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암석의 파괴인성계수와 균열감응도의 해석

백 환 조[1)]

Notch Sensitivity Analysis for the Rock Fracture Toughness

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요 약 암석의 파괴인성계수(fracture toughness)는 균열의 성장에 대한 암석의 저항을 나타낸다. 실험실에서 측정한 파괴인성계수는 일반적인 암석의 불균질성이나 이방성 외에도 시험편의 형상이나 하중조건에 의하여 크게 영향을 받는다. 따라서, 제한된 수의 시험편을 사용하여 측정된 파괴인성계수는 자료의 분산이 심하므로 실제 적용에 있어서 문제가 된다. 균열감응도란 파괴인성계수의 측정에 사용되는 시험편의 형상에 따라 결정되는 지수로서, 시험편의 파괴가 균열의 성장에 의한 것인지, 혹은 인장강도에 의한 것인지를 판별하는 기준이 된다. 이러한 균열감응도를 파악하여 암석의 파괴인성계수 측정에 유효한 시험편의 크기나 초기균열 길이의 범위를 설정할 수 있다. 이는 또한 실험실에서 측정된 파괴인성계수의 유효성 여부를 판별하는 기준으로 사용될 수 있다. 본 논문에서는 암석의 파괴인성계수의 측정에 흔히 사용되는 몇 가지 형태의 시험편들에 대하여 균열감응도를 계산하고 이에 따른 초기균열 길이의 범위를 제시하고자 한다.

1. Introduction

An understanding of the mechanics or mechanisms of rock fracture is a key element in the solution of many engineering problems involving rock structures. The application of linear elastic fracture mechanics (LEFM) principles has been proven to be an effective approach in rock engineering and geoscience fields. The mode I fracture toughness, K_{Ic}, represents the ability of material against the opening mode fracture propagation. Recently, fracture toughness and related fracture mechanics parameters of rock materials have been widely applied to various rock engineering problems, including the hydraulic fracturing (Takahashi and Abe, 1987; Rummel, 1976), stability analysis of rock slopes and underground structures (Ingraffea, 1979), and interpretation of the tunnel boring machine performance. Therefore, a standardized procedure for testing and data interpretation for rock fracture toughness has been suggested by the International Society for Rock Mechanics (ISRM, 1988), for which special attention was given to the difficulties in obtaining the true fracture mechanics parameter for the wide variety of rock materials. Rock specimens are usually cored from drill holes, and core-based specimens are more cost-effective for determining rock fracture toughness. Accordingly, the single-edge-cracked round-bar-in-bending (SECRBB), semi-circular bending (Chong and Kuruppu, 1984), notched Brazilian discs (Szendi-Horvath, 1980; Guo *et al.*, 1993), and the chevron bend (CB) specimens are most practical specimen types. Other core-based specimen types include the burst cylinder method, modified ring test, and the round compact tension (Clifton *et al.*, 1976; Thiercelin and Roegiers, 1986; Sun and Ouchterlony, 1986).

However, the fracture toughness of inhomogeneous and anisotropic rock materials, which generally violate the fundamental assumptions of LEFM, often depends on the specimen geometry and test method adopted. This fact is often attributed to improper initial notch length, along with other test variables. In this paper, the notch sensitivity analysis is applied to the CB and SECRBB specimens, and the proper ranges for each specimen type are suggested.

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2. Fracture Mechanics of Aggregative Materials

Griffith (1920) suggested a fracture criterion based on the energy balance in connection with the theory of instability: in a cracked body under loading, small increases in crack length at the point of instability result in zero free energy changes, the decrease in free energy caused by the driving force being offset by increases due to resisting forces. Due to the difficulties involved in measuring the energy of fracture experimentally, Griffith theory did not gain much attention until the correlation between the energy of fracture and the stress intensity factor parameter was made by Irwin (1957). In LEFM, the governing parameter is the stress intensity factor, K, which represents the magnitude of singularity of the stress field at a loaded crack tip. In a tensile loading, the basic relation equates K to a critical value, K_{Ic}, which is taken as a material property called the plane strain fracture toughness.

Due to the high stresses near the crack tip, most materials exhibit some types of nonlinearity prior to fracturing. LEFM is applicable only if the region of the nonlinearity at the crack tip is much smaller than that dominated by the elastic crack tip stress singularity. The nonlinear behavior is usually due to plastic flow in metallic materials and to microcracking in aggregative materials, such as rock and concrete. However, the various mechanisms of energy dissipation in aggregative materials have not been fully identified. The measured fracture toughness of aggregative materials often depends on the initial crack length and other specimen dimensions. This observation has raised concern about the fracture toughness as an intrinsic property of aggregative materials.

2.1 Rock Fracture Toughness

For quasi-brittle rock materials, crack propagation is the major cause of rock failure in many cases, and the assessment of the fracture toughness and other fracture mechanics parameters is important in analyzing behavior of structures involving rock materials. Although the rock material is generally inhomogeneous and anisotropic, the size of rock masses encountered in rock engineering projects or in geoscience applications is often assumed essentially semi-infinite. Under these conditions, the application of LEFM seems to be appropriate. Consequently, test methods based on LEFM for determining fracture toughness of rock materials using sub-sized core specimen have been suggested by the ISRM(1988).

The rock fracture toughness may be affected substantially by the test conditions, as well as rock microstructures. Effects of these test variables should be identified for a reliable interpretation of rock fracture toughness data. In rocks where the main nonlinear deformation mechanism is microcracking, the effect of the plastic constraint on crack propagation is far less significant compared to that in metallic materials. Hence, there is no distinct transition from plane stress to plane strain conditions, and the fracture toughness of rock would be independent of specimen thickness (Schmidt and Lutz, 1979). However, it is reasonably recommended that the specimen thickness be greater than the size of fracture process zone, and for core-based cylindrical specimens, the specimen diameter be 10 times greater than the grain size for a representative fracture toughness measurement (ISRM, 1988). Considering NX-size specimens usually cored in the field, the grain size of the specimen should be less than 5.4 mm, which raises concern about the validity of fracture toughness measurements on coarse-grained rock materials. For these rock types, other fracture mechanics parameters and test procedures typically applied to concrete or mortar would produce more reasonable data.

2.2 Specimen Geometries

Among the various specimen geometries listed above, the SECRBB and CB specimens are the simplest and most practical test geometries for determining rock fracture toughness values. The CB specimen has been suggested as a reference geometry by the ISRM, along with the SR specimen. Baek (1994) found that the SECRBB specimen yields fracture toughness values consistently about 20 to 25% lower, depending on the rock types tested, than those determined by the CB specimen. However, he also noticed considerable subcritical crack growth and a rising R-curve behavior in his specimens, which violated the basic assumptions for the CB specimen type. The SECRBB specimen type is therefore still promising for rock fracture toughness measurement, considering the consistency of data and simplicity of test geometry. Accordingly, the notch sensitivity analysis for these two specimen types will be discussed here.

2.2.1 The SECRBB Specimen

The geometry of the SECRBB specimen is shown in Fig. 1. Assuming a linear elastic behaviour, the compliance of the specimen, C, is defined as:

$$C = \frac{\delta_F}{F} \tag{1}$$

where δ_F is the load point displacement (LPD) and F is the applied load. Within the limits of the slender beam theory, the compliance of a SECRBB specimen is given by:

$$C_{secrbb} = \frac{S^3}{48\,E \cdot I_{secrbb}} = \frac{1}{ED} \cdot \frac{4(S/D)^3}{3\pi f(\alpha)} \tag{2}$$

where S is the support span, and $f(\alpha)$ is a dimensionless function derived from I_{secrbb}, the moment of inertia of the SECRBB cross-section. The mode I fracture toughness for SECRBB specimen (S/D=3.33) is then given by (Bush, 1976):

$$K_{Ic} = \sqrt{\frac{F^2}{4(aD - a^2)^{0.5} \cdot D^2} \cdot \frac{\partial(cED)}{\partial(a/D)}} = \frac{F}{D^{1.5}} \cdot Y'_{secrbb} \tag{3}$$

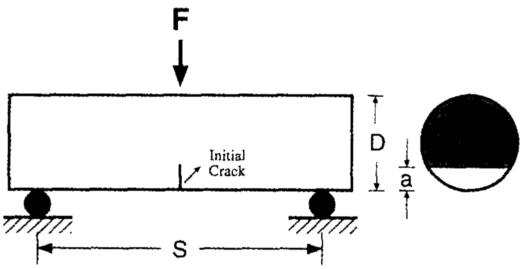

Fig. 1. Test geometry for the SECRBB specimen.

From a compliance calibration for the Ekeberg marble, Ouchterlony(1981) obtained Y'_{secrbb} as:

$$Y'_{secrbb} = 10.62\,\alpha^{0.5}\,(1 + 19.65\alpha^{4.5})^{0.5}/(1-\alpha)^{0.25} \tag{4}$$

which is valid for $0 \leq \alpha \leq 0.6$, and S/D=3.33.

2.2.2 The Chevron Bend Specimen

The Chevron bend specimen is one of the two specimen geometries suggested by the ISRM. Chevron-notched specimen have several advantages over other specimen types, especially for materials exhibit brittle fracturing: the ligament shape enables a stable crack propagation from a self-produced sharp crack. A stable crack growth up to certain distance from the initial chevron tip produces a naturally sharp crack, so that the fatigue pre-cracking requirement of ASTM E399 can be omitted. Furthermore, assuming a flat R-curve for generally brittle rock materials, fracture toughness is calculated from the maximum load and initial specimen dimensions.

The geometry of the CB specimen is shown in Fig. 2. In level I testing, only the maximum load, F_{max}, is recorded and the fracture toughness is calculated by (ISRM, 1988):

$$K_{Ic} = A_{min} \cdot \frac{F_{max}}{D^{1.5}} \tag{5}$$

With the notations given in Fig. 2, the dimensionless factor, A_{min} is given as:

$$A_{min} = \left\{1.84 + 7.15\left(\frac{a_0}{D}\right) + 9.85\left(\frac{a_0}{D}\right)^2\right\} \cdot \left(\frac{S}{D}\right) \tag{6}$$

3. Notch Sensitivity Analysis

Two different types of failure mechanism act within a notched rock specimen under loading. The first is a strength failure governed by the maximum tensile stress at the crack tip and the tensile strength of the rock material. The second is the failure due to crack extension, governed by the fracture toughness of the rock material. Therefore, it is necessary to determine which failure mechanism is critical at the instant of specimen failure.

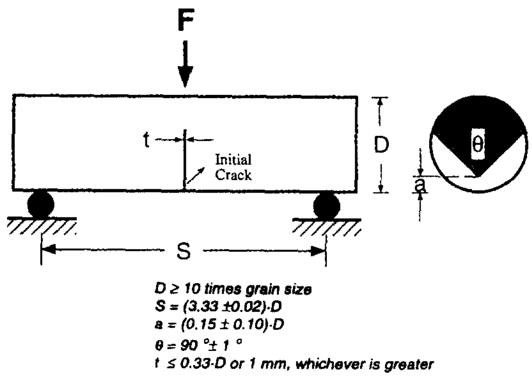

Fig. 2. Test geometry for the CB specimen.

3.1 The Brittleness Number

Carpinteri (1982) employed the concept of notch sensitivity in analyzing fracture toughness test results of aggregative materials, like concrete or mortar. Assuming homogeneous, isotropic and linear elastic material behavior, he applied the dimensional analysis for physical similitude and scale modelling to define a non-dimensional brittleness number, s, given by;

$$s = \frac{K_{Ic}}{\sigma_t \cdot w^{0.5}} \tag{7}$$

where w is the specimen height.

The stress intensity factor for the three point bend specimen, such as the CB and the SECRBB specimens, is given in general form as;

$$K_I = \frac{F \cdot S}{b \cdot w^{1.5}} \cdot f\left(\frac{a}{w}\right) \tag{8}$$

where S is the support span, b is the specimen width, and f(a/w) is a dimensionless stress intensity factor coefficient. At the critical load F_c, the generalized force, F_{crack}, required for the crack extension failure is;

$$F_{crack} = \frac{F_c \cdot S}{\sigma_t \cdot b \cdot w^2} = \frac{s}{f(a/w)} \tag{9}$$

Assuming that the linear elastic slender beam theory is still applicable for the notched three-point bend specimen, the generalized force required for ultimate strength failure, $F_{strength}$, is;

$$F_{strength} = \frac{F_c \cdot S}{\sigma_t \cdot b \cdot w^2} = \frac{2}{3} \cdot \left(1 - \frac{a}{w}\right)^2 \tag{10}$$

It should be noted that the two generalized forces given above are both dimensionless and, therefore, are directly comparable. The notch sensitivity is then defined as a function of the brittleness number, s, and the crack length ratio, a/w. At a certain combination of these two parameters, if the generalized force for crack extension is smaller than that of ultimate strength failure, crack extension is more critical and the specimen is notch sensitive. A valid fracture toughness measurement for a given specimen geometry can be assured only if the specimen is notch sensitive. Carpinteri (1982) concluded that some recurring experimental inconsistencies, that is, the variance of the measured fracture toughness values with the crack length, specimen size, and test geometry, can be explained by the notch sensitivity of the specimen type adopted.

3.2 Notch Sensitivity of the CB and SECRBB Specimens

The coordinates of the centroid, $\bar{x}$ and $\bar{y}$, and the moment of inertia with respect to the y direction, I_y, of a plane area shown in Fig. 3 are calculated from the theory of elasticity as:

$$\bar{x} = \frac{Q_y}{A} = \frac{\int x \, dA}{\int dA}, \quad \bar{y} = \frac{Q_x}{A} = \frac{\int y \, dA}{\int dA} \tag{11}$$

and,

$$I_y = \int x^2 \, dA \tag{12}$$

where Q_x and Q_y are the first moments of the area about the x- and y- axis, respectively, and A is the area of the plane.

As shown in Fig. 4, the distance y' from the new centroid of the ligament plane of the notched specimens to the crack tip is calculated as:

$$y' = D \cdot h(\alpha) \tag{13}$$

where D is the specimen diameter and α is the crack length ratio, that is, a/D. The dimensionless function h(α) depends on the specimen geometry and is given as:

$$h(\alpha)_{cb} = 0.56 - 0.51\alpha - 0.02\alpha^2 - 0.06\alpha^3$$
$$h(\alpha)_{secrbb} = 0.50 - 0.85\alpha + 1.11\alpha^2 - 1.75\alpha^3 \quad (14)$$

where the subscripts cb and secrbb represent the CB and SECRBB specimen, respectively. For the notched cross-section, the moment of inertia of the ligament is given by:

$$I = \pi D^4 \cdot i(\alpha) \quad (15)$$

where the dimensionless function i(α) also depends on the specimen geometry, such that:

$$i(\alpha)_{cb} = 0.01 - 0.04\alpha + 0.05\alpha^2 - 0.02\alpha^3$$
$$i(\alpha)_{secrbb} = 0.02 - 0.02\alpha - 0.09\alpha^2 + 0.27\alpha^3 \quad (16)$$

Assuming that the elastic slender beam theory is applicable for the notched beam, the tensile stress developed at the notch tip in the three-point bend specimen is:

$$\sigma_t = \frac{1}{4} \cdot \frac{F \cdot S}{I} \cdot \bar{y} \quad (17)$$

From Eqs. (13), (15), and (17), the generalized force for the ultimate strength failure, $F_{strength}$ of the CB and SECRBB specimens (S/D=3.33) is given by:

$$F_{strength} = \frac{F_{max}}{\sigma_t \cdot D^2} = \frac{4\pi}{3.33} \cdot \frac{i(\alpha)}{h(\alpha)} \quad (18)$$

From Eqs. (3) and (5), the fracture toughness formulas for the CB and SECRBB specimens (S/D =3.33) can be written as:

$$K_{Ic} = \frac{F_{max}}{D^{1.5}} \cdot f(\alpha) \quad (19)$$

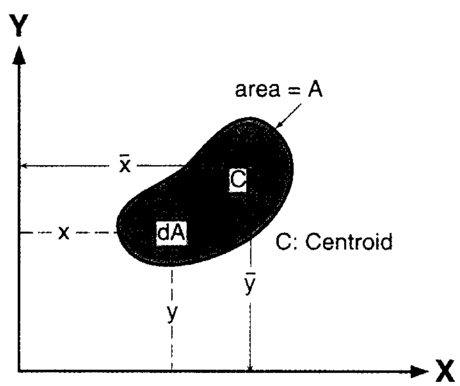

Fig. 3. Centroid of a plane area.

The dimensionless stress intensity factor, f(α) is given in Eq. (4) for the SECRBB specimen, and in Eq. (6) for the CB specimen. The generalized force for crack extension, F_{crack}, is given as:

$$F_{crack} = \frac{F_{max}}{\sigma_t \cdot D^2} = \frac{s}{f(\alpha)} \quad (20)$$

where the brittleness number, s, for the cylindrical specimen is;

$$s = \frac{K_{Ic}}{\sigma_t \cdot D^{0.5}} \quad (21)$$

3.3 Notch-Sensitive Ranges of the Initial Crack Length

Fig. 5 shows the generalized crack extension force curves, for different brittleness numbers, and the generalized strength failure curve for the CB specimen. The range of the crack length ratio in the figure corresponds to the valid range of the dimensionless stress intensity factor for this specimen type. The region where the crack propagation failure curves are below the ultimate strength failure curve defines the notch-sensitive

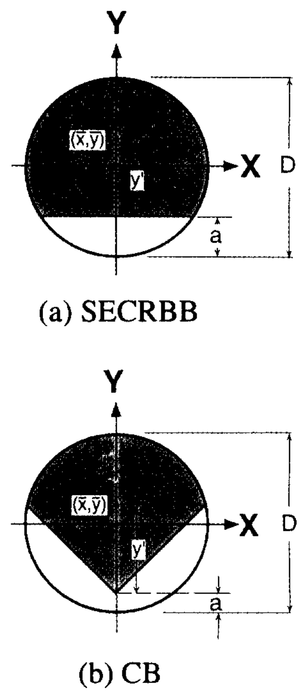

Fig. 4. Notched Cross-Section of the SECRBB and CB specimens.

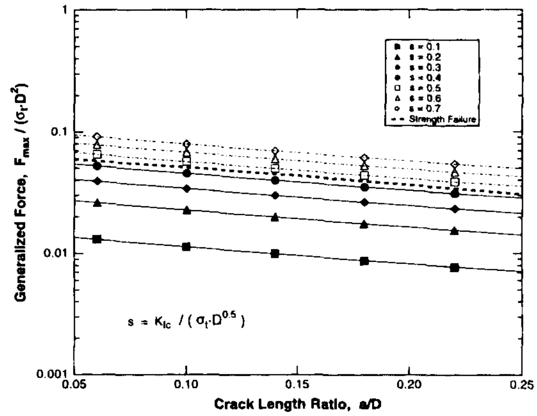

Fig. 5. Generalized Force Curves versus Crack Length Ratio for the CB Specimen.

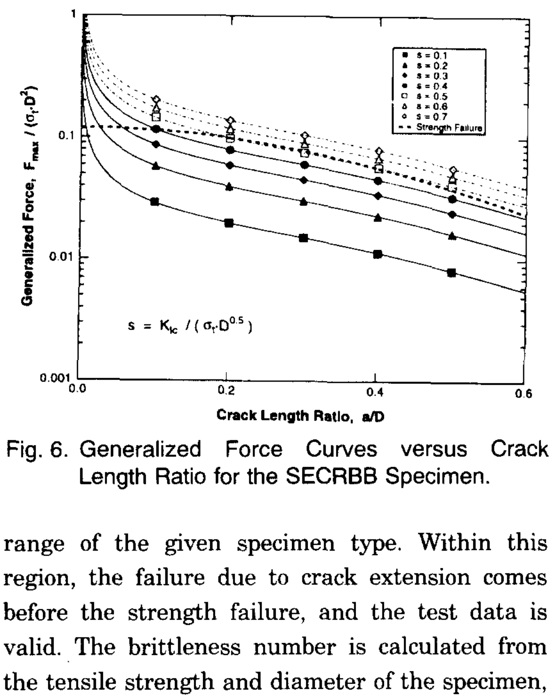

Fig. 6. Generalized Force Curves versus Crack Length Ratio for the SECRBB Specimen.

range of the given specimen type. Within this region, the failure due to crack extension comes before the strength failure, and the test data is valid. The brittleness number is calculated from the tensile strength and diameter of the specimen, and the measured fracture toughness which depends on the crack length ratio. As shown in this figure, the brittleness number calculated from the measured fracture toughness of the test specimen should be less than 0.45; otherwise, test result is regarded as invalid, according to the notch sensitivity.

A similar plot for the SECRBB specimen is given in Fig. 6. Consider a SECRBB specimen for which the brittleness number is calculated as 0.5. If the crack length ratio is in the range of 0.2 to 0.3, fracture toughness testing on this specimen may be accepted, since the crack extension failure curve is tangent to the strength failure curve within this range. However, for other crack length ratios, for which the strength failure is more critical and the test result is rejected.

3.4 Applications of the Notch Sensitivity Analysis

The notch sensitivity analysis with a fixed crack length ratio of the specimens can be applied to discriminate invalid fracture toughness values in screening stage. For example, the initial crack length ratio, a_0/D, of the CB specimen ranges from 0.05 D to 0.25 D, following the size requirements in the ISRM suggested test methods. The notch sensitivity analysis for the CB specimen shown in Fig. 5 indicates that the brittleness number should be less than 0.45 for this range of crack length ratio. Assuming that the tensile strength of a rock material is a true material property, the maximum 'acceptable' fracture toughness values can be set prior to the testing. Table 1 shows results of the fracture toughness test on the Ellenberger dolostone, for which the tensile strength, σ_t, was measured to be 13.66 ± 3.74 MPa.

Table 1 shows that the maximum acceptable fracture toughness values increase with increasing specimen diameter. This means that the possibility of valid fracture toughness measurements is greater with larger specimens. The measured fracture toughness values of the smaller diameter specimens are higher than the maximum acceptable values. It seems that 25.4 mm diameter specimens of the Ellenberger dolostone were too small for a valid fracture

Table 1. The maximum acceptable and measured fracture toughness values of the Ellenberger dolostone.

Specimen diameter [mm]	Maximum acceptable K_{Ic} [$MN/m^{1.5}$]	Measured K_{Ic} range [$MN/m^{1.5}$]
25.4	1.24	1.33-1.50
50.8	1.75	1.45-1.82
76.2	2.15	1.50-1.81

toughness measurement. Other specimen sizes produced apparently reasonable values of fracture toughness, and it can be concluded that measuring fracture toughness of the Ellenberger dolostone requires specimen size much greater than 25.4 mm.

4. Conclusions

From the notch sensitivity analysis of typical test geometries for the rock fracture toughness measurements, the following conclusions can be derived.

① Within the valid ranges for the initial crack length ratio of the CB specimen suggested by the ISRM, the specimen is notch sensitive at failure if the calculated brittleness number is less than 0.45 for the rock type tested.

② For the SECRBB specimen type, with the initial crack length ratio from 0.1 to 0.6, the brittleness number of the specimen should be less than 0.4.

③ The notch sensitivity analysis with a fixed crack length ratio of the specimens can be applied to discriminate invalid fracture toughness values in screening stage. However, the rough assumptions employed for the notch sensitivity analysis need further investigation.

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