Intersymbol Interferences Due to Mismatched Roll-off Factors and Sampling-Time Jitter in a Gaussian Noise Channel

*Seungkeun Park, *Jin Dam Mok, and **Sangsin Na

Abstract

This paper presents two results on intersymbol interferences in baseband digital communication over an additive white Gaussian noise channel - the interferences due to mismatched square-root raised-cosine filters, in which the filters have different roll-off factors, and/or due to sampling-time jitter. The result for the mismatched filters is that even the jitter-free sampling causes intersymbol interference and it is negligibly small for a wide range of signal-to-noise ratio up to 10 dB, for the roll-off factor ranging from 0.2 to 0.5, the mismatch loss being within 0.1 dB from the optimum at around 10^{-6}. For jitter interference an approximation formula for the bit error probability is derived in case of the matched filters, which shows how the roll-off factors and the amount of jitter affect the system performance. The formula is reasonably accurate.

I. Introduction

Consider a baseband digital communication system described in Figure 1. The source emits, with equal probability, either 1 or -1 in every T second and its output denoted by $\{X_k\}_{k=-\infty}^{\infty}$ is assumed to be independent identically distributed (iid) random sequence with the alphabet $\{1, -1\}$. The transmitter is basically a pulse-shaping filter with the impulse response $h_s(t)$. The output $s(t)$ of the transmitter pulse-shaping filter excited by the source sequence is transmitted over the channel, which adds a continuous-time random noise $n(t)$ to $s(t)$. The receiver consists of a receiver filter, a sampler, and a decision block. The receiver filter with the impulse response $h_r(t)$ takes in the signal $r(t)$, the sum of the transmitted signal $s(t)$ and the channel noise $n(t)$. The sampler samples the output $z(t)$ of the receiver filter at nominal time $t=lT$ to produce $z(lT)$, which is compared with a preset threshold γ to estimate the transmitted source sequence. We will use $\gamma=0$ in the analysis since it is the optimum threshold in the sense that it gives the smallest average bit error probability.

Problem Statement The decision for $\hat{X}_k$ is based on the sampled output of the receiver filter $h_r(t)$. In this paper we are concerned with performance degradation due to two different kinds of interference reflected in the bit error probability $P(E) \triangleq \Pr(\hat{X}_k \neq X_k)$ of the baseband antipodal communication system where a raised-cosine pulse-shaping filter is used. We pursue this problem in the presence of a Gaussian channel noise. The pulse-shaping filter $h_s(t)$ and the receiver filter $h_r(t)$ in this paper are individually of a square-root raised-cosine characteristic and the overall response $h_o(t) \triangleq h_s(t) * h_r(t)$ constitutes a raised-cosine filter.

(1) Intersymbol Interference due to Mismatched Filters

One of the two problems dealt with in the paper is the effect of interference due to the mismatch of the roll-off factors of the transmitter and the receiver filters. For this mismatch interference we deal separately with perfect sampling-timing, which we will call jitter-free sampling, and with imperfect sampling-timing. The advantage of separate dealing is that one can see the direct cause and effect from a clearer viewpoint. This kind of mismatch occurs when a receiver is to be used for various transmitters with different roll-off factors in their respective pulse-shaping filters. A practical situation where the mismatch between the transmitter and the receiver roll-off factors occurs is the digital broadcasting systems via satellites. Here several satellites broadcast through pulse-shaping filters with different roll-off factors, while subscribers' receiver uses a filter with a fixed roll-off factor, e.g., 0.35. The fixed roll-off factor at the receiver is amenable to the mass production of the receiver sets at the sacrifice of bit error probability.

* Department Korea Electronics and Telecommunications Research Institute
** School of Electrical and Electronics Engineering, Ajou University

Manuscript Received: August 11, 1997.

(2) Intersymbol Interference due to Sampling-Time Jitter in a Matched Filtered System

The other problem considered is the intersymbol interference due to sampling-time jitter in a matched filtered system. In a practical digital communication system, imperfect timing is bound to occur in the sampling of the receiver filter output. Hence the sampling-time will not be at integer multiples of the source symbol duration T. This sampling-time jitter causes the system to suffer intersymbol interference and to lose the performance in error probability. This paper captures jitter as a random variable and quantifies the effect of jitter in the form of an approximation formula for the bit error probability. This formula is obtained from the linear approximation of the Taylor series expansion of the impulse response of the overall filter.

Previous work for sampling-time jitter has been reported in the literature, e.g., [1, 2, 3] among others. For the case of binary signaling over an additive white Gaussian noise channel, Lindsey and Simon in [1] discuss the jitter effect on the bit error probability as a function of signal-to-noise ratio for various degree of jitter. There the correlator synchronization error effect is discussed for baseband signals in an integrate-and-dump type of environment: nonreturn-to-zero pulses, the Manchester code, return-to-zero signals, and the Miller code. Since the channel bandwidth is implicitly assumed to be unlimited, the effect of jitter is limited at most to three source bits. Huang and et al. in [2] deals with pulses in connection with intersymbol interference and jitter. Raised cosine and/or square-root raised cosine filters are discussed as a method of relieving the adverse effect of sampling-time jitter and a general introduction can be found in [3] among others.

This paper resembles [1] in that the purpose and results are in the same vein, but differs from the previous work including [1] in that herein is studied (i) the effect of mismatched square-root raised-cosine filters in jitter-free sampling and (ii) the sampling-time jitter in a square-root raised-cosine filtered system, as opposed to an integrate-and-dump environment. And to the best of authors' knowledge, there has been no report on the study of the effect of the mismatch in square-root raised-cosine filters on the bit error probability and the derivation of an approximation formula for the intersymbol interference in the context considered herein.

The rest of the paper is organized as follows: in Section 2 preliminaries are discussed; in Section 3 the mismatched square-root raised-cosine filters with jitter-free sampling are considered and numerical results are presented; in Section 4 intersymbol interference due to jitter is considered and an approximation formula is derived for the bit error probability; and the conclusions follow in Section 5.

Ⅱ. Preliminaries

The goals of this paper are to study the effect of the roll-off factor mismatch on the error probability $P(E) \triangleq \Pr(\hat{X}_k \neq X_k)$ in the case of jitter-free sampling, and to derive an approximation formula for the intersymbol interference due to jitter in the case of matched filters. To derive an expression for $P(E)$ consider the output $s(t)$ of the pulse-shaping filter $h_s(t)$ in Figure 1 due to the source sequence $\{X_k\}$:

$$s(t) = \sum_{k=-\infty}^{\infty} X_k h_s(t-kT).$$

Then the receiver filter output $z(t)$ is given as follows:

$$\begin{aligned} z(t) &= (s(t)+n(t)) * h_r(t) \\ &= \sum_{k=-\infty}^{\infty} X_k h_s(t-kT) * h_r(t) + n(t) * h_r(t), \end{aligned}$$

where $n(t)$ is assumed to be an additive white Gaussian noise with the double-sided power spectral density function $S_n(f) = \frac{N_0}{2}$ and to be independent of the source $\{X_k\}$. For notational convenience let $h_o(t)$ and $n_r(t)$ be defined as follows: $h_o(t) \triangleq h_s(t) * h_r(t)$, and $n_r(t) \triangleq n(t) * h_r(t)$. Then, since $h_s(t-kT) * h_r(t) = h_o(t-kT)$, the receiver filter output $z(t)$ can be written as

$$z(t) = \sum_{k=-\infty}^{\infty} X_k h_o(t-kT) + n_r(t). \qquad (1)$$

Without the loss of generality due to the strict sense stationarity of $\{X_k\}$ and $n(t)$, we can carry out the analysis for random variable X_0 and the resulting bit error probability will be valid for an arbitrary k. Therefore, we will assume that the source bit X_0 is transmitted and that the decision for X_0 is based on $z(t)$ sampled at $t=0$. The assumption of sampling at $t=0$ would be appropriate for a system with accurate sampling-time acquisition. Then the decision for X_0 will be

$$\hat{X}_0 = \begin{cases} 1, & \text{if } z(0) > 0, \\ -1, & \text{if } z(0) < 0. \end{cases}$$

The decision variable $z(0) = z(t)|_{t=0}$ can be broken into several terms: from (1)

$$
\begin{aligned}
z(0) &= \sum_{k=-\infty}^{\infty} X_k h_o(-kT) + n_r(0) \\
&= X_0 h_o(0) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} X_k h_o(-kT) + n_r(0),
\end{aligned}
\tag{2}
$$

where the first term is due to the corresponding source message bit X_0, the second the source bits other than X_0, and the last the noise. The second term is the intersymbol interference. At the design stage the overall filter $h_o(t)$ is chosen so that $h_o(t-kT)$ at the nominal sampling-time $t = mT$, i.e., jitter-free sampling-time, is zero except $t=kT$. This condition is the Nyquist condition for zero intersymbol interference [3]. A class of filters satisfying the Nyquist condition is raised-cosine filters, whose impulse responses are of the form of

$$
h(t) = \frac{\sin(\pi t/T)}{\pi t/T}\,\frac{\cos(\pi \alpha t/T)}{1-4\alpha^2 t^2/T^2},
\tag{3}
$$

where α is called the roll-off factor and it takes on values from the interval $[0, 1]$.

III. Mismatch Intersymbol Interference

We have found out that the overall response does not satisfy the Nyquist condition for zero intersymbol interference when the filters of the transmitter and the receiver have different roll-off factors. To see this let α and β denote the roll-off factors of the transmitter and the receiver square-root raised-cosine filters, respectively. Then overall frequency response $H_o(f)$ is the product $H_\alpha(f)H_\beta(f)$ of the transmitter filter $H_\alpha(f)$ and the receiver filter $H_\beta(f)$, each of which is of a square-root raised-cosine characteristic. The overall impulse response $h_o(f)$ is given as follows.

$$
\begin{aligned}
\frac{h_o(\tau T)}{\sqrt{E_b}} &= (1-M)\frac{\mathrm{sinc}\big((1-M)\tau\big)}{1-16M^2\tau^2} \\
&+ \frac{1}{2}\frac{(1-m)\tau + \frac{|\alpha-\beta|}{4M}}{(\tau+\frac{1}{4M})}\,\mathrm{sinc}\big((1-m)\tau + \frac{|\alpha-\beta|}{4M}\big) \\
&+ \frac{1}{2}\frac{(1-m)\tau - \frac{|\alpha-\beta|}{4M}}{(\tau-\frac{1}{4M})}\,\mathrm{sinc}\big((1-m)\tau - \frac{|\alpha-\beta|}{4M}\big) \\
&- \frac{1}{2}m\sin(\pi\tau)\Big(\mathrm{sinc}\big(m(\tau+\frac{\alpha+\beta}{4\alpha\beta})\big) - \mathrm{sinc}\big(m(\tau-\frac{\alpha+\beta}{4\alpha\beta})\big)\Big) \\
&+ \frac{1}{2}m\cos(\pi\tau)\Big(\mathrm{sinc}\big(m(\tau+\frac{\beta-\alpha}{4\alpha\beta})\big) + \mathrm{sinc}\big(m(\tau-\frac{\beta-\alpha}{4\alpha\beta})\big)\Big),
\end{aligned}
\tag{4}
$$

where $M \triangleq \max\{\alpha, \beta\}$, $m \triangleq \min\{\alpha, \beta\}$, and $\tau \triangleq t/T$. This expression reduces to (3), when $E_b=1$ and $\alpha=\beta$, as it should.

We note that, when $\alpha \neq \beta$, $h(lT)$ can be nonzero for nonzero integer n. Therefore, in such cases, it violates the Nyquist condition for zero intersymbol interference and hence incurs intersymbol interference even for jitter-free sampling. As a specific example, let's consider the case of $\alpha=1$ and $\beta=0.5$. For jitter-free sampling, i.e., $t_l=lT$, the overall response sampled at such time instants $h_o(lT)$ equals

$$
\frac{\sqrt{E_b}}{4}(1+\cos(l\pi))\Big(\mathrm{sinc}\big(\frac{l+\frac{1}{2}}{2}\big) + \mathrm{sinc}\big(\frac{l-\frac{1}{2}}{2}\big)\Big),
$$

from which it can be seen that $h_o(lT)=0$ for $l=\pm 2i+1$ for integer i but nonzero otherwise. For example,

$$
\begin{aligned}
h_o(0) &= \frac{\sin(\frac{\pi}{8})}{\frac{\pi}{8}}\sqrt{E_b} \approx 0.9745\sqrt{E_b}, \\
h_o(\pm 2T) &= \Big(\frac{\sin(\frac{9}{8}\pi)}{\frac{9}{4}\pi} + \frac{\sin(\frac{7}{8}\pi)}{\frac{7}{4}\pi}\Big)\sqrt{E_b} \approx 0.0155\sqrt{E_b}.
\end{aligned}
$$

The bit error probability $P(E)$ including the roll-off factor mismatch intersymbol interference in case of jitter-free sampling is obtained from (2) and (4), using the fact that $n_r(0)$ is Gaussian with mean 0 and variance $\frac{N_0}{2}$ - this is found through a standard linear system approach, conditioning on iid equiprobable random vector $(X_{-L}\cdots X_{-1}\, X_1\cdots X_L)$, and taking the limit as the truncation length L of filters grows arbitrarily large.

$$
P(E) = \lim_{L\to\infty} P_L(E),
$$

where

$$
\begin{aligned}
P_L(E) &\triangleq \frac{1}{2^{2L}} \sum_{\mathbf{x}=(x_{-L}\cdots x_{-1}x_1\cdots x_L)} P_L(E|\mathbf{x}), \\
P_L(E|\mathbf{x}) &\triangleq Q\left(\frac{h_o(0) + \sum_{k=1}^{L} h_o(kT)(x_{-k}+x_k)}{\sqrt{\frac{N_0}{2}}}\right),
\end{aligned}
\tag{5}
$$

where the sum is taken over all the possible sequences $\mathbf{X} = (x_{-L}\cdots x_{-1}x_1\cdots x_L) \in \{-1, 1\}^{2L}$, the $2L$-tuple Cartesian product of $\{-1, 1\}$. We expect that $P(E) \approx P_L(E)$ for a large L.

Figure 2 shows $P_L(E)$ with $L=6$. This choice of L effectively equals $L=\infty$. The receiver roll-off factor β is chosen to be 0.35, a common practice in digital satellite communication. The transmitter roll-off factor α is chosen to be 0.2, 0.35 and 0.5. There are three curves closely spaced together. The matched case $\alpha=\beta=0.35$ reduces $P_L(E)$ to the optimum bit error probability $Q(\sqrt{\frac{2E_b}{N_0}})$ for binary signaling, which is represented by the bottom curve. From these curves we can say that the mismatch intersymbol interference is negligibly small for $\frac{2E_b}{N_0} \lesseqgtr 10$ dB, at which the mismatch interference is approximately within

20% of the optimum bit error probability in the case of $\alpha=0.2$, and 12% in the case of $\alpha=0.5$. However, the intersymbol interference grows with the signal-to-noise ratio. The reason for the growth is explained as following. For a moderate signal-to-noise ratio, the error due to the channel noise is dominant in the bit error probability over that due to the mismatch interference. Referring to (4) and (5), we note that the mismatch intersymbol interference occurs in the same proportion to E_b/N_0. On the other hand the noise affects less and less as the signal-to-noise ratio improves. Therefore, the mismatch interference gradually gains dominance over the channel noise as the signal-to-noise ratio increases.

Even with the growing mismatch interference, the resulting loss in terms of the signal-to-noise ratio at $P_L(E)=10^{-6}$ is within 0.1 dB for $\alpha=0.2$ and 0.5. Since this loss is equivalent to the energy increase of 2 to 3% ($10^{\frac{0.1}{10}}=1.0233$), one can decide favorably to pursue hardware simplicity at the cost of this slight increase in the signal energy.

From these observations, it is concluded that the mismatch interference can be ignored for a wide range of the signal-to-noise ratio and the signal energy can be bargained for hardware simplicity.

Ⅳ. Jitter Intersymbol Interference and Its Approximation

Imperfect timing due to jitter in the sampling of the receiver filter output causes the system to suffer intersymbol interference and hence to lose the performance in error probability. A reasonable approach to capturing jitter is to model the sampling-time as a random sequence. An example of such approach is to take the sampling time $\{t_m\}_{m=-\infty}^{\infty}$ as

$$t_m = mT + J_m,$$

where $\{J_m\}_{m=-\infty}^{\infty}$ is an iid Gaussian random sequence with mean 0 and variance σ_j^2.

We are concerned with the evaluation of the bit error probability $P(E) \triangleq \Pr(\hat{X}_k \neq X_k)$ with the following transmitter and receiver filters: $h_s(t)=\sqrt{E_b}\, h_{\surd}(t)$, and $h_r(t)=h_{\surd}(t)$, where $h_{\surd}(t)$ is the impulse response of the square-root raised-cosine filter and E_b accounts for the energy of the transmitted signal per bit. Then the overall filter impulse response $h_o(t)$ is given by

$$h_o(t) = h_s(t) * h_r(t) = \sqrt{E_b}\,h(t),$$

where $h(t)$ is a raised-cosine filter given in (3). Then we proceed with stationarity of $\{X_k\}$ and $n(t)$ as in jitter-free sampling in Section 3 to evaluate

$$P(E) = \Pr(\hat{X}_0 \neq X_0) = \Pr\left(z(t_0) < 0 \mid X_0 = 1\right),$$

where the decision variable $z(t_0)$ is the sampled output of $z(t)$ sampled at $t_0=0\cdot T+J_0$. In the remainder we let J denote the random variable J_0 for the notational simplicity. Then

$$z(J) = X_0 h_o(J) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} X_k h_o(J-kT) + n_r(J),$$

where $n_r(J)$ is a Gaussian random variable with mean 0 and variance $\frac{N_0}{2}$.

If we take the truncation of the filters into account, the decision variable $z(J)$ in (4) becomes

$$z(J) = X_0 h_o(J) + \sum_{\substack{k=-L \\ k\neq 0}}^{L} X_k h_o(J-kT) + \tilde{n}_r(J), \tag{6}$$

where $\tilde{n}_r(J)$ is a Gaussian random variable obtained from the truncated receiver filter:

$$\tilde{n}_r(J) = \tilde{h}_r(t) * n(t)\big|_{t=J},$$

where $\tilde{h}_r(J)$ is the truncated receiver filter. In the paper we will use the truncation length $L=6$ so that $\tilde{n}_r(J)$ may be virtually equal to $n_r(J)$.

Now we apply the Taylor series expansion of $h_o(t)$ at $t=kT$:

$$h_o(t) = \sum_{n=0}^{\infty} \frac{h_o^{(n)}(kT)}{n!}(t-kT)^n,$$

where $h_o^{(n)}(kT)$ is the n-th order derivative of $h_o(t)$ with respect to t evaluated at $t=kT$. Keeping the first two terms of the series will yield a Taylor-series approximation $\tilde{h}_o(t)$ of $h_o(t)$, i.e.,

$$\tilde{h}_o(t) \triangleq h_o(kT) + h_o'(kT)(t-kT) \quad \text{around } t=kT$$

The decision variable $z(J)$ in (6) can be approximated by

$$\tilde{z}(J) \triangleq X_0 h_o(0) + \sum_{k=1}^{L} h_o(kT)(X_{-k}+X_k) + J\sum_{k=1}^{L} h_o'(kT)(X_{-k}-X_k) + \tilde{n}_r(J).$$

We now define $\tilde{P}_L(E)$ to be the following approximation of the bit error probability $P(E)$:

$$\tilde{P}_L(E) \triangleq \Pr\left(\tilde{z}(J)<0\right).$$

Then, by conditioning on $(X_{-L}\cdots X_{-1}X_1\cdots X_L)=\mathbf{X}$ we find

$$\tilde{P}_L(E) = \sum_{\mathbf{x}=(x_{-L}\cdots x_{-1}x_1\cdots x_L)} P\left(\tilde{z}(J)<0 \mid \mathbf{x}\right)P(\mathbf{x})$$

$$= \frac{1}{2^{2L}} \sum_{x=(x_{-L}\cdots x_{-1}x_1\cdots x_L)} P(\tilde{z}(J) < 0|x),$$

where the sum is taken over all the possible sequences X $=(x_{-L}\cdots x_{-1}x_1\cdots x_L)\in\{-1, 1\}^{2L}$ and where is used the iid equiprobable property of source $\{X_k\}$. Since J and $\tilde{n}_r(J)$ are independent zero-mean Gaussian random variables with respective variances σ_J^2 and σ_n^2, $\Pr(\tilde{z}(J)<0\,|\,X)$ is found to be

$$Q\left(\frac{\sqrt{\frac{2E_b}{N_0}}\left(h_o(0)+\sum_{k=1}^{L}h_o(kT)(x_{-k}+x_k)\right)}{\left[\frac{2E_b}{N_0}\left(\sigma_J\sum_{k=1}^{L}h'_o(kT)(x_{-k}-x_k)\right)^2+1\right]^{\frac{1}{2}}}\right)$$

It is worth mentioning several special cases, where the above expression reduces to simpler formulas.

(a) Matched roll-off factors and jitter-free sampling

(b) Matched roll-off factors

(c) Jitter-free sampling

(a) Matched roll-off factors and jitter-free sampling In this case we have $h_o(t)$ a raised-cosine filter and therefore $h_o(0) = \sqrt{E_b}$ and $h_o(kT)=0$ for all $k=\pm1, \pm2, \ldots$ Also since jitter-free sampling means that $\sigma_J=0$, we have the optimum

$$\tilde{P}_L(E)=Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

(b) Matched roll-off factors This case corresponds to $h_o(0)=\sqrt{E_b}$ and $h_o(kT)=0$ for all $k=\pm1, \pm2, \ldots$ The effect of intersymbol interference due to jitter can be studied.

(c) Jitter-free sampling This case corresponds to $\sigma_J=0$. In fact this case has been considered in detail in the previous section.

In the following two subsections we focus on two cases where first the roll-off factors are matched $\alpha=\beta$ and secondly the mismatched roll-off factors are combined with sampling-jitter.

4.1 The Case of Matched Roll-off Factors

In this subsection we focus on the case of matched roll-

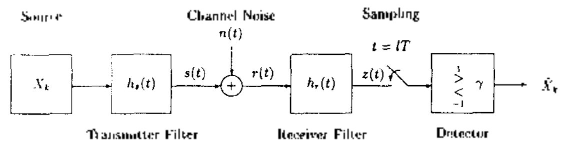

Figure 1. A block diagram of a digital communiation system

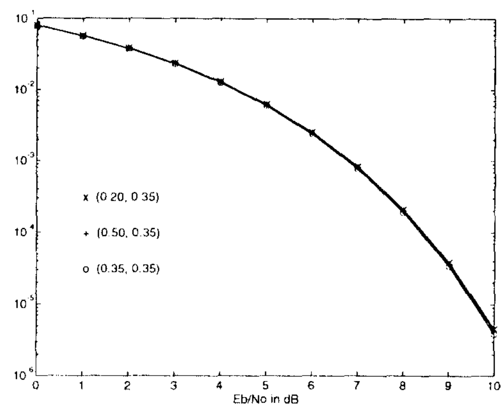

Figure 2. The bit error probability $P_L(E)$ of mismatched raised-cosine filter pairs with the truncation length $L=6$

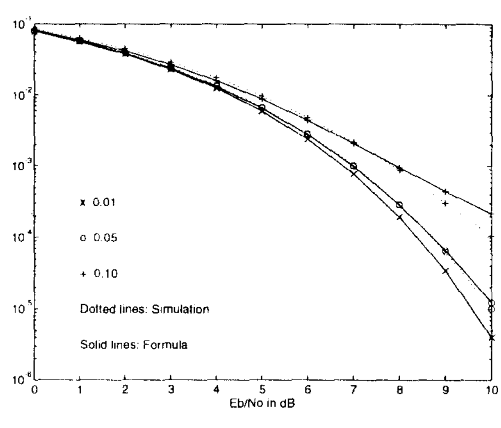

Figure 3. The bit error probability $P(E)$ from simulation and its approximation $\tilde{P}(E)$ for the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}$ =0.01, 0.05 and 0.1 : the roll-off factor α=0.35 and the truncation length $L=6$

Table 1. The $h_o(kT)$ and $h_o'(kT)$ for the raised-cosine filter with roll-off factor α

	$k=0$	$k=\pm1,\pm2,\ldots,\pm L$
$h_o(kT)$:	$\sqrt{E_b}$	0
$h'_o(kT)$:	0	$\sqrt{E_b}\frac{(-1)^k\cos(\alpha\pi k)}{kT(1-4\alpha^2k^2)}$ [1]

[1] In case of $2\alpha k=\pm1$, use the limit value

$$\lim_{\alpha\to\pm\frac{1}{2k}}\sqrt{E_b}\frac{(-1)^k\cos(\alpha\pi k)}{kT(1-4\alpha^2k^2)}=\sqrt{E_b}\frac{(-1)^k\pi}{4kT}$$

off factors. For the raised-cosine filter $h_o(t)=h(t)$ we obtain Table 1. Using the table yields

$$\tilde{P}_L(E)=\frac{1}{2^{2L}}\sum_{\mathbf{x}=(x_{-L}\cdots x_{-1}x_1\cdots x_L)}\tilde{P}_L(E|\mathbf{x}),$$

where

$$\tilde{P}_L(E|\mathbf{x})=Q\left(\left[\frac{E_b}{E_b\left(\frac{\sigma_J}{T}\sum_{k=1}^{L}\frac{(-1)^k\cos(\alpha\pi k)(x_{-k}-x_k)}{k(1-4\alpha^2k^2)}\right)^2+\sigma_n^2}\right]^{\frac{1}{2}}\right).$$

In case that the truncation length L is large, for example $L\geq 4$, σ_n^2 essentially equals $\frac{N_0}{2}$. With this assumption $\tilde{P}_L(E)$ is rewritten as follows.

$$\tilde{P}_L(E|\mathbf{x})=Q\left(\frac{\sqrt{\frac{2E_b}{N_0}}}{\left[\frac{2E_b}{N_0}\left(\frac{\sigma_J}{T}\sum_{k=1}^{L}\frac{(-1)^k\cos(\alpha\pi k)(x_{-k}-x_k)}{k(1-4\alpha^2k^2)}\right)^2+1\right]^{\frac{1}{2}}}\right). \quad (7)$$

Figure 3 shows the accuracy of $\tilde{P}_L(E)$ in (7) for the case of the roll-off factor $\alpha=0.35$, a common practice in digital satellite communication, and the truncation length $L=6$. The amount of jitter is specified in terms of the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}$, for which the following three values are chosen: 0.01, 0.05 and 0.1. The three solid lines are obtained from the evaluation of the approximation formula (7), while the dotted lines are obtained from an extensive simulation using random number generation. Figure 3 shows general agreement between the approximation formula and the simulation results for $\frac{\sigma_J}{T}=0.01$ and 0.05 for a wide range of the signal-to-noise ratio of 0 to 10 dB. (For the case of $\frac{\sigma_J}{T}=0.01$ the formula and the simulation are virtually indistinguishable and hence only the solid line is visible in the figure. These two also virtually coincide with the optimum bit error probability $Q(\sqrt{\frac{2E_b}{N_0}})$, the reason for which will be explained subsequently.) For $\frac{\sigma_J}{T}=0.01$ the approximation stays well within 2% of the simulation result for $\frac{E_b}{N_0}=0$ to 10 dB, the whole range that has been considered. For $\frac{\sigma_J}{T}=0.05$ the approximation stays within 5% of the simulation result for $\frac{E_b}{N_0}$ up to 9 dB and at $\frac{E_b}{N_0}=10$ it is 30% greater than the simulation. Even for the case of $\frac{\sigma_J}{T}=0.1$ the two lines are in good agreement for $\frac{E_b}{N_0}$ up to 8 dB, after which the discrepancy grows and reaches the factor of 2 at $\frac{E_b}{N_0}=10$ dB.

From these observations we conclude that the approximation formula is reasonably accurate for a wide range of the signal-to-noise ratio up to 10 dB and for the jitter-to-symbol duration ratio up to considerably large 0.05. Upon accepting this conclusion, we can take a closer look at the formula $\tilde{P}_L(E)$ in (7) for the various factors involved in the intersymbol interference induced by jitter. These factors are (a) the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}$, (b) the signal-to-noise ratio $\frac{E_b}{N_0}$, (c) the roll-off factor α, and (d) the truncation length L.

Among these factors, the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}$ plays the most important role in the sense that, if it is zero, then all the other factors are completely nullified - this case corresponds to jitter-free sampling and reduces the approximation formula to the optimum bit error probability of binary antipodal signalling. For a nonzero $\frac{\sigma_J}{T}$ all the other factors claim their roles. However, if $\frac{\sigma_J}{T}$ is so small that the first term of the square-rooted the denominator in (7) - the intersymbol interference term - is negligible compared with the second term - the channel noise term, then for a wide range of the signal-to-noise ratios $\tilde{P}_L(E)\approx Q(\sqrt{\frac{2E_b}{N_0}})$, one example of which is shown in Figure 3 for $\frac{\sigma_J}{T}=0.01$. For a significantly large $\frac{\sigma_J}{T}$ (i) the effect of intersymbol interference is still small, compared with the channel noise, for a small signal-to-noise ratio, e.g., $\frac{E_b}{N_0}\lessapprox 3$ dB, and hence the approximation formula is close to the optimum bit error probability, but (ii) it grows with $\frac{E_b}{N_0}$ and becomes dominant over the channel noise. (Also the approximation formula begins to lose the accuracy because the combined effect of the jitter and $\frac{E_b}{N_0}$ is too great to be captured in the first-order Taylor series approximation. This phenomenon is noticeable in Figure 3 at around 10 dB.)

The roll-off factor also affects the intersymbol interference. For example, with the same source sequence $(x_{-L}\cdots x_{-1}x_1\cdots x_L)$, the contributions to the intersymbol interference, in the worst case of $x_{-k}=-x_k$ for $k=1,\ldots,L$, is for $\alpha=1$ is much smaller than that for $\alpha=0$ for $L=6$, these two are 0.5878 and 24.01 respectively. This example clearly shows that a larger roll-off factor has a smaller overall intersymbol interference. Of course this has been the principal idea of introducing raised-cosine filters.

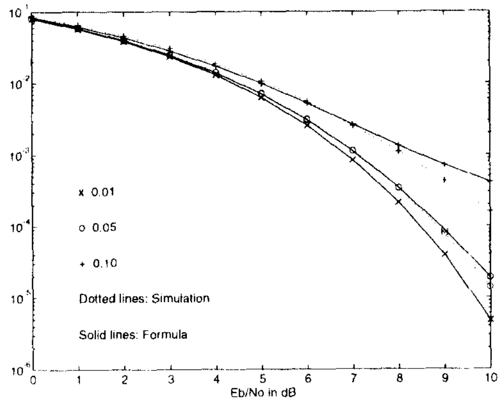

Figure 4. The bit error probability $P(E)$ from simulation and its approximation $\tilde{P}(E)$ for the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}=0.01$, 0.05 and 0.1 : the roll-off factor pair (0.2, 0.35) and the truncation length $L=6$

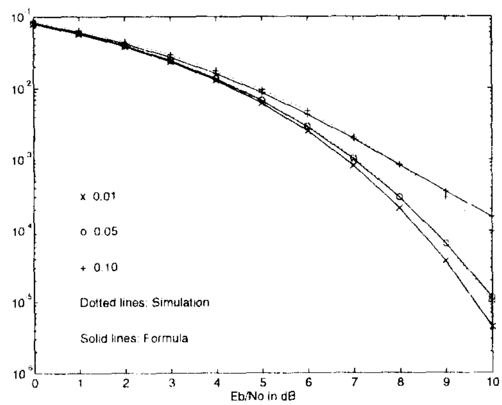

Figure 5. The bit error probability $P(E)$ from simulation and its approximation $\tilde{P}(E)$ for the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}=0.01$, 0.05 and 0.1 : the roll-off factor pair (0.5, 0.35) and the truncation length $L=6$

The truncation length L also affects the intersymbol interference as it is the upper limit in the summation in the intersymbol interference term in (7). The larger the truncation length, the larger intersymbol interference may result. This makes sense because a larger truncation length in the time domain is equivalent to a smaller bandwidth in frequency and a smaller bandwidth is more likely to cause more interference.

4.2 The Case of Mismatched Roll-off Factors with Sampling-Jitter

For the case where the roll-off factors are mismatched and sampling-jitter exists, we present the numerical results in Figures 4 and 5. All the the results are for $L=6$, and the roll-off factor pairs are (0.2, 0.35) and (0.5, 0.35). The corresponding values for $h_o(kT)$ and $h_o'(kT)$ are tabulated in Table 2. These figures show that the overall performance trend is the same as in the case of the matched roll-off factor $\alpha=0.35$ in Figure 3, and that the approximation formula seems accurate for up to 10 dB for the case of $\frac{\sigma}{T}=0.05$ (especially so for larger receiver roll-off factors 0.35 and 0.5) and up to 8 dB for $\frac{\sigma}{T}=0.1$. We note that the mismatched performance of $\beta=0.5$ is better than the matched performance of $\beta=0.35$ in the case of $\frac{\sigma}{T}=0.1$, which is rather surprising in that a mismatched filter outperforms a matched one. This performance inversion is due to large sampling jitter because it does not happen for small jitter. This finding makes us believe that in case of a large degree of jitter the larger receiver roll-off factor is better than the matched roll-off factor.

V. Conclusions

This paper addresses intersymbol interferences in a raised-cosine filtered digital communication system : the mis-

Table 2. The values of $h_o(kT)$ and $h_o'(kT)$ for various roll-off factor pairs (α, β)

	(0.2, 0.35)		(0.35, 0.35)		(0.5, 0.35)	
k	$\frac{h_o(kT)}{\sqrt{E_b}}$	$\frac{h_o'(kT)}{\sqrt{E_b}}$	$\frac{h_o(kT)}{\sqrt{E_b}}$	$\frac{h_o'(kT)}{\sqrt{E_b}}$	$\frac{h_o(kT)}{\sqrt{E_b}}$	$\frac{h_o'(kT)}{\sqrt{E_b}}$
0	9.9343×10^{-1}	0	1	0	9.9539×10^{-1}	0
1	5.3807×10^{-3}	-9.3906×10^{-1}	0	-8.9018×10^{-1}	2.6174×10^{-3}	-8.5158×10^{-1}
2	-2.3570×10^{-3}	3.8708×10^{-1}	0	3.0614×10^{-1}	1.2978×10^{-3}	2.5026×10^{-1}
3	-1.1465×10^{-3}	-1.8371×10^{-1}	0	-9.6548×10^{-2}	-3.2972×10^{-3}	-5.0012×10^{-2}
4	3.6812×10^{-3}	8.1073×10^{-2}	0	1.1294×10^{-2}	1.9937×10^{-3}	-1.1483×10^{-2}
5	-4.4075×10^{-3}	-2.7349×10^{-2}	0	1.2571×10^{-2}	5.0804×10^{-4}	1.4474×10^{-2}
6	3.4099×10^{-3}	1.5866×10^{-3}	0	-9.5258×10^{-3}	-1.5007×10^{-3}	-2.6802×10^{-3}

match intersymbol interferences in case of jitter-free sampling and the jitter intersymbol interferences in case of matched- and mismatched-filter systems.

The mismatch intersymbol interference occurs when the roll-off factors of the transmitter and receiver square-root raised-cosine filters are different. We have observed (i) that, for the signal-to-noise ratio $\frac{E_b}{N_0} \leqq 10$ dB, error due to the channel noise is dominant over that due to the mismatch interference and that the mismatch interference occurs in the same proportion to $\frac{E_b}{N_0}$, which gains dominance over the channel noise as the signal-to-noise ratio increases, and (ii) that, for this range of the signal-to-noise ratio, the intersymbol interference can be ignored for roll-off factors between 0.2 and 0.5 and therefore (iii) the signal energy can be bargained for hardware simplicity.

The jitter intersymbol interference occurs when the sampling-time is out of synchronization with the bit duration. An approximation formula for bit error probability is derived from the Taylor series approximation of the overall filter response. This formula is reasonably accurate for a wide range of the signal-to-noise ratio $\frac{E_b}{N_0} \leqq 10$ dB, and for a considerable jitter-to-symbol duration ratio $\frac{\sigma_J}{T} \leqq 0.05$. Within the accuracy of the approximation formula, we have identified the factors that affect the amount of intersymbol interference: these factors are the jitter-to-symbol duration ratio $\frac{\sigma_J}{T}$; the signal-to-noise ratio $\frac{E_b}{N_0}$; the the roll-off factor α of the raised-cosine filter; and the truncation length L.

The accuracy of the approximation formula will make it useful in studying the intersymbol interference as it can replace time-consuming simulations.

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▲Seungkeun Park

Mr. Park was born in Seoul, received the B.S. and M.S. in statistics from the University of Korea in 1991 and 1993, respectively. Since 1993 he has been at ETRI, where he is currently a member of engineering staff. His research interests are in the area of RF spectrum, nonlinear estimation, and radio regulation.

▲Jin Dam Mok

Mr. Mok was born in Seoul, received the B.S. and M.S. in electronics engineering from Younse University in 1980 and 1982, respectively. Since 1981 he has been at ETRI, where he is currently a principal member of engineering staff. His research interests are in the area of IMT-2000, radio regulation, wireless standardization.

▲Sangsin Na

Dr. Na was born in illo, Chunnam, received the B.S. in electronics engineering from Seoul National University, and the M.S. and Ph.D. in electrical engineering from the University of Michigan, Ann Arbor, U.S.A. in 1985 and 1989, respectively. From 1989 to 1991 he was at Electrical Engineering Department, the University of Nebraska, Lincoln, U.S.A. Since 1991 he has been at the School of Electrical and Electronics Engineering at Ajou University, where he is currently an associate professor.

His research interests are in the area of data compression source coding, digital communications, and signal processing.