Convergence Analysis of the Modified Adaptive Sign (MAS) Algorithm Using a Mixed Norm Error Criterion

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Abstract

In this paper, a modified adaptive sign (MAS) algorithm based on a mixed norm error criterion is proposed. The mixed norm error criterion to be minimized is constructed as a combined convex function of the mean-absolute error and the mean-absolute error to the third power. A convergence analysis of the MAS algorithm is also presented. Under a set of mild assumptions, a set of nonlinear evolution equations that characterizes the statistical mean and mean-squared behavior of the algorithm is derived. Computer simulations are carried out to verify the validity of our derivations.

I. Introduction

The adaptive least mean square (LMS) algorithm [1] and the sign algorithm [2] have received a great deal of attention during the last two decades and are now widely used in variety of applications due to there simplicity and relatively robust performance. The algorithms attempt to minimize the mean-squared and mean-absolute estimation errors at each iteration, respectively. Meanwhile, the adaptive filtering algorithms that are based on high order error power (HOEP) conditions [3]-[7] have been proposed and their performances have been investigated. Despite their potential advantages, these HOEP algorithms are much less popular comparing to the LMS and sign algorithms in practice since they can be very sensitive to the stability.

The paper by Walach and Widrow [3] seems to be the first one dealing with the HOEP conditions in the stochastic gradient adaptive signal processing. They presented convergence analyses of the adaptive least mean fourth (LMF) algorithm and its family. The performance of the LMF algorithm is then compared with that of the LMS algorithm for different plant noise densities in the system identification mode. By evaluating the ratio between the misadjustment of the LMS algorithm and that of the LMF algorithm, it was shown that the LMF algorithm has substantially less noises in the filter coefficients than the conventional LMS algorithm for the same speed of convergence, except the case when the plant measurement noise of the unknown system has a Gaussian distribution. The necessary condition for the convergence of the mean and mean-squared behavior of the LMF algorithm was also derived. The results in [3], however, are somewhat restrictive due to the employment of the wild assumption that the filter coefficients are already close to the optimal values.

Douglas and Meng [4] examined a family of adaptive algorithms based on general error criteria (or non-mean-square error criteria) for which the error function to be minimized was modeled as an arbitrary memoryless odd-symmetric nonlinear function. It was shown that, in the system identification mode, using an error criterion optimized for the plant noise density can significantly improved the overall performance and reduce the fluctuations in the coefficient estimates. Pei and Tseng [5] also investigated the performances of the HOEP criteria for adaptive FIR filters, and several important observations were made. They showed that the HOEP criteria yield the same optimum solution for any high order power when the input signals are Gaussian processes. It was also shown that the sign algorithm is preferred when the signals are corrupted by an impulsive noise.

Kim, et al. [6], presented a convergence analysis of the least mean-absolute third (LMAT) algorithm by deriving equations to characterize the statistical mean and mean-squared behavior of the algorithm. They also investigated the steady-state responses of the LMAT algorithm and compared the performance of the LMAT algorithm with that of the LMS algorithm [7]. It was shown that the LMAT algorithm is able to converge faster than the LMS algorithm in many practical situations.

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More recently, the mixed norm based adaptive algorithms have been introduced in order to achieve the combined benefits of different error norm criteria. Chamber, et al. [8] proposed the least mean mixed norm (LMMN) adaptive algorithm that combines the LMS and the LMF algorithms. A mixing parameter is used to control the proportions of the two error norms. The LMMN algorithm shows faster initial convergence than the LMF algorithm, and has less misadjustment than the LMS algorithm. They also investigated the convergence and steady-state properties of the algorithm in [9], and confirmed their observations by computer simulations.

Chamber and Avlonitis [10] also proposed another mixed norm adaptive algorithm that is suitable for system identification modes. The algorithm combines, this time, the LMS and the sign algorithms. It was shown that the proposed algorithm is robust to the presence of significant impulsive noise in the desired response of the filter, while maintaining good accuracy in the steady-state.

In this paper, a new adaptive algorithm, called the modified adaptive sign (MAS) algorithm, is proposed by combining the sign and the LMAT algorithm. The mixed norm error criterion to be minimized is constructed as a combined convex function of the mean-absolute error and the mean-absolute error to the third power. Both error functions are the perfect convex functions with respect to the filter coefficient vector, and therefore the MAS algorithm does not have local minima. A statistical convergence analysis of the MAS algorithm is also presented. Under a set of mild assumptions, a set of nonlinear evolution equations that characterizes the statistical mean and mean-squared behavior of the MAS algorithm is derived. Computer simulations are carried out to verify the validity of our derivations.

The MAS algorithm basically makes use of the fact that the sign algorithm is relatively slow but normally stable, while the LMAT algorithm is relatively fast but sensitive to the stability. A motivation to the MAS algorithm is therefore to retain the fast convergence property of the LMAT algorithm and the robustness of the sign algorithm simultaneously.

II. Problem Statement

Now, consider the problem of adaptively estimating the primary input signal $d(n)$ using the reference input $x(n)$. Let $H(n)$ denote the adaptive filter weight vector of size N. Define the reference input vector $X(n)$ as

$$X(n) = \left[x(n), x(n-1), \cdots, x(n-N+1)\right]^T, \tag{1}$$

where $[\cdot]^T$ denotes the transpose of $[\cdot]$. The Mixed norm to be minimized is of the form

$$\nabla(n) = \alpha\, E\left\{|e(n)|\right\} + (1-\alpha)\, E\left\{|e(n)|^3\right\}, \tag{2}$$

where $E\{\cdot\}$ denote a statistical expectation of $\{\cdot\}$ and $\alpha \in [0, 1]$ is a mixing parameter. Note that, for α being 1 and 0, equation (2) becomes the sign and the LMAT algorithm, respectively. It can be readily expected that the performance of the MAS algorithm lies intermediate between the two algorithms according to the choice of the parameter α.

The MAS algorithm update the coefficient vector $H(n)$ using

$$H(n+1) = H(n) + \mu X(n)\left[\alpha + 3(1-\alpha)e^2(n)\right] sign\{e(n)\}, \tag{3}$$

where μ is the adaptive step-size of positive value,

$$sign\{e(n)\} = \begin{cases} 1 & \text{if} \quad e(n) \ge 0 \\ -1 & \text{otherwise,} \end{cases} \tag{4}$$

and $e(n)$ is the estimation error at time n given by

$$e(n) = d(n) - H^T(n)\, X(n). \tag{5}$$

Let H_{opt} denote the optimal coefficient vector given by

$$H_{opt} = R_{XX}^{-1}\, R_{dX}, \tag{6}$$

where

$$R_{XX} = E\left\{X(n)\, X^T(n)\right\}, \tag{7}$$

and

$$R_{dX} = E\left\{d(n)\, X(n)\right\}. \tag{8}$$

Also, define the coefficient misalignment vector $V(n)$ as

$$V(n) = H(n) - H_{opt}, \tag{9}$$

and its autocorrelation matrix $K(n)$ as

$$K(n) = E\left\{V(n) V^T(n)\right\}. \tag{10}$$

Using (9) in (3), we get

$$V(n+1) = V(n) + \mu X(n)\left[\alpha + 3(1-\alpha)e^2(n)\right] sign\{e(n)\}. \tag{11}$$

The optimal estimation error $e_{min}(n)$ is given by

$$e_{min}(n) = d(n) - X^T(n) H_{opt}. \tag{12}$$

Combining (5), (9), and (12), it follows that

$$e(n) = e_{min}(n) - X^T(n) V(n). \tag{13}$$

Finally, let

$$\sigma_e^2(n) = E\{e^2(n)\} \tag{14}$$

and

$$\xi_{min} = E\{e_{min}^2(n)\} \tag{15}$$

denote the mean-squared error power and minimum mean-squared error power, respectively.

Convergence analysis of the MAS algorithm is very complicated due to the existence of the nonlinear and high order powered error signal in the coefficient update equation. We thus make the following assumptions in order to make our analysis mathematically more tractable.

Assumption 1: $d(n)$ and $X(n)$ are zero-mean, wide-sense stationary, and jointly Gaussian random processes.
Assumption 2: The input pair $\{d(n), X(n)\}$ at time n is independent of $\{d(n), X(n)\}$ at time k, if $n \neq k$.

A direct consequence of Assumption 1 is that the estimation error $e(n)$ in (5) is also zero-mean and Gaussian when conditioned on the coefficient vector $H(n)$ (or equivalently on $V(n)$). Assumption 2 is the commonly employed "independence assumption"[2] and is valid if μ is chosen to be sufficiently small. One direct consequence of Assumption 2 is that $H(n)$ is independent of the input pair $\{d(n), X(n)\}$ since $H(n)$ depends only on the inputs at time $n-1$ and before.

Ⅲ. Convergence Analysis

Taking the statistical expectation on both sides of (11) gives

$$\begin{aligned} E\{V(n+1)\} = E\{V(n)\} &+ \mu\alpha E\{X(n) sign\{e(n)\}\} \\ &+ 3\mu(1-\alpha) E\{X(n) e^2(n) sign\{e(n)\}\}. \end{aligned} \tag{16}$$

The second and the last expectation on the right-hand side (RHS) of (16) can be simplified using the fact that for an arbitrary Borel function $G(\cdot)$ and jointly Gaussian random variables x_1 and x_2 [11]

$$E\{x_1 G(x_2)\} = \frac{E\{x_2 G(x_2)\} E\{x_1 x_2\}}{E\{x_2^2\}} \tag{17}$$

Thus, using (17) in conjunction with Assumption 1, it was shown that [2]

$$E\{X(n) sign\{e(n)\}\} \approx -\sqrt{\frac{2}{\pi}} \frac{1}{\sigma_e(n)} R_{XX} E\{V(n)\}, \tag{18}$$

where

$$E\{e^2(n) | V(n)\} = \sigma_{e|V}^2(n). \tag{19}$$

Note in (18) that we have made use of the approximation

$$\sigma_{e|V}(n) \approx \sigma_e(n). \tag{20}$$

Also, using (17) in conjunction with Assumption 1 once again, we have

$$\begin{aligned} &E\{X(n) e^2(n) sign\{e(n)\}\} \\ &= E\{E[X(n) e^2(n) sign\{e(n)\} | V(n)]\} \\ &\approx -2\sqrt{\frac{2}{\pi}} \sigma_e(n) R_{XX} E\{V(n)\} \end{aligned} \tag{21}$$

We have made use of the followings in deriving (21):

$$E\{|e(n)|^3 | V(n)\} = 2\sqrt{\frac{2}{\pi}} \sigma_{e|V}^3(n), \tag{22}$$

$$E\{X(n) e(n) | V(n)\} = -R_{XX} V(n), \tag{23}$$

and the approximation

$$E\{\sigma_{e|V}(n) V(n)\} \approx \sigma_e(n) E\{V(n)\}. \tag{24}$$

Note in (23) that we have made use of the independence assumption (i.e., Assumption 2) as well as the orthogonality principle.

Therefore, using (18) and (21) in (16), we have the mean behavior for the coefficient misalignment vector of the MAS algorithm as

$$E\{V(n+1)\} = \left[I_N - \mu\alpha\sqrt{\frac{2}{\pi}} \frac{1}{\sigma_e(n)} R_{XX} \right.$$

$$-6\mu(1-\alpha)\sqrt{\frac{2}{\pi}}\,\sigma_e(n)\,R_{XX}\Bigg]E\{V(n)\}, \tag{25}$$

where I_N denotes the $N\times N$ identity matrix.

From (25), it is easy to show that the mean behavior of the coefficient misalignment vector $E\{V(n)\}$ converges to the zero vector (or equivalently, $E\{H(n)\}$ is convergent to H_{opt}) if the convergence parameter μ is selected to be

$$0<\mu<\frac{\sqrt{2\pi}\,\sigma_e(n)}{\left[\alpha+6(1-\alpha)\sigma_e^2(n)\right]\lambda_{\max}}, \qquad \forall n, \tag{26}$$

where $\lambda_{\max}$ represents the maximum eigenvalue of the matrix R_{XX}. Notice, unfortunately, that there exist the time-varying functions $\sigma_e^2(n)$ and $\sigma_e(n)$ in the upper bound of the condition for μ as was the cases of the LMF and the LMAT algorithms [2], [6]. Since $\sigma_e^2(n)$ can be often large at the beginning of adaptation processes, we see that the convergence of the MAS algorithm may still depend on the choice of initial conditions. Unlike the LMF and the LMAT algorithms, however, the upper bound for μ of the MAS algorithm contains these time-varying functions in both the numerator and the denominator, the risk can be significantly reduced according to the choice of the mixing parameter α.

We next derive an expression for the mean-squared estimation error $\sigma_e^2(n)$. employing (13) in (14) and using Assumption 2, it follows

$$\begin{aligned}\sigma_e^2(n) &= \xi_{\min}+E\left\{V^T(n)\,X(n)\,X^T(n)V(n)\right\}\\ &\quad -2E\left\{V^T(n)\,X(n)e_{\min}(n)\right\}\\ &= \xi_{\min}+tr\left\{K(n)\,R_{XX}\right\},\end{aligned} \tag{27}$$

where $\xi_{\min}$ is obtained by using (12) in (15) so that

$$\xi_{\min}=E\left\{d^2(n)\right\}-H_{opt}^T\,R_{dX}, \tag{28}$$

and by Assumption 2 as well as the orthogonality principle,

$$E\left\{V^T(n)\,X(n)\,e_{\min}(n)\right\}=0, \tag{29}$$

and

$$E\left\{V^T(n)\,X(n)\,X^T(n)\,V(n)\right\}=tr\left\{K(n)\,R_{XX}\right\}. \tag{30}$$

Here, $K(n)$ is defined in (10), and $tr\{\cdot\}$ denotes the trace of $\{\cdot\}$. Notice from (27) that if $K(n)$ converges, so does $\sigma_e^2(n)$.

Finally, we need an expression for $K(n)$ to complete the analysis. Substituting (11) in (10) leads to

$$\begin{aligned}K(n+1) &= K(n)+\mu^2\alpha^2R_{XX}\\ &\quad +\mu\alpha\,E\left[V(n)\,X^T(n)\,sign\{e(n)\}\right]\\ &\quad +\mu\alpha\,E\left[X(n)V^T(n)\,sign\{e(n)\}\right]\\ &\quad +3\mu(1-\alpha)\,E\left[V(n)\,X^T(n)\,e^2(n)\,sign\{e(n)\}\right]\\ &\quad +3\mu(1-\alpha)\,E\left[X(n)V^T(n)\,e^2(n)\,sign\{e(n)\}\right]\\ &\quad +6\mu^2\alpha(1-\alpha)\,E\left[X(n)\,X^T(n)\,e^2(n)\right]\\ &\quad +9\mu^2(1-\alpha)^2\,E\left[X(n)\,X^T(n)\,e^4(n)\right]\end{aligned} \tag{31}$$

We make use of the followings to simplify (31):

$$E\left[V(n)\,X^T(n)\,sign\{e(n)\}\right]\approx-\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_e(n)}K(n)\,R_{XX}, \tag{32}$$

$$E\left[X(n)V^T(n)\,sign\{e(n)\}\right]\approx-\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_e(n)}R_{XX}\,K(n), \tag{33}$$

$$E\left[V(n)\,X^T(n)\,e^2(n)\,sign\{e(n)\}\right]\approx-2\sqrt{\frac{2}{\pi}}\,\sigma_e(n)\,K(n)\,R_{XX}, \tag{34}$$

$$E\left[X(n)V^T(n)\,e^2(n)\,sign\{e(n)\}\right]\approx-2\sqrt{\frac{2}{\pi}}\,\sigma_e(n)\,R_{XX}\,K(n), \tag{35}$$

$$E\left[X(n)V^T(n)\,e^2(n)\right]\approx\left[\sigma_e^2(n)\,I_N+2\,R_{XX}\,K(n)\right]R_{XX}, \tag{36}$$

and

$$E\left[X(n)V^T(n)\,e^4(n)\right]\approx3\sigma_e^2(n)\left[\sigma_e^2(n)\,I_N+3\,R_{XX}\,K(n)\right]R_{XX} \tag{37}$$

In (32) through (37), we have made use of the following approximations as well as the approximation in (20):

$$E\left\{\sigma_{e|V}(n)V(n)V^T(n)\right\} \approx \sigma_e(n)K(n), \quad (38)$$

$$E\left\{\sigma_{e|V}^4(n)\right\} \approx \sigma_e^4(n), \quad (39)$$

and

$$E\left\{\sigma_{e|V}^2(n)R_{XX}V(n)V^T(n)R_{XX}\right\}$$
$$\approx \sigma_e^2(n)R_{XX}K(n)R_{XX}, \quad (40)$$

Therefore, combining (32) through (37) with (31), we obtain the expression for the mean-squared behavior of the coefficients of the MAS algorithm as

$$\begin{aligned} K(n+1) = {} & K(n) - 2\mu^2 R_{XX} \\ & - \mu\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_e(n)}\left[\alpha + 6(1-\alpha)\sigma_e^2(n)\right] \\ & \left[K(n)R_{XX} + R_{XX}K(n)\right] \\ & + 6\mu^2\alpha(1-\alpha)\left[\sigma_e^2(n)I_N + 2R_{XX}K(n)\right]R_{XX} \\ & + 12\mu^2(1-\alpha)^2\sigma_e^2(n)\left[\sigma_e^2(n)I_N \right. \\ & \left. + 3R_{XX}K(n)\right]R_{XX}. \end{aligned} \quad (41)$$

This now completes our analysis of the MAS algorithm. It is very difficult to obtain a sufficient condition for the convergence of $K(n)$ from (41). We are currently working on finding the condition, and hopefully publish the results some other time.

Ⅳ. Experimental Results

In order to demonstrate the motivation to the MAS algorithm and the validity of our derivations in the previous section, we present some experimental results for which the MAS algorithm is used in the third-order adaptive predictor. The primary input d(n) is modeled as an autoregressive process given by

$$d(n) = \zeta(n) + 0.9d(n-1) - 0.1d(n-2) - 0.2d(n-3), \quad (42)$$

where $\zeta(n)$ is a white Gaussian process with zero-mean and variance such that the variance of $d(n)$ is 1. The results are produced by taking the ensemble averages over 50 independent runs using 5,000 samples.

We first confirm our motivation to the MAS algorithm by plotting the trace values of the covariance matrix $K(n)$ for the sign, the MAS, and the LMAT algorithms. The result is depicted in Figure 1, where the convergence parameter μ is selected to be 0.007 for each algorithm. Curves 1, 2, and 3 represent $tr\{K(n)\}$ for the sign, MAS, LMAT algorithms, respectively. From Figure 1, we can exactly see that the sign algorithm is normally stable in spite of slow convergence, while the LMAT algorithm is relatively fast convergent despite keen sensitiveness to stability. The MAS algorithm, however, retains fast convergence as well as robustness as expected.

We next check the validity of our derivations presented in the previous section using the same third-order adaptive predictor. The parameter μ is selected to be 0.005 this time. Figure 2 illustrates the theoretical and empirical results for the mean behavior $E\{H(n)\}$ of the third-order adaptive coefficients, and Figure 3 shows those for the mean-squared behavior of the coefficients by plotting the

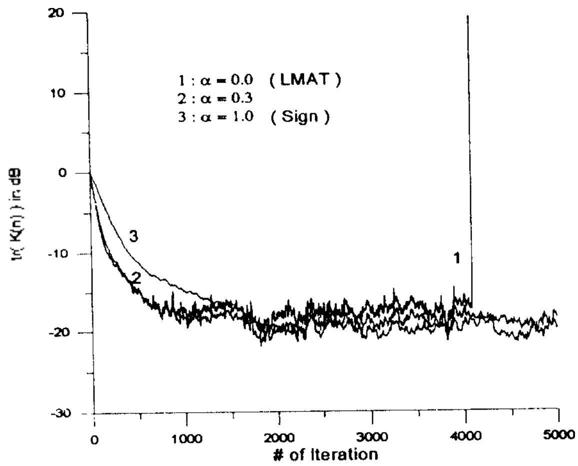

Figure 1. Simulation results of normalized $tr\{K(n)\}$ in dB to confirm the motivation to the MAS algorithm.

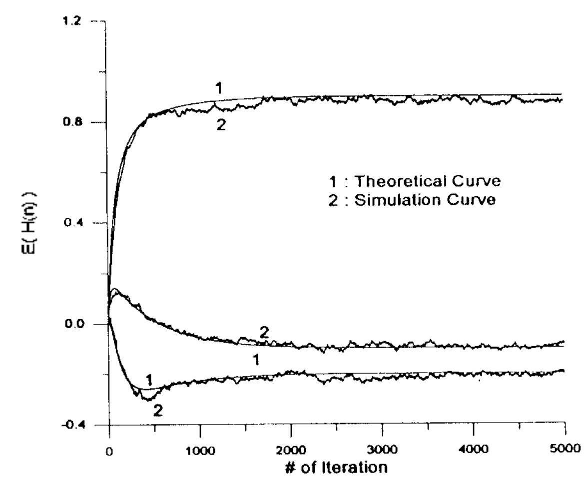

Figure 2. Mean behavior of the coefficients, $E\{H(n)\}$.

normalized $tr\{K(n)\}$ in dB. It can be observed that the theoretical curves agree with the simulation ones fairly well.

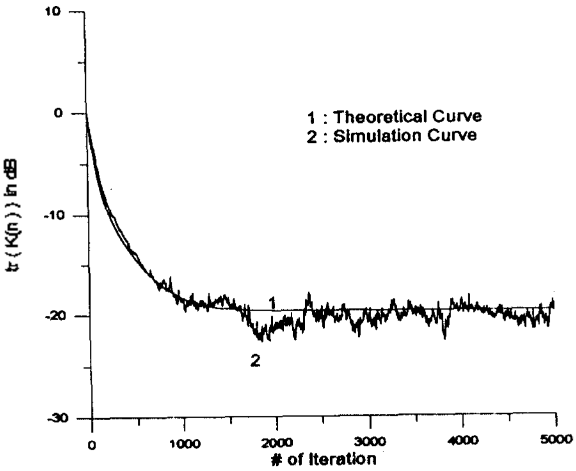

Figure 3. Mean-squared behavior of the coefficients, normalized $tr\{K(n)\}$ in dB.

V. Concluding Remarks

In this paper, the MAS algorithm based on the newly mixed norm error condition was proposed, and its statistical convergence properties were analytically investigated. When the input signals involved are zero-mean, wide-sense stationary, and Gaussian, a set of nonlinear evolution equations that characterizes the mean and mean-squared behavior of the algorithm was derived. Experimental results were presented in order to confirm our motivation to the MAS algorithm and to check the validity of our derivation. It was observed that the MAS algorithm is able to retain both fast convergene and robustness successfully as expected, and that our theoretical results matche's very well with simulation ones very well.

We are currently working on finding the mean-squared convergence condition. We shall also examine performances of the MAS algorithm in various signal environments and compare with other competing algorithms.

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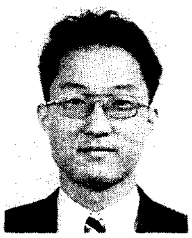

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