# A Frequency-Variable Ultrasonic Vibrator with Conical Horn for Ultrasonic Machining

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## Abstract

A frequency-variable ultrasonic vibrator is designed, which is made of a multi-layered PZT vibrator with a conical horn. Transmission line equations for the conical horn are derived and analyzed using the equivalent circuit method in order to analyze the characteristics of the vibrator. The controllability of the driving characteristics by varying the electrical impedance is confirmed by the experimental results of the free admittance characteristics and the vibrational velocity distributions.

## I. Introduction

The vibrating force of ultrasonic waves has been widely utilized in machining fields such as welding and cutting [1]. It can also be used in the ultrasonic vibrators for surgical operations[2-4]. These ultrasonic equipments generally provide good processed states, energy saving and easy automation[5]. However, resonant frequencies of the conventional ultrasonic equipments cannot be controlled their resonant frequency. In case of processing materials of different characteristic impedances, different ultrasonic equipments will be needed because the optimum driving frequency of ultrasonic vibrators is different for different characteristic impedance of the material to be processed[6].

It is known that the resonant frequencies of a thickness mode piezoelectric vibrator with high electromechanical coupling vary notably according to the electrical impedance connected to the electrical terminals[7]. This effect is utilized for the frequency control of multi-layered ultrasonic transducers[8-9]. We derived recently a new transmission-line model for a thickness mode piezoelectric vibrator which includes an electrical impedance connected to the electrical terminals. By use of the new transmission-line model, we analyzed the bolt-clamped type frequency-controllable ultrasonic transducer[10-11].

A solid horn is used as an amplitude transformer in ultrasonic equipments for dynamic engineering applications as mentioned above. For this purpose, the horn of conical type is most commonly used[12]. The conical horn has been analyzed by several authors[13-15].

In this paper, a frequency-variable ultrasonic vibrator is designed, which is made of multi-layered PZT vibrator with a conical horn. Transmission-line equations for the conical horn are derived and analyzed using the equivalent circuit in order to analyze the characteristics of the vibrator. The driving characteristics of the device are confirmed to be varied by the electric impedance by the experimental results of the free admittance characteristics and the vibrational velocity distributions.

## II. Theory

### 2.1 A New Transmission-line Model Equivalent Circuit

A new transmission-line model equivalent circuit was investigated for the case where an electrical impedance $Z_e$ is connected to the electrodes of the piezoelectric vibrator as shown in Fig. 1.

The electric charge $Q_i$ due to the external stress is as follows[16];

$$Q_i = \frac{h}{j\omega}(v_l - v_o) \tag{1}$$

where $h$ is the piezoelectric h constant, and $v_o$, $v_l$ are the particle velocities on the two surfaces ($x=0$ and $x=l_b$), respectively.

A portion of the electric charge $Q_i$ moves across the electrical impedance, and then the charge remaining in the electrodes is as follows;

$$Q_{eff} = \frac{Q_i}{j\omega C_o Z_e + 1} = \sigma Q_i \tag{2}$$

where $C_o$ is the clamped capacitance of the piezoelectric

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vibrator.

Thus, the mechanical force due to the electric charge $Q_{eff}$ is $F_e = hQ_{eff}$. From this relation, a new transmission-line equation including the load condition in the electrodes is obtained as follows;

$$\begin{bmatrix} p_l \\ v_l \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} p_o \\ v_o \end{bmatrix} \tag{3}$$

$$\text{where } \boldsymbol{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix},$$

$$P_{11} = P_{22} = \frac{\dfrac{z_o}{j\tan\gamma l_b} + z}{\dfrac{z_o}{j\sin\gamma l_b} + z}, \quad P_{12} = \frac{z_o^2 + \dfrac{2z_o(1-\cos\gamma l_b)}{j\sin\gamma l_b} z}{\dfrac{z_o}{j\sin\gamma l_b} + z},$$

$$P_{21} = \frac{1}{\dfrac{z_o}{j\sin\gamma l_b} + z}, \quad z = -\frac{\sigma C_o h^2}{j\omega}$$

In these equations, $p_o$, $p_l$ are the forces on the two surfaces ($x=0$ and $x=l_b$), and the characteristic impedance and wave number of the piezoelectric vibrator are denoted by $z_o$ and $\gamma$, respectively. Defining $F_A = -zv_l$ and $F_B = -zv_o$, Eqn. (3) may be rewritten as

$$\begin{bmatrix} p_l - F_B \\ v_l \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \boldsymbol{B} \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_o + F_A \\ v_o \end{bmatrix} \tag{4}$$

$$\text{where } \boldsymbol{B} = \begin{bmatrix} \cos\gamma l_b & jz_o \sin\gamma l_b \\ j\dfrac{1}{z_o}\sin\gamma l_b & \cos\gamma l_b \end{bmatrix},$$

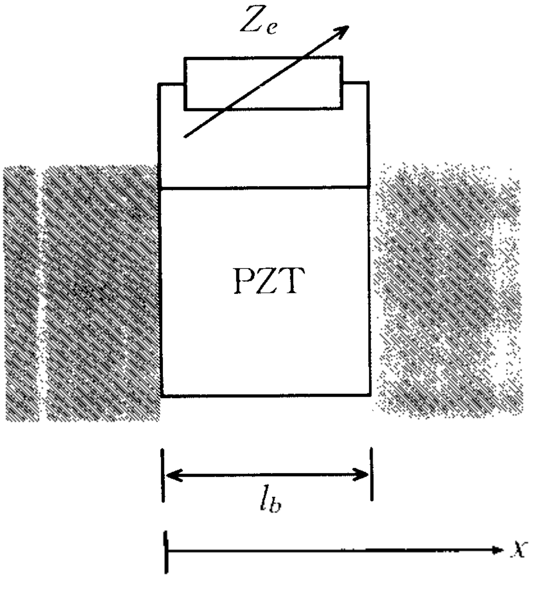


Figure 1. Thickness mode piezoelectric vibrator and electrical impedance connected to electrical terminals.

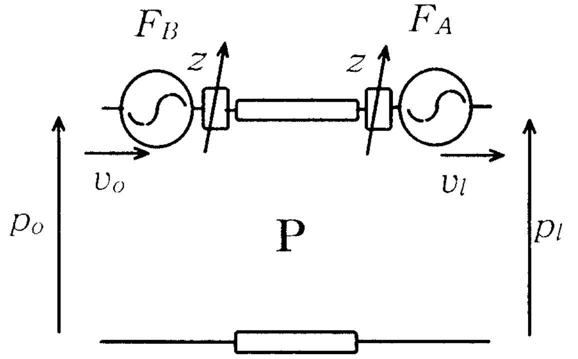


Figure 2. New transmission-line model equivalent circuit including electrical impedance effect.

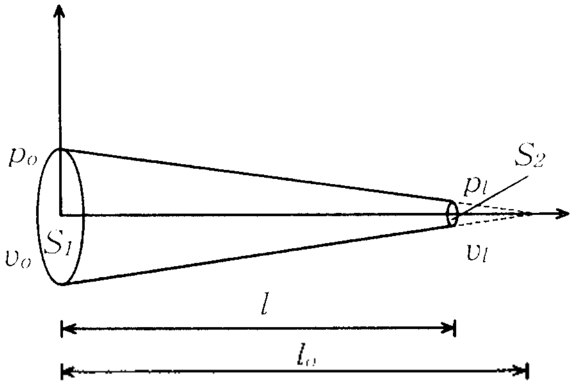


Figure 3. Horn for amplification of vibrational displacement amplitude.

From this equation, a new transmission-line model equivalent circuit can be constructed as shown in Fig. 2.

In Fig. 1 and Eqn. (2), we can investigate the effect of the electrical load $Z_e$. In the case of $Z_e = \infty$, open electrical terminals, the quantity $z$ is naught in Eqn. (4). As a special case, when the electrical load is an impedance of inductance $L_e$, the quantity $\sigma$ in Eqn. (2) can be found as follows;

$$\sigma = \frac{1}{1-\omega^2 C_o L_e} \tag{5}$$

The closer the value of $L_e$ is to $1/(\omega^2 C_o)$, the larger the value of the impedance $z$ in the equivalent circuit is. Thus, the acoustic characteristics of the piezoelectric vibrator, and thus the resonant frequency of the transducer can be varied.

An elastic horn used in ultrasonic devices is shown in Fig. 3. In this paper, to analyze the horn using the equivalent circuit method, a transmission-line model equivalent circuit for the horn is derived as follows;

$$\begin{bmatrix} p_l \\ v_l \end{bmatrix} = \frac{l}{l-l_o} C \begin{bmatrix} p_o \\ v_o \end{bmatrix} \tag{6}$$

where

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$C_{11} = \frac{l}{l_o} \sqrt{\frac{S_2}{S_1}} \sin \alpha l + \frac{S_2}{S_1} \cos \alpha l,$$

$$C_{12} = j \left\{ \left( \frac{c\sqrt{S_1 S_2}}{\omega \alpha l_o} + \frac{cS_2}{\omega} \right) \sin \alpha l \right.$$

$$\left. + \left( - \frac{c\sqrt{S_1 S_2}}{\omega l_o} + \frac{cS_2}{l_o \alpha \omega} \right) \cos \alpha l \right\},$$

$$C_{21} = -j \frac{\omega}{cS_1 \alpha} \sin \alpha l, \quad C_{22} = \frac{1}{\alpha l_o} \sin \alpha l - \cos \alpha l.$$

In these equations, $\alpha$ and $c$ are the wave number and the elastic constant of the conical horn respectively, and $S_1$, $S_2$ are the cross sectional areas of the two ends of the horn.

## 2.2 Calculation of Free Admittance

As an example of the electrical load effect mentioned above, the frequency-controllable ultrasonic vibrator, which has a driving layer and two frequency control layers, is suggested. As shown in Fig. 4, an electric source is connected to the driving layer(**B**) and the electrical impedance are connected to the frequency control layers(**P**). These layers are sandwiched by a tail mass(**A**) and the conical horn(**C**). The equivalent circuit of this ultrasonic vibrator, based on the new transmission-line model, is shown in Fig. 5. The matrix of the transmission-line equations for the tail mass is given as follows;

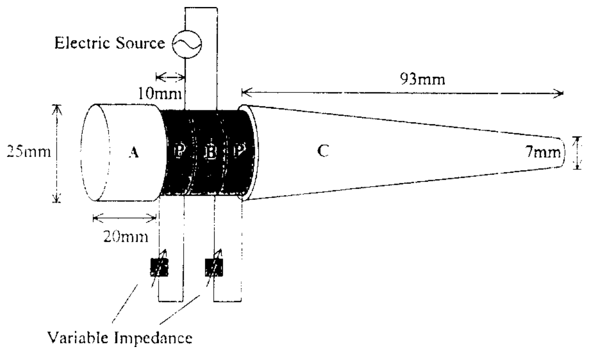


Figure 4. Schematic of frequency-variable ultrasonic vibrator.

$$\mathbf{A} = \begin{bmatrix} \cos \beta l_a & jz_a \sin \beta l_a \\ j\frac{1}{z_a} \sin \beta l_a & \cos \beta l_a \end{bmatrix} \tag{7}$$

where $l_a$ is the length of the tail mass, and the characteristic impedance and the wave number of the tail mass are denoted by $z_a$ and $\beta$, respectively.

Using this equivalent circuit, all terms in the fundamental equations of electroacoustic transducers[17] can be obtained as follows;

Mechanical impedance:

$$z_t = \frac{p_o}{v_o} = -\frac{Q_{12}}{Q_{11}} \tag{8}$$

Force factor:

$$A = -\frac{v_o}{I} z_t \tag{9}$$

$$= \frac{h}{j\omega} \left\{ \frac{Q_{22}^{-1}}{Q_{12}^{-1}} (V_{22} - U_{22}) + (-V_{12} + U_{12}) \right\} z_t$$

Clamped impedance:

$$Z_d = \frac{V}{I} = \frac{1}{j\omega C_o} - \frac{1}{j} \left( \frac{h}{\omega} \right)^2 \left[ \left\{ \frac{Q_{12}^{-1}}{Q_{22}^{-1}} (V_{12} - U_{12}) \right. \right.$$

$$\left. \left. + (-V_{22} + U_{22}) \right\} (V_{21} - U_{21}) + B_{21} \right] \tag{10}$$

where

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} = \boldsymbol{P} \cdot \boldsymbol{A} \tag{11}$$

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = \boldsymbol{B} \cdot \boldsymbol{U} \tag{12}$$

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \boldsymbol{P} \cdot \boldsymbol{V} \tag{13}$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \frac{l}{l-l_o} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \cdot X \tag{14}$$

and $Q_{12}^{-1}$ and $Q_{22}^{-1}$ are the elements of the inverse matrix of $\boldsymbol{Q}$.

The free impedance $Z_f$ can be obtained by substituting all terms obtained above into the following equation.

$$Z_f = Z_d + Z_a - \frac{A^2}{z_r + z_t} \tag{15}$$

The free admittance can also be obtained as the inverse of the free impedance. From the free admittance, the frequency characteristics can be found.

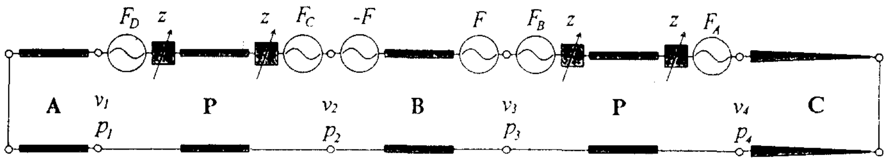


Figure 5. Equivalent circuit of frequency-variable ultrasonic vibrator.

### 2.3 Calculation of Velocity Distributions

It is very convenient to calculate the velocity distributions or the force distributions in the vibrator using the new transmission-line model equivalent circuit derived in this study. Substituting a resonance frequency $f_o$ obtained from Eqn. (15) into Eqn. (14) and rewriting the equation in term of functions of distance, the relations between forces and velocities at the two acoustic terminals are obtained as follows;

$$\begin{bmatrix} p_5(x) \\ v_5(x) \end{bmatrix} = \begin{bmatrix} Q_{11}(x) & Q_{12}(x) \\ Q_{21}(x) & Q_{22}(x) \end{bmatrix} \begin{bmatrix} p_o \\ v_o \end{bmatrix} \tag{16}$$

Applying the boundary condition $p_o = 0$, the force and velocity distributions can be easily obtained as follows;

$$p_5(x) = Q_{12}(x)\, v_o \tag{17}$$

$$v_5(x) = Q_{22}(x)\, v_o \tag{18}$$

## Ⅲ. Theoretical and Experimental Results

### 3.1 Fabrication

Fig. 6 shows a photograph of the frequency-variable ultrasonic vibrator. It has three pairs of 5*mm*-thick piezoelectric layers. The polarized direction in each piezoelectric layer is put to contact with each other head to head. Thus, each pair acts like one 10*mm*-thick piezoelectric layer. The piezoelectric layers are sandwiched by an aluminum column and an aluminum horn, and clamped together with a steel bolt. A surgical vibrator is fixed at the end of the horn which amplifies the amplitude of the vibration. As an electrical load connected to the electrical terminals of frequency control layers, a coil made of a wire wound on a bakelite cylinder is used. The inductance of the coil is varied by moving a core through the cylinder.

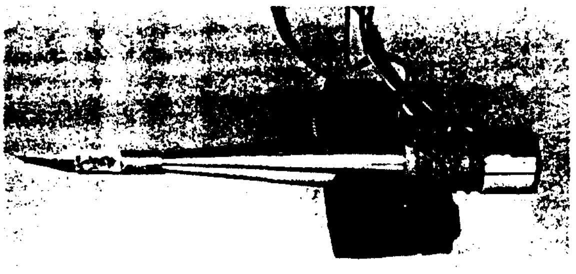


Figure 6. Photograph of the frequency-variable ultrasonic vibrator.

### 3.2 Results of Free Admittance

The experimental results of the free admittance characteristics are shown in Fig. 7 with the theoretical results. The experimental results agree well with the theoretical results. In this figure, the resonant frequency of the ultrasonic vibrator can be varied widely by the inductance value $L_e$ of the coil connected to the electric terminals of the frequency control piezoelectric layers. In the case of $L_e = \infty$, the admittance characteristics show three resonant modes of 20kHz, 33kHz and 48kHz in the frequency range shown. The resonant frequencies of 20kHz and 33kHz move to 24kHz and 37kHz, respectively, when the value of inductance $L_e$ is 62mH. By changing the inductance value to 33mH, the resonant frequencies can be changed to 30kHz and 44kHz. These results clearly show the possibility of the frequency control of the ultrasonic vibrator suggested in this study.

### 3.3 Measurement of Velocity Distributions

To measure the velocity distributions of the frequency-variable ultrasonic vibrator, the non-contact measurement method[18] can be used. The velocity distributions, when the ultrasonic vibrator is driven at the resonant frequencies found from Fig. 7, are shown in Fig. 8.

In this figure, the experimental results are marked by the symbol of ■. The theoretical results are calculated with Eqn. (18) in which the value of $v_o$ is the velocity value at $x = 0$ in the experiment. These results reconfirm the possibility of the control of the velocity distribution by varying the inductance.

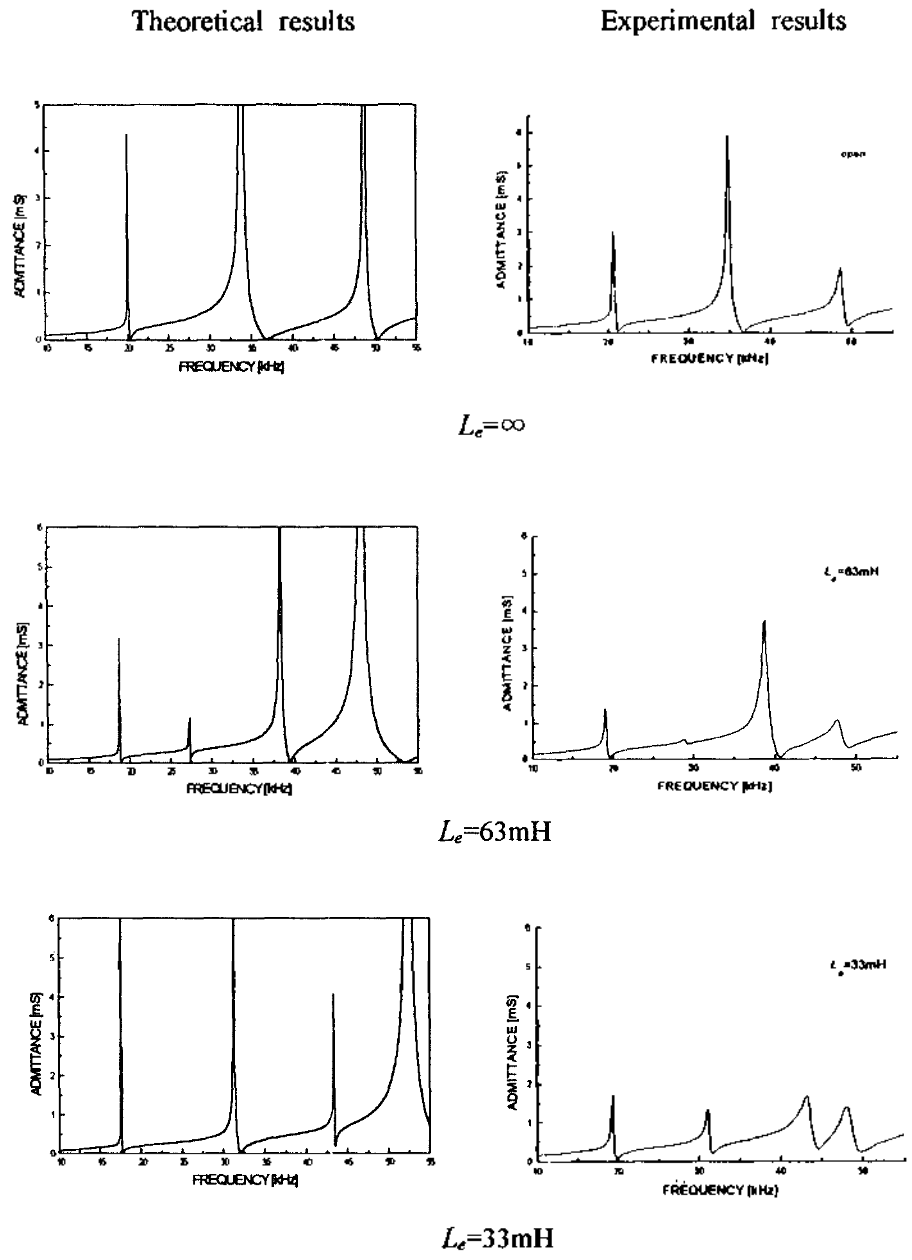
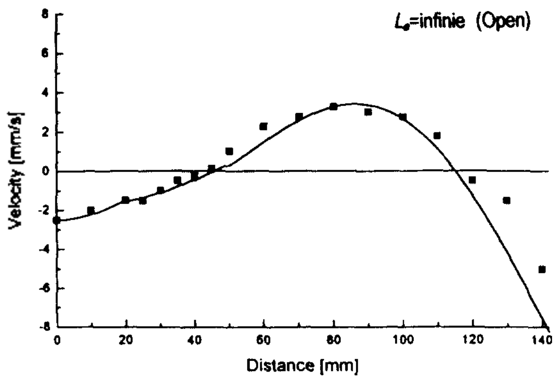


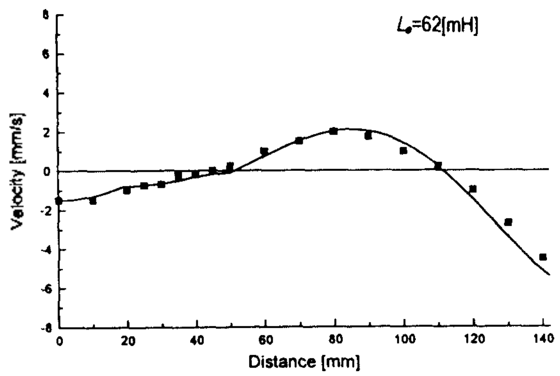
Figure 7. Free admittance characteristics.

## Ⅳ. Conclusion

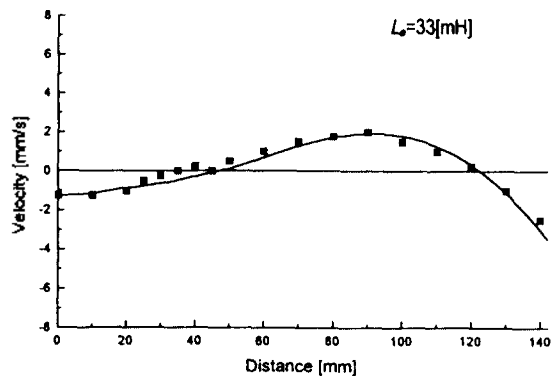
A frequency-variable ultrasonic vibrator for machining is designed, which is made of a multi-layered PZT vibrator with a conical horn. Transmission-line equations for the conical horn are derived and analyzed using the equivalent circuit method, in order to analyze the characteristics of the vibrator. The experimental results agree well with the theoretical results. It has be confirmed that the resonant frequency of the ultrasonic vibrator can be varied widely by changing the electrical impedance connected to the electrical terminals of piezoelectric layers.



(a) $f_o$=33kHz



(b) $f_o$=37kHz



(c) $f_o$=30kHz

Figure 8. Vibrational velocity distribution.

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