Adaptive Formulation of the Transition Matrix of Markovian Mobile Communication Channels

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Abstract

This study models mobile communication channels as a discrete finite Markovian process, and a Markovian jump linear system having parallel Kalman filter type is applied. What is newly proposed in this paper is an equation for obtaining the transition matrix according to sampling time by using a weighted Gaussian sum approximation and its simple calculation process. Experiments show that the proposed method has superior performance and reuires computation compared to the existing MJLS using tthe ransition matrix given by a statistical method or from priori information.

I. Introduction

Recently, development in chipsets has activated researches on equalizer algorithms requiring much computational burden, a typical example of which is the Maximum Likelihood Sequence Estimation Technique [1]. Lawrence applied the Kalman filter which is one of Maximum Likelihood Sequence Estimation Techniques to an adaptive equalizer algorithm [2]. After that, studies on adaptive equalizer using the Kalman filter have been progressed to produce a fast Kalman filter algorithm [3], and further to introduce parallel Kalman filter algorithm using Gaussian sum approximation [4]. However, fast Kalman filter algorithm has weaknesses in stability. And Gaussian sum approximation is computationally complex and burdensome because of the parallel Kalman filter number which is exponentially increasing according to time variation, thus requiring a suboptimal scheme. In mobile communication, the base station is relatively free compared to a mobile station in terms of space issue and complication in calculation process, and therefore, adaptive equalizer algorithm requiring a large amount of calculation may be applied thereto. That is, an equalizer having complicated and large computational capability can be used solely for reverse channels. Since the standards for mobile communication at present classify channel coefficients into urban, rural, etc., the channel environment of a mobile station is randomly varied to be urban or rural depending on time.

These channels are defined to be Markovian mobile communication channels in this paper, and Markovian jump linear system(MJLS) model, in which a channel coefficient is assumed to be a discretely finite random variable, is established and applied to the mobile communication environment. Particularly, in the existing Markovian jump linear system, the transition matrix has been given by a statistical method or from priori information; whereas, this paper presents, and shows along with experimental results, an equation which enables estimation of transition matrix, which is the core of Markovian jump linear system model, by using observation errors of Gaussian sum approximation according to time.

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II. Kalman Filter Algorithm

The Kalman filter is composed of a state equation and an observation equation shown in Equations (1) and (2), where the subscript 'k' is the time sampling:

$$X_k = FX_{k-1} + Gu_{k-1} \tag{1}$$

$$y_k = CX_k + v_k \tag{2}$$

In Equation (1), X_k is a state vector showing (D+1)-tap delay, and u_k is a binary random sequence in which probabilities of generating 0 and 1 are p and 1-p and its variance is p(1-p). The structures of F and G are as follows:

$$F = \begin{bmatrix} 0 & \cdots\cdots & \cdots\cdots & 0 \\ 1 & 0 & \cdots\cdots & 0 \\ 0 & 1 & \cdots\cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots\cdots & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 \\ 0 \\ : \\ : \\ 0 \end{bmatrix}$$

In Equation (2), y_k is an observation variable, and v_k is an observation noise, white Gaussian noise whose mean is 0 and variance is R. Also, channel coefficient vector C is defined in Equation (3), where the superscript T means transposition:

$$C^T = \begin{bmatrix} c_1 \\ c_2 \\ : \\ : \\ c_n \end{bmatrix} \tag{3}$$

The Kalman filter algorithm can be obtained from Equations (1) and (2) as follows:

$$X^{mean}_{k/k-1} = FX^{mean}_{k-1/k-1} + Gp$$
$$P_{k/k-1} = FP_{k-1/k-1}F^T + Q$$
$$e_k = y_k - CX^{mean}_{k/k-1}$$
$$K_k = P_{k/k-1}C^T[CP_{k/k-1}C^T + R]^{-1} \tag{4}$$
$$X^{mean}_{k/k} = X^{mean}_{k/k-1} + K_k e_k \tag{5}$$
$$P_{k/k} = P_{k/k-1} - K_k C P_{k/k-1}$$

where,

$$y^{k-1} \equiv \{y_0, y_1, \cdots, y_{k-1}\}$$
$$Q \equiv p(1-p)I$$
$$X^{mean}_{k-1/k-1} \equiv E(X_{k-1}/y^{k-1})$$
$$P_{k/k-1} \equiv E[(X_k - X^{mean}_{k/k-1})(X_k - X^{mean}_{k/k-1})^T/y^{k-1}]$$
$$P_{k/k} \equiv E[(X_k - X^{mean}_{k/k})(X_k - X^{mean}_{k/k})^T/y^k]$$

As is shown in Equation (4), the inverse matrix is obtained during the calculation process of Kalman gain (K_k). Generally, the greater the matrix size is, the more complicated and difficult the calculation of inverse matrix is. However, development of symmetry of covariance $P_{k/k}$ and chipsets makes it possible to obtain an estimator at real time using the Kalman filter. In Equation (5), Kalman gain (K_k) assumes the role of a weighted value with respect to an error. That is, when Kalman gain (K_k) is large, the state estimator depends on information of the observed value. And the decision processes of a receiver are as follows:

$$u(k-D) = 1, \; X^{mean}_{k/k}(1+D) \geq p$$
$$u(k-D) = 0, \; X^{mean}_{k/k}(1+D) < p$$

However, a model composed of Equations (1) and (2) has a weakness that the channel coefficient is fixed. In other words, the channel coefficient in Equation (3) has a constant during the estimation process; however, the channel coefficient in mobile communication may be assumed to be a random variable since channel environment is variously changed by fading.

The model described in this paper is to propose a Markovian jump linear system algorithm in which the channel coefficient is assumed to be a random variable having a discrete finite coefficient, for a finite channel environment.

Ⅲ. Proposed MJLS Algorithm

The model presented in this paper uses Equstion (6) instead of Equation (2)

$$y_k = C(\Theta_k)X_k + v_k \tag{6}$$

The sequence $\Theta_k := \{\theta_1, \cdots, \theta_m\}$ is a Markovian random variable having a discrete and finite value, where the priori transition probability is defined Equation (7):

$$p^{k,k+1}_{ij} = \Pr\{\Theta_{k+1} = \theta_j / \Theta_k = \theta_i\}, \quad \forall\ 1 \leq i, j \leq m \tag{7}$$

What is most important in a state estimator of Bayesian approach method is a posteriori probability density function. If the probability density function $f(X_{k-1}/y^{k-1})$ is assumed to be known, the total probability theorem will yield Equation(8)

$$f(X_{k-1}/y^{k-1}) = \sum_{i=1}^{m} f(X_{k-1}/\Theta_{k-1}, y^{k-1}) f(\Theta_{k-1}/y^{k-1}) \tag{8}$$

In the above equation, $f(X_{k-1}/\Theta_{k-1}, y^{k-1})$ is assumed to be a Gaussian probability density function $G(X_{k-1}; X^{mean}_{k-1/k-1}, P_{k-1/k-1})$ with mean X^{mean}_{k-1} and covariance $P_{k-1/k-1}$. Further, if $f(\Theta_{k-1}/y^{k-1})$ is expressed in terms of the weighted value $\alpha^{(i)}_{k-1/k-1}$, we have Equation (9) as follows:

$$f(X_{k-1}/y^{k-1}) = \sum_{i=1}^{m} \alpha^{(i)}_{k-1/k-1} G(X_{k-1}; X^{mean(i)}_{k-1/k-1}, P^{(i)}_{k-1/k-1}) \tag{9}$$

Accordingly, a state estimator can be obtained by recursively determining posteriori probability density function $f(X_k/y^k)$ by using $f(X_{k-1}/y^{k-1})$.

The posteriori pdf $f(X_k/y^k)$ is updated with the following algorithm [5].

step 1 : $f(\Theta_{k-1}/y^{k-1}) \rightarrow f(\Theta_k/y^{k-1})$
step 2 : $f(X_{k-1}/\Theta_{k-1}, y^{k-1}) \rightarrow f(X_k/\Theta_{k-1}, y^{k-1})$
step 3 : $f(X_k/\Theta_{k-1}, y^{k-1}) \rightarrow f(X_k/\Theta_k, y^{k-1})$
step 4 : $f(\Theta_k/y^{k-1}) \rightarrow f(\Theta_k/y^k)$
step 5 : $f(X_k/\Theta_k, y^{k-1}) \rightarrow f(X_k/\Theta_k, y^k)$

Recursive calculation may be summarized as follows.

(step 1)

$$\alpha^{(j)}_{k/k-1} = \sum_{i=1}^{m} p_{ij}^{k-1,k} \alpha^{(i)}_{k-1/k-1}$$

(step 2)

$$G(X_k ; X^{mean(i)}_{k/k-1}, P^{(i)}_{k/k-1})$$

where the average is $X^{mean(i)}_{k/k-1} = FX^{mean(i)}_{k-1/k-1} + Gp$, and covariance is $P^{(i)}_{k/k-1} = FP^{(i)}_{k-1/k-1}F^T + Q$.

(step 3)

$$G(X_k ; X^{mean(j)}_{k/k-1}, P^{(j)}_{k/k-1})$$

where the average $X^{mean(j)}_{k/k-1}$ and covariance $P^{(j)}_{k/k-1}$ are:

$$\omega^{(ij)}_{k/k-1} = \frac{p_{ij}^{k-1,k} \alpha^{(i)}_{k-1/k-1}}{\alpha^{(j)}_{k/k-1}}$$

$$X^{mean(j)}_{k/k-1} = \sum_{i=1}^{m} \omega^{(ij)}_{k/k-1} X^{mean(i)}_{k/k-1}$$

$$P^{mean(j)}_{k/k-1} = \sum_{i=1}^{m} \omega^{(ij)}_{k/k-1} \{ P^{(i)}_{k/k-1} + (X^{mean(j)}_{k/k-1} - X^{mean(i)}_{k/k-1})(X^{mean(j)}_{k/k-1} - X^{mean(i)}_{k/k-1})^T \}$$

(step 4)

$$\alpha^{(j)}_{k/k} = \frac{\alpha^{(j)}_{k/k-1} G(y_k ; C(\Theta) X^{mean(j)}_{k/k-1}, C(\Theta) P^{(j)}_{k/k-1} C^T(\Theta) + R)}{\sum_{j=1}^{m} \alpha^{(j)}_{k/k-1} G(y_k ; C(\Theta) X^{mean(j)}_{k/k-1}, C(\Theta) P^{(j)}_{k/k-1} C^T(\Theta) + R)}$$

$$p_{ij}^{k,k+1} = \alpha^{(j)}_{k/k}, \quad \forall i = 1, 2, \cdots, m$$

(step 5)

$$G(X_k ; X^{mean(j)}_{k/k-1}, P^{(j)}_{k/k-1})$$

where the average $X^{mean(j)}_{k/k-1}$ and covariance $P^{(j)}_{k/k}$ are:

$$K^{(j)}_k = P^{(j)}_{k/k-1} C(\Theta)^T \{ C(\Theta) P^{(j)}_{k/k-1} C^T(\Theta) + R \}^{-1}$$
$$X^{mean(j)}_{k/k} = X^{mean(j)}_{k/k-1} + K^{(j)}_k (y_k - C(\Theta) X^{mean(j)}_{k/k-1})$$
$$P^{(j)}_{k/k} = P^{(j)}_{k/k-1} - C(\Theta) P^{(j)}_{k/k-1}$$

The result of the above calculation brings in a posteriori probability density function as shown in Equation (10):

$$f(X_k/y^k) = \sum_{j=1}^{m} \alpha^{(j)}_{k/k} G(X_k ; X^{mean(j)}_{k/k}, P^{(j)}_{k/k}) \tag{10}$$

From these equations, a state estimator and covariance are obtained as follows:

$$X^{mean}_{k/k} = \sum_{j=1}^{m} \alpha^{(j)}_{k/k} X^{mean(j)}_{k/k}$$

$$P_{k/k} = \sum_{j=1}^{m} \alpha^{(j)}_{k/k} \{ P^{(j)}_{k/k} + (X^{mean(j)}_{k/k} - X^{mean}_{k/k})(X^{mean(j)}_{k/k} - X^{mean}_{k/k})^T \}$$

And their decision processes are:

$$u(k-D) = 1, \quad X^{mean}_{k/k}(1+D) \geq p$$
$$u(k-D) = 0, \quad X^{mean}_{k/k}(1+D) < p$$

An algorithm using the MJLS is more complicated than the Kalman filter. The reason for it is that the channel coefficient is assumed to be a random variable in the model. Particularly, in step 3, the Gaussian sum approximation equation is approximated to be a Gaussian probability density function in order to avoid exponential increase of Gaussian sum approximation. And in this paper, the transition matrix which varies according to time is obtained by using observation errors of the Gaussian sum approximation as in Equation (11) contrary to the existing MJLS algorithm in which the transition matrix is given by a statistical method or from priori information.

$$p_{ij}^{k,k+1} = \alpha^{(j)}_{k/k}, \quad \forall i = 1, 2, \cdots, m \tag{11}$$

If $\alpha^{(j)}_{k/k} = 1$, only the jth Kalman filter operates.

Ⅳ. Simulation

In mobile communication, the channel environment is largely divided into two: urban area and rural area. Particularly in radio communication, the channel coefficient is given as a standard according to urban or rural area. The channel coefficients of the urban and rural areas for conducting experiments are shown below. They are in the form of a Markov chain.

- Urban : $C(\theta_1) = (-0.077\ -0.355\ 0.059\ 1.0\ 0.059\ -0.273)$
- Rural : $C(\theta_2) = (-0.05\ -0.1\ 0.2\ 1.0\ 0.2\ -0.15)$

Also, S/N used in the experiments is:

$$S/N(dB) = 10 \log_{10} \frac{\sum_{t=1}^{k} (C^T X_{t/t}^{mean})^2}{\sum_{t=1}^{k} (y_t - C^T X_{t/t}^{mean})^2}$$

Figure 1 shows results of Kalman filtering with urban channel coefficient(the circled curve), rural channel coefficients(the * curve) mismatched Kalman filterindg and conventional MJLS algorithm(the +curve). Table 1 lists experimental conditions for Figure 1. Figure 1 shows that the conventional MJLS has superior performance to the mismatched Kalman filter.

Figure 2 shows the roles of weighting $\alpha_{0/0}^{(i)}$ and the transition matrix of the initial distribution in the condition given Table 2. MJLS types A and B are for the cases of different weighted values $\alpha_{0/0}^{(i)}$ of the initial distribution, but their results are almost the same. MJLS types B and C are for the case in which the transition matrix is differently designated, and they show that the conventional MJLS performance depends on the probability corresponding to actual channel coefficient in the transition matrix.

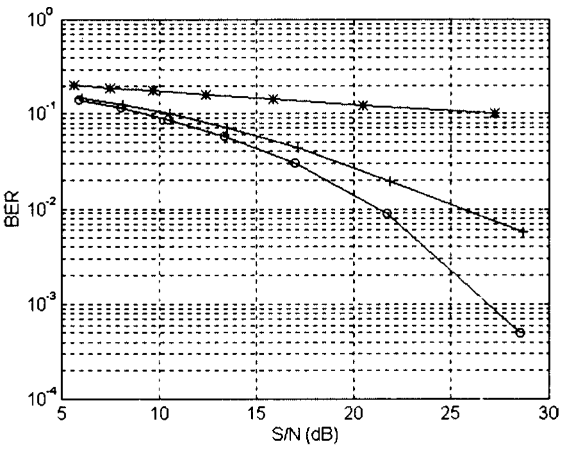

Figure 1. Bit Error Ratio for several types of binary system

Table 1. Simulation Conditions for Fig. 1

Algorithm(symbol)	Conditions
matched KF(o)	*channel coefficients* = $C(\theta_1)$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), P_{0/0} = O_{6\times6}$
mismatched KF(*)	*channel coefficients* = $C(\theta_2)$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), P_{0/0} = O_{6\times6}$
existing MJLS(+)	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.8, a_{0/0}^{(2)} = 0.2,\ p_{11} = 0.95, p_{22} = 0.95, p_{12} = p_{21} = 0.05$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$

Figure 3 shows comparison of using adaptive formulation of transition matrix proposed in this paper to the Kalman filter in which the transition matrix having coincidence probability of channel coefficient of 0.99 in the

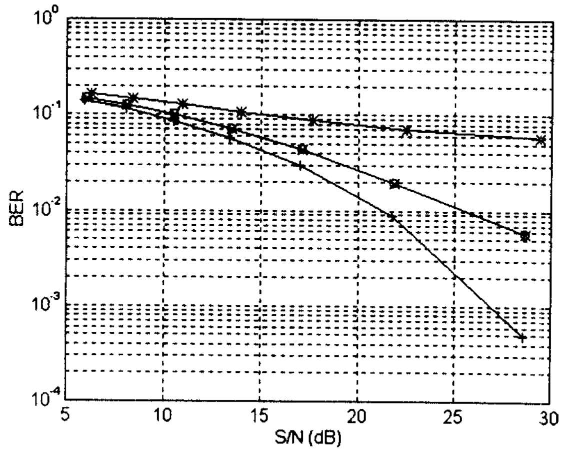

Figure 2. Bit Error Ratio for existing MJLS

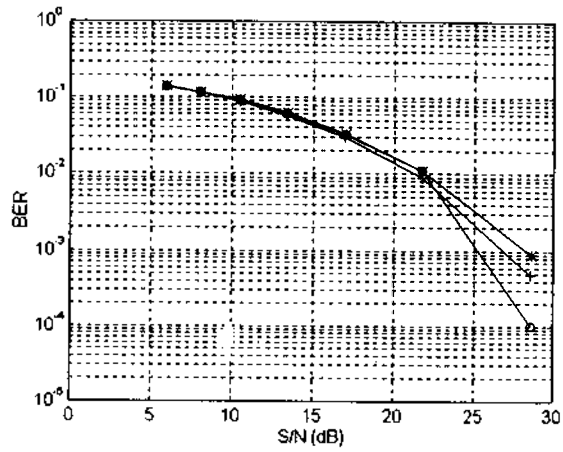

Figure 3. Proposed MJLS versus other types curve

Table 2. Simulation Conditions for Fig. 2

Algorithm(symbol) - Type	Condition
matched KF(•)	*channel coefficients* = $C(\theta_1)$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), P_{0/0} = O_{6\times6}$
Existing MJLS(o) - A	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.1, a_{0/0}^{(2)} = 0.9,\ p_{11} = 0.95, p_{22} = 0.95, p_{12} = p_{21} = 0.05$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$
Existing MJLS(x) - B	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.8, a_{0/0}^{(2)} = 0.2,\ p_{11} = 0.95, p_{22} = 0.95, p_{12} = p_{21} = 0.05$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$
EXisting MJLS(+) - C	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.8, a_{0/0}^{(2)} = 0.2,\ p_{11} = 0.3, p_{22} = 0.3, p_{12} = p_{21} = 0.7$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$

Table 3. Simulation Conditions for Fig. 3

Algorithm(symbol)	Condition
matched KF(•)	*channel coefficients* = $C(\theta_1)$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), P_{0/0} = O_{6\times6}$
Existing MJLS(•)	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.8, a_{0/0}^{(2)} = 0.2,\ p_{11} = 0.99, p_{22} = 0.99, p_{12} = p_{21} = 0.01$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$
Proposed MJLS(o)	*channel coefficients* = $C(\theta_1), C(\theta_2)$ $a_{0/0}^{(1)} = 0.8, a_{0/0}^{(2)} = 0.8,\ p_{11}^{0,1} = 0.2, p_{22}^{0,1} = 0.2, p_{12}^{0,1} = p_{21}^{0,1} = 0.8$ $p_{ij}^{k,k+1} = a_{k/k}^{(j)} \quad \forall i = 1, 2, \cdots, m$ $X_{0/0}^{(1)} = (0,0,0,0,0,0), X_{0/0}^{(2)} = (0,0,0,1,0,0), P_{0/0}^{(1)} = P_{0/0}^{(2)} = O_{6\times6}$

existing MJLS algorithm is assumed to be an urban channel coefficient. Its experimental conditions are shown in Table 3. The results of Figure 3 show that the proposed MJLS algorithm has superior performance to the existing MJLS. Also, the proposed MJLS has one type of Kalman filter from sampling time $k = 52$, i.e., $\alpha^{(1)}_{51/51} = 1$. Whereas, the conventional MJLS system has to use two Kalman filters for the continuous observation values. It means that the proposed MJLS has simpler calculation processes compared to the existing MJLS.

V. Conclusions

Proposed in this paper is an improved MJLS algorithm shown in an equation where change of the channel environment of mobile communication into an urban or rural channel coefficient according to space of a mobile station is established to be a discrete finite random variable in its model, and the transition matrix is changed according to sampling time. The MJLS algorithm is an algorithm which can be applied to a finite number of channel environments, and is dependent on the transition matrix. Particularly, while the transition matrix is given by a statistical method or from priori information in the conventional MJLS algorithm, the transition matrix and other calculation processes are obtained simply in the MJLS algorithm proposed in this paper by using a weighted value $\alpha^{(j)}_{k/k}$ of Gaussian sum approximation which varies according to sampling time. The simulation results show that the proposed MJLS algorithm is superior. Inasmuch as the proposed MJLS algorithm can be applied to Markovian model which has discrete finite channel environment, it can be also extended not only to urban or rural channel environment discussed in this paper but also to other channel environments such as tunnel. However, development of faster algorithms is still needed in the proposed algorithm as Kalman filtering, which requires computing power, is used.

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