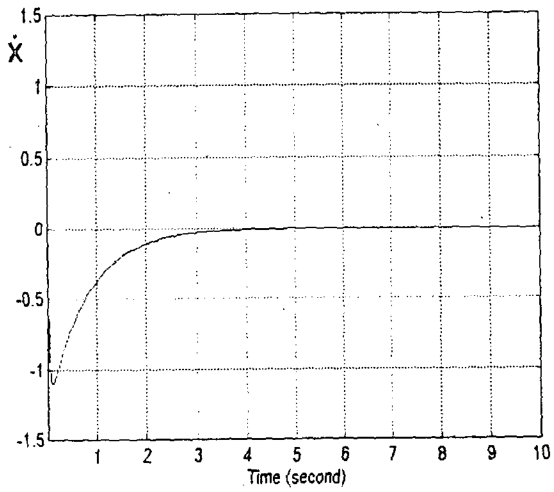

Fig. 11. State variable $\dot{x}$ (case $d=0$).

Fig. 10. and Fig. 11. illustrate the simulation results.

V. Conclusion

In this paper, we have presented the robust stability analysis method for the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model. Both input-output stability and Lyapunov stability can be analyzed by the proposed analysis method.

Compared with the previous researches on the similar topics, our paper has something to do with Ray's works [7-8]. Ray introduced circle criterion and modified circle criterion to analyze the stability of the fuzzy control system. However, the fuzzy controller used in his works was nothing but a simple SISO nonlinear function in the early stage of fuzzy control theory. In addition, he failed to treat uncertainty and nonlinearity of the plant by using the nominal and linear plant model. But, in our paper, we have thoroughly dealt with the essential and practical issues of current fuzzy control theory such as robustness, complex nonlinear modeling and nonlinear cancellation.

In his previous works [10-12], Tanaka also treated these issues in an excellent way but there are some differences and disadvantages in comparison with our method. The controller structure 'PDC(parallel distributed compensation)' which is suggested in Tanaka's papers, can linearize the nonlinear plant only in the small subset of the state space. In the remainder part of the state space, interpolation or 'fuzzy blending' is used to generate the controller output. On the other hand, our fuzzy feedback linearization regulator has the controller structure that can linearize the whole state space. Therefore, our controller outperforms Tanaka's PDC in the aspect of the performance robustness to nonlinearity of the plant. Besides, different from Tanaka's method, our stability analysis method is a graphical analysis method. Therefore, the stability margins of the design parameters can be visualized in Gershigorin plots. With the help of these stability margins, a robust stable fuzzy feedback linearization regulator can be easily designed without repetitive and tedious trial and error as in PDC.

A simple example illustrates the effectiveness of the proposed stability analysis method. Further works are still under investigation to apply the proposed method to the more general nonlinear systems, e. g., multivariable nonlinear and output feedback control case.

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Appendix A

- **A computing method for maximum and minimum sector bounds**

basic assumption : $\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) = 1 \qquad \max_{\boldsymbol{x}} h_i(\boldsymbol{x}(t)) = 1$

The maximum and minimum sector bounds of $a_{Nj}(t)$ can be computed from Eqn. (13) and Eqn. (14)

$$\max_t (a_{Nj}(t)) = (\hat{a}_j - a_{Lj}) + \max_t \left\{ \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta a_{ij}(t) \right\} + \max_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) d_{ij} \right\} \quad \textbf{(13)}$$

$$\min_t (a_{Nj}(t)) = (\hat{a}_j - a_{Lj}) + \min_t \left\{ \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta a_{ij}(t) \right\} + \min_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) d_{ij} \right\} \quad \textbf{(14)}$$

where $e_{ij} = \hat{a}_j - a_{ij}$

The second terms of Eqn. (13) and Eqn. (14) can be computed using the following property.

$$\min_i (\Delta a_{ij}(t)) \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))\Delta a_{ij}(t)) \le \max_i (\Delta a_{ij}(t)) \quad \textbf{(15)}$$

The third terms of Eqn. (13) and Eqn. (14) can be computed from Eqn. (16) and Eqn. (17)

$$\max_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t))\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t))b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))e_{ij} \right\} =$$

$$\max\left\{ \frac{\Delta\overline{b^{nn}}}{\underline{b^p}} \cdot \overline{e_j^{nn}},\ \frac{\Delta\underline{b^n}}{\underline{b^p}} \cdot \underline{e_j^n},\ \frac{\Delta\underline{b^n}}{\overline{b^n}} \cdot \overline{e_j^{nn}},\ \frac{\Delta\overline{b^{nn}}}{\overline{b^n}} \cdot \underline{e_j^n},\ \frac{\Delta\underline{b^{nn}}}{\overline{b^p}} \cdot \overline{e_j^n},\ \frac{\Delta\overline{b^n}}{\overline{b^p}} \cdot \underline{e_j^{nn}},\ \frac{\Delta\overline{b^n}}{\underline{b^n}} \cdot \overline{e_j^n},\ \frac{\Delta\underline{b^{nn}}}{\underline{b^n}} \cdot \underline{e_j^{nn}} \right\} \quad \textbf{(16)}$$

$$\min_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t))\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t))b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))e_{ij} \right\} =$$

$$\min\left\{ \frac{\Delta\underline{b^{nn}}}{\overline{b^p}} \cdot \underline{e_j^{nn}},\ \frac{\Delta\overline{b^n}}{\overline{b^p}} \cdot \overline{e_j^n},\ \frac{\Delta\overline{b^n}}{\underline{b^n}} \cdot \underline{e_j^{nn}},\ \frac{\Delta\underline{b^{nn}}}{\underline{b^n}} \cdot \overline{e_j^n},\ \frac{\Delta\overline{b^{nn}}}{\underline{b^p}} \cdot \underline{e_j^n},\ \frac{\Delta\underline{b^n}}{\underline{b^p}} \cdot \overline{e_j^{nn}},\ \frac{\Delta\underline{b^n}}{\overline{b^n}} \cdot \underline{e_j^n},\ \frac{\Delta\overline{b^{nn}}}{\overline{b^n}} \cdot \overline{e_j^{nn}} \right\} \quad \textbf{(17)}$$

where, $b_i^p = \{ b_i \mid b_i > 0 \}$

$b_i^n = \{ b_i \mid b_i < 0 \}$

$e_{ij}^{nn} = \{ e_{ij} \mid e_{ij} \ge 0 \}$

$e_{ij}^n = \{ e_{ij} \mid e_{ij} < 0 \}$

$\underline{b^p} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))b_i^p \le \overline{b^p}$

$\underline{b^n} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))b_i^n \le \overline{b^n}$

$\underline{e_j^{nn}} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))e_{ij}^{nn} \le \overline{e_j^{nn}}$

$\underline{e_j^n} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t))e_{ij}^n \le \overline{e_j^n}$

$\Delta\overline{b^{nn}} = \max_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) \ge 0 \}$

$\Delta\overline{b^n} = \max_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) < 0 \}$

$\Delta\underline{b^{nn}} = \min_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) \ge 0 \}$

$\Delta\underline{b^{nn}} = \min_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) < 0 \}$

Appendix B

· A computing method for $G(s)$

To compute $G(s)$ from Eqn. (6), we divide Eqn. (6) into the linear and the nonlinear part as

$$x^{(n)} - \boldsymbol{a}_L{}^T \cdot \boldsymbol{x} = \boldsymbol{a}_N(t)^T \cdot \boldsymbol{x} + d \quad \textbf{(18)}$$

Using the following state-space representation of (19), we can compute $G(s)$ from Eqn. (20)

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}_L \boldsymbol{x} + \boldsymbol{B}_L \boldsymbol{v} \\ \boldsymbol{z} &= \boldsymbol{C}_L \boldsymbol{x} \\ \boldsymbol{v} &= -\boldsymbol{w} + \boldsymbol{d} \\ \boldsymbol{w} &= \boldsymbol{A}_N(t)\, \boldsymbol{z} \end{aligned} \quad \textbf{(19)}$$

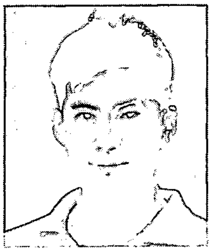
where,

$$\boldsymbol{A}_L = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ & & \vdots & & \\ a_{L1} & a_{L2} & a_{L3} & \cdots & a_{Ln} \end{bmatrix}, \quad \boldsymbol{B}_L = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ & & \vdots & & \\ -1 & -1 & -1 & \cdots & -1 \end{bmatrix},$$

$$\boldsymbol{C}_L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\boldsymbol{A}_N(t) = \begin{vmatrix} a_{N1}(t) & 0 & 0 & \cdots & 0 \\ 0 & a_{N2}(t) & 0 & \cdots & 0 \\ 0 & 0 & a_{N3}(t) & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & a_{Nn}(t) \end{vmatrix} \text{ and } \boldsymbol{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ -d \end{bmatrix}$$

$$G(s) = C_L(sI - A_L)^{-1} B_L \quad \textbf{(20)}$$



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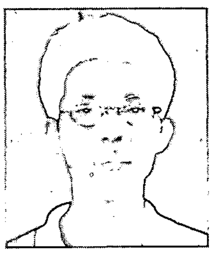

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