韓國水資源學會誌論文集
第30卷 第3號 · 1997年 6月
pp. 225~233

유한한 Bandwidth를 갖는 비선형 불규칙 파열에서의 Threshold Crossing Rate, 위상분포와 파군특성

Threshold Crossing Rate, Phase Distribution and Group Properties of Nonlinear Random Waves of Finite Bandwidth

조 용 준*
Cho, Yong Jun

Abstract

The nonlinear effects on the statistical properties of wave groups in terms of the average number of waves in a group and the mean number of waves in a high run is studied in this paper utilizing the complex envelope and total phase function, random variable transformation technique and perturbation method. It turns out that the phase distribution is modified significantly by nonlinearities and it shows systematic excess of values near the mean phase and the corresponding symmetrical deficiency on both sides away from the mean. For the case of threshold crossing rate, it turns out that threshold crossing rate reaches its maxima just below the mean water level rather than zero and considerable amount of probability mass is shifted toward the larger values of water surface elevation as nonlinearity is getting profound. Furthermore, the mean waves in a high run associated with nonlinear wave are shown to have larger values than the linear counterpart. Similar trend can also be found in the average number of waves in a group.

요 지

부체의 장주기동요, 연안구조물과 계류선의 파괴 등과 같은 역기능으로 인해 파군현상의 정확한 해석은 시급한 과제라 하겠다. 본고에서는 연안계에서 발생하는 파랑과 가장 근접한 비선형불규칙 파랑계를 산정하여 해안구조물의 피로거동에 지대한 영향을 미치는 파군당 파랑의 수와 해안구조물의 first excursion failure mode를 결정하는 high run에서의 파랑수를 중심으로 파군현상을 해석하였다. 해석과정에 mapping technique과 유의파경사를 perturbation parameter로 한 섭동이론이 사용되었다. 해석결과 Gaussian wave계에서 균등분포하는 것으로 알려진 위상함수는 평균값 주위에 집중분포하였고, 그 정도는 비선형성이 증가할수록 심화되었다. threshold crossing rate의 경우 비선형성이 심화될수록 평균해수위보다 큰 쪽으로 분포형이 이동하였으며 파군당 파랑수와 high run에서의 파랑수도 비슷한 경향을 보여 최근 설치범위가 심해쪽으로 확대되는 해안구조물의 경우 피로거동에 대한 보강이 요망된다.

keywords : group property, nonlinear random waves, phase distribution, threshold crossing rate

* 서울시립대학교 토목공학과 조교수

1. Introduction

The grouping of high waves is animportant parameter in many engineering problems associated with port development, which may influence long period oscillation of moored vessels and other floating structures and surf beat. After the Gaussian model was first developed in well known papers by Rice (1944), particular attention to properties of wave groups in Gaussian noise was paid by Longuet-Higgins (1957). Recent interests in this subject (Goda, 1983) have been stimulated by the suggestion that exceptional damage to ships, coastal defences or offshore structures may be caused by the occurrence of a run of successive high waves. Further reason for interest is the relation of wave groups with the formation of wave breaking. Longuet-Higgins (1984) had obtained the expressions for wave group length and length of a high run using two apparently distinct approaches in the context of Gaussian waves: first, by a wave envelope function and, later, by treating the sequence of wave heights as a Markov chain. It was shown that two approaches are roughly equivalent and spectral bandwidth has a significant influence on the wave group length and the length of a high run. On the other hand, the probable effect of nonlinearity on the formation of wave group in random wave field of finite bandwidth remains uncertain due to the complicated form of non-linear random waves which was first developed by Longuet-Higgins in 1963. In a case when the underlying frequency spectrum is narrow, the stochastic representation of nonlinear sea surface is reduced to a familiar form in which each realization is an amplitude modulated second order Stokes wave (Tayfun, 1980; 1986).

In contrast with the intricate complexity of the expression of nonlinear waves of finite bandwidth, such an approximation constitutes a simpler formulation to study numerically or analytically the nonlinear effects on the statistical description of wave properties. But considering the side band instability of Stokes wave, the narrow band assumption at the site away from the generating area is no longer valid. The search for a way simpler than that of Longuet-Higgins (1963) to describe non-linear waves of finite bandwidth was recently carried out by Tung et al. (1989). Based on the studies of Tayfun (1980, 1986), Tung et al. (1989) proposed a simple but accurate ex-pression for second order nonlinear wave elevation for waves of moderate bandwidth. This wave model was more elaborated by Cho and Yoon (1992) to analyze the extreme dis-tributions of wave elevation. It turns out that as nonliearity is getting profound, these extreme distributions deviate from the linear counter-part in an increasing manner. The general character of this deviation is in the form of a spreading of the density mass toward the larger and smaller values of crest. Hence, it can be deducible from this tendency that there should be a significant modification in wave group properties when nonlinearity be in-troduced. The objective here is to gain some theoretical insight into the nature of nonlinear effects on the statistical properties of wave group in terms of the wave group length and the length of a high run, quantities that are of great importance in the design of ships, coastal defences or offshore structures. In this paper, our attention is centered on deep water waves only.

2. Review of Wave Group Theory

It is known that for stationary random process $\zeta(t)$ of arbitrary bandwidth, the average number of waves in a group G and mean number of waves in a high run H (Lin, 1967) are

$$G = N_\zeta(\zeta_0)/N_A(A_0) \tag{1}$$

$$H = N_\zeta(\zeta_o)Q\ (A_o)/N_A(A_o) \tag{2}$$

where

$$N_\zeta(\zeta_o)=\int_0^\infty \dot{\zeta} f_{\zeta\dot{\zeta}}(\zeta_o, \dot{\zeta})d\dot{\zeta} \tag{3}$$

$$N_A(A_o)=\int_0^\infty \dot{A} f_{A\dot{A}}(A_o, \dot{A})d\dot{A} \tag{4}$$

$$Q(A_o)=\int_{A_0}^\infty f_A(A)dA \tag{5}$$

In Eqs. (3), (4) and (5), $f_{\zeta\dot{\zeta}}(\cdot,\cdot)$and $f_{A\dot{A}}(\cdot,\cdot)$ are the joint probability density function of ζ and $\dot{\zeta}$ [overdot denotes time derivative] and the joint probability density function of A and $\dot{A}$, respecively, and $N_\zeta(\zeta_0)$ represents the number of upcrossings by ζ of a given level ζ_o per unit time and $N_A(A_0)$ is the number of up-crossings of a given level A_0 per unit time by the wave envelope A and $Q(A_o)$ is the exceedance probility of a given level A_0 by the wave envelope. To apply Eqs. (1) and (2) to nonlinear random waves, it is necessary to have $f_{\zeta\dot{\zeta}}(\zeta_0, \dot{\zeta})$ and $f_{A\dot{A}}(A, \dot{A})$ which in turn require a nonlinear wave model of finite bandwidth.

3. Envelope and Phase Process of Nonlinear Random Waves

Consider infinitely long-crested waves of arbitrary bandwidth in deep water. The surface displace displacement is given by (Longuet-Higgins, 1963)

$$\zeta=\sum_{i=0}^\infty a_i\cos\chi_i+\frac{1}{2g}\sum_{i=0}^\infty\sum_{j=0}^\infty a_ia_j\omega_i^2\cos(\chi_i+\chi_j)$$
$$-\frac{1}{2g}\sum_{i=0}^\infty\sum_{j>i}^\infty a_ia_j(\omega_j^2-\omega_i^2)\cos(\chi_j-\chi_i) \tag{6}$$

in which $\chi_i=k_ix-\omega_it+\varepsilon_i$, k_i is the wave number, $\omega_i=(gk_i)^{1/2}$ is wave frequency, ε_i is random phase uniformly distributed over the interval $(0, 2\pi)$ and a_i is the amplitude of the component wave. Upon introducing the following random processes,

$$\eta_1=\frac{1}{(M_0)^{1/2}}\sum_{i=0}^\infty a_i\cos\chi_i \tag{7}$$

$$\eta_2=\frac{1}{(M_2)^{1/2}}\sum_{i=0}^\infty a_i\omega_i\sin\chi_i \tag{8}$$

$$\eta_3=-\frac{1}{(M_4)^{1/2}}\sum_{i=0}^\infty a_i\omega_i^2\cos\chi_i \tag{9}$$

$$\eta_4=\frac{1}{(M_0)^{1/2}}\sum_{i=0}^\infty a_i\sin\chi_i \tag{10}$$

$$\eta_5=-\frac{1}{(M_2)^{1/2}}\sum_{i=0}^\infty a_i\omega_i\cos\chi_i \tag{11}$$

$$\eta_6=-\frac{1}{(M_4)^{1/2}}\sum_{i=0}^\infty a_i\omega_i^2\sin\chi_i \tag{12}$$

it was shown that the nondimensional nonlinear wave elevation ζ_1 can be written as (Tung et al., 1989)

$$\zeta_1=\zeta/(m_0)^{1/2}$$
$$\cong(\frac{M_0}{m_0})^{1/2}(\eta_1-\frac{1}{2}\varepsilon\eta_1\eta_3+\frac{1}{2}\varepsilon\eta_4\eta_6) \tag{13}$$

where M_i and m_i are ith spectral moments of the linear and nonlinear wave elevation, respectively, and $\varepsilon=(M_4)^{1/2}/g$. For a monochromatic waves of amplitude a and frequency ω, $M_4=a^2\omega^4/2$ soth at $\varepsilon=ak/2$is a small quantity. For the problem under consideration, ε will be used as a perturbation parameter. The Hilbert transfer of ζ_1, $\hat{\zeta}_1$, can be represented by

$$\hat{\zeta}_1=\zeta/(m_0)^{1/2}\cong(\frac{M_0}{m_0})^{1/2}(\eta_4-\varepsilon\eta_4\eta_3). \tag{14}$$

On this basis, one defines the complex process

$$W=\zeta_1+i\hat{\zeta}_1=Ae^{i\varphi} \tag{15}$$

where i is the imaginary unit. Hence the nondimensional wave envelope and phase process are defined by

$$A=(\zeta_1^2+\hat{\zeta}_1^{\ 2})^{1/2} \tag{16}$$

$$\varphi=\tan^{-1}\hat{\zeta}_1/\zeta_1 \tag{17}$$

4. Joint Distribution of Wave Envelope and its First Derivative

Our task is to obtain the joint distribution of wave envelope, its first derivative to be used in Eqs. (1) and (2). To this end, we carry out the differentiation of nonlinear wave elevation and its Hilbert transform with respect to time. We first note that

$$\eta_2 = \dot{\eta}_1 \left(\frac{M_0}{M_2}\right)^{1/2} \quad (18)$$

$$\eta_3 = \ddot{\eta}_1 \left(\frac{M_0}{M_4}\right)^{1/2} \quad (19)$$

$$\eta_5 = \dot{\eta}_4 \left(\frac{M_0}{M_2}\right)^{1/2} \quad (20)$$

$$\eta_6 = \ddot{\eta}_4 \left(\frac{M_0}{M_4}\right)^{1/2} \quad (21)$$

and to the first order of ε, $m_0 = M_0$, $m_2 = M_2$ and $m_4 = M_4$. Based on this facts, nondimensional wave elevation ζ_1 can be rewritten, to the order of ν,

$$A\cos\varphi = \zeta/(m_0)^{1/2}$$
$$\cong \eta_1 - \frac{1}{2}\varepsilon\eta_1\eta_3 + \frac{1}{2}\varepsilon\eta_4\eta_5 \quad (22)$$

where

$$\nu = ((M_0 M_2/M_1^2) - 1)^{1/2}, \qquad 0 < \nu < 1 \quad (23)$$

is a measure of the bandwidth of the frequency spectrum which, for all practical purpose, is a small quantity. Then, it follows that, to the order of ν,

$$\dot{A}\cos\varphi - A\dot{\varphi}\sin\varphi = \dot{\zeta}/(m_2)^{1/2}$$
$$\cong \eta_2 - 2\varepsilon\eta_2\eta_3 \quad (24)$$

$$A\sin\varphi = \hat{\zeta}/(M_0)^{1/2}$$
$$\cong \eta_4 - \varepsilon\eta_4\eta_3 \quad (25)$$

$$\dot{A}\sin\varphi + A\dot{\varphi}\cos\varphi = \dot{\hat{\zeta}}/(m_2)^{1/2}$$
$$\cong \eta_5 - \varepsilon\eta_3\eta_5 - \varepsilon\eta_2\eta_6 \quad (26)$$

The random variables A, $\dot{A}$, φ and $\dot{\varphi}$ are seen to be functions of η_1, η_2, η_3, η_4, η_5 and η_6 which are random variables having zero mean and unit standard deviation. Furthermore, the pairs (η_1, η_5, η_3) and (η_4, η_2, η_6) are statistically independent, each of which is jointly Gaussian. Therefore, the joint distribution of η_1, η_2, η_3, η_4, η_5 and η_6 is given by

$$f_{\eta_1\eta_2\eta_3\eta_4\eta_5\eta_6}(\cdot,\cdot,\cdot,\cdot,\cdot,\cdot)$$
$$= f_{\eta_1\eta_5\eta_3}(\cdot,\cdot,\cdot) f_{\eta_4\eta_2\eta_6}(\cdot,\cdot,\cdot) \quad (27)$$

where

$$f_{\eta_1\eta_5\eta_3}(\cdot,\cdot,\cdot) \quad (28)$$
$$= \frac{1}{(2\pi)^{3/2}|S_1|^{1/2}} \exp\left[-\frac{1}{2|S_1|}\sum_{j=1}^{3}\sum_{k=1}^{3}|S_1|_{jk}\eta_j\eta_k\right]$$

$$f_{\eta_4\eta_2\eta_6}(\cdot,\cdot,\cdot) \quad (29)$$
$$= \frac{1}{(2\pi)^{3/2}|S_2|^{1/2}} \exp\left[-\frac{1}{2|S_2|}\sum_{j=1}^{3}\sum_{k=1}^{3}|S_2|_{jk}\eta_j\eta_k\right]$$

In Eqs. (28) and (29), $|S_i|_{jk}$, the cofactor of the element in the jth row and kth column of the matrix of covariances S_i, is given, respectively, by

$$S_1 = \begin{vmatrix} E[\eta_1^2] & E[\eta_1\eta_5] & E[\eta_1\eta_3] \\ E[\eta_5\eta_1] & E[\eta_5^2] & E[\eta_5\eta_3] \\ E[\eta_3\eta_1] & E[\eta_3\eta_5] & E[\eta_3^2] \end{vmatrix} \quad (30)$$

and

$$S_2 = \begin{vmatrix} E[\eta_4^2] & E[\eta_4\eta_2] & E[\eta_4\eta_6] \\ E[\eta_2\eta_4] & E[\eta_2^2] & E[\eta_2\eta_6] \\ E[\eta_6\eta_4] & E[\eta_6\eta_2] & E[\eta_6^2] \end{vmatrix} \quad (31)$$

where $E[\cdot]$ is used to denote the expected value ofquantity enclosed in the brackets. By introducing the auxiliary random variables,

$$\alpha = \eta_3 \quad (32)$$

$$\beta = \eta_6 \quad (33)$$

the joint distribution of A, $\dot{A}$, φ and $\dot{\varphi}$, $f_{A\dot{A}\varphi\dot{\varphi}}(\cdot)$, can be obtained by the standard method of transformation of random variables (Papoulis, 1965). That is,

$$f_{AA\varphi\dot\varphi}(\cdot) = f_{\eta_1,\eta_5,\eta_3}(\eta_1,\eta_5,\eta_3) f_{\eta_4,\eta_2,\eta_6}(\eta_4,\eta_2,\eta_6) \left| J\left(\frac{\eta_1\ \eta_2\ \eta_3\ \eta_4\ \eta_5\ \eta_6}{A\ \varphi\ \dot A\ \dot\varphi\ \alpha\ \beta}\right)\right| \quad (34)$$

where J is the Jacobian of the variable transformation. From Eqs. (24), (25), (26), (28) and (29), and following the perturbation technique (Huang et al., 1983), it may be shown that, to the order of ε,

$$|J| = A + \frac{9}{2}\varepsilon\alpha A^2 \quad (35)$$

Performing the integration with respect to α and β,

$$f_{AA\varphi\dot\varphi}(\cdot,\cdot,\cdot,\cdot) \quad (36)$$
$$= \int\int f_{AA\varphi\dot\varphi\alpha\beta}(\cdot,\cdot,\cdot,\cdot,\cdot,\cdot)d\alpha d\beta$$

the joint distribution of wave envelope, phase process and their first derivative can be obtained. From Eqs. (24) and (25), and following the similar procedure in the transformation of random variables, the joint distribution of nondimensional wave elevation and its first derivative is given by

$$f_{\zeta\dot\zeta}(\zeta,\dot\zeta) = \frac{1}{2\pi}\{1 + 0.5\varepsilon(7\rho_1 - \rho_2\rho_3)\zeta + 0.5\varepsilon(\rho_2\rho_3 - 4\rho_1)\cdot(\zeta\dot\zeta^2 - \frac{\varepsilon}{2}\rho_1\zeta^3)\Big\} \exp\Big\{-\frac{1}{2}(\zeta^2 + \dot\zeta^2)\Big\} \quad (37)$$

where the correlation coefficients ρ_i are

$$\rho_1 = E[\eta_1\eta_3] = E[\eta_4\eta_6] \quad (38)$$

$$\rho_2 = E[\eta_2\eta_6] = -E[\eta_3\eta_5] \quad (39)$$

$$\rho_3 = E[\eta_2\eta_4] = -E[\eta_1\eta_5] \quad (40)$$

As $\varepsilon=0$, the joint distribution in Eq. (37) is reduced to jointly Gaussian as expected.

5. Average Number of Waves in a Group, Mean number of Waves in a High Run

Substituting Eq. (37) into Eq. (3), we can obtain the number of up-crossings by non-dimensional wave elevation of a given level per unit time $N_\zeta(\zeta_0)$. That is, $N_\zeta(\zeta_0)$ is given by

$$N_\zeta(\zeta_0) = \frac{1}{2\pi}\Big\{1 + \frac{\varepsilon}{2}(\rho_2\rho_3 - \rho_1)\zeta_o - \frac{\varepsilon}{2}\rho_1\zeta_0^3\Big\} \cdot \exp(-\frac{1}{2}\zeta_0^2) \quad (41)$$

Integrating Eq.(36) with respect to A, $\dot A$ and $\dot\varphi$ yields the phase distribution in the form

$$f_\varphi(\varphi) = \frac{1}{2\pi}\Big\{1 + \frac{\varepsilon}{4}\sqrt{2\pi}\rho_1\cos\pi\Big\} \quad (42)$$

As $\varepsilon=0$, the phase distribution in Eq. (42) is reduced to the well known uniform distribution. From the marginal distribution of wave envelop and its first derivative obtainable from Eqs. (36), (41), (1) and (2), average number of waves in a group G and mean number of waves in a high run H are given, respectively, by

$$G = \frac{1}{\sqrt{2\pi}\sqrt{1-\rho_3^2}A_0}\Big\{1 + \frac{\varepsilon}{2}(\rho_2\rho_3 - \rho_1)\zeta_0 - \frac{\varepsilon}{2}\rho_1\zeta_0^3\Big\} \cdot \exp\Big\{-\frac{1}{2}(\zeta_0^2 - A_0^2)\Big\} \quad (43)$$

and

$$H = \frac{1}{\sqrt{2\pi}\sqrt{1-\rho_3^2}A_0}\Big\{1 + \frac{\varepsilon}{2}(\rho_2\rho_3 - \rho_1)\zeta_0 - \frac{\varepsilon}{2}\rho_1\zeta_0^3\Big\} \cdot \exp\Big\{-\frac{1}{2}\zeta_0^2\Big\} \quad (44)$$

6. Numerical results

To quantify the above results, we must specify the wave spectrum from which the quantities ρ_1, ρ_2, ρ_3 and ε may be calculated. In this study, we shall use the Wallops spectrum proposed by Huang et al. in 1981 based on the hydrodynamic analysis rather than data fitting which takes the form

$$\Phi(\omega) = \frac{\alpha g^2}{\omega^m\omega_0^{5-m}}\exp\Big\{-\frac{m}{4}(\frac{\omega_0}{\omega})^4\Big\} \quad (45)$$

where

$$m = \left| \frac{\log(2\pi^2\xi^2)}{\log 2} \right| \quad (46)$$

is the absolute value of the slope of the spectrum on the log-log scale in the high frequency range and

$$\xi = M_0^{\frac{1}{2}}/L_0 = \sigma k/(2\pi) = \varepsilon/(2\pi) \quad (47)$$

is the significant slope, L_0 being the wave length whose frequency ω_0 corresponds to the peak of the single peak Wallops spectrum. In Eq. (45), the coefficient α is given by

$$\alpha = \frac{(2\pi\xi)^2 m^{(m-1)/4}}{4^{(m-5)/4}} \frac{1}{\Gamma[(m-1)/4]} \quad (48)$$

where $\Gamma[\cdot]$ is the gamma function (Abramowitz and Stegun, 1968). From (45), it may be shown that

$$\rho_1 = -\frac{\Gamma[(m-3)/4]}{\Gamma^{1/2}[(m-1)/4]\Gamma^{1/2}[(m-5)/4]} \quad (49)$$

$$\rho_2 = -\frac{\Gamma[(m-4)/4)]}{\Gamma^{1/2}[(m-3)/4]\Gamma^{1/2}[(m-3)/4]} \quad (50)$$

$$\rho_3 = \frac{\Gamma[(m-2)/4]}{\Gamma^{1/2}[(m-1)/4]\Gamma^{1/2}[(m-3)/4]} \quad (51)$$

$$\varepsilon = 2\pi\xi \left[\frac{m\Gamma[(m-5)/4]}{4\Gamma[(m-1)/4]}\right]^{1/2} \quad (52)$$

so that ε, ρ_1, ρ_2 and ρ_3 are solely dependent on the value of ξ which was shown (Huang et al., 1981) to rarely exceed 0.03 in the ocean. In Figs. 1 and 2, the number of up-crossings in Eq. (41) is plotted for varying ξ and $N_\zeta(\zeta_0)$ of linear waves is also included for the comparison. Here, it is obvious that threshold crossing rate reaches its maxima just below the mean water level rather than $\zeta_0 = 0$, and considerable amount of probability mass is shifted toward the larger values of ζ_0 as nonlinearity is getting profound: this is consistent with the vertically asymmetric property of nonlinear waves which are known to have more sharp peaks and shallower troughs than the linear counterpart. The phase distribution is plotted in Fig. 3 for ξ = 0.01, 0.015, 0.02, 0.025 and 0.03. It turns out that the phase distribution is modified significantly by nonlinearities, and it shows a systematic excess of values near the mean phase and corresponding symmetrical deficiency on both sides away from the mean. In Figs. 4 and 5, the average number of waves in a group G in Eq. (43) is plotted for varying ζ_0 and A_0 with $\xi = 0.03$, respectively.

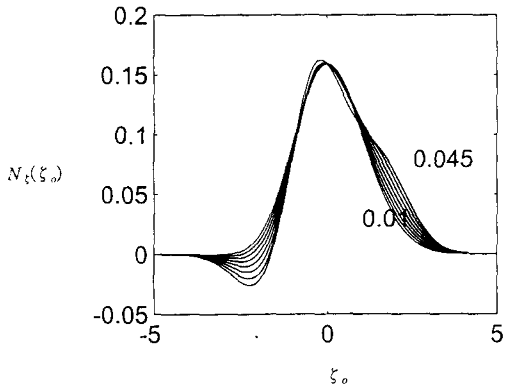

Fig. 1. Threshold Crossing Rate for Varying ξ

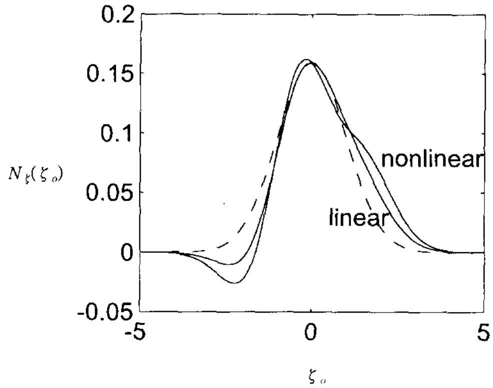

Fig. 2. Threshold Crossing Rate for $\xi = 0.03$ and 0.045 with a Linear Counterpart (solid line: nonlinear, dashed line: linear)

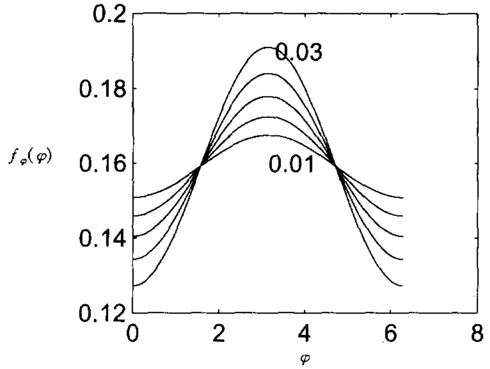

Fig. 3. Phase Distribution for Varying ξ

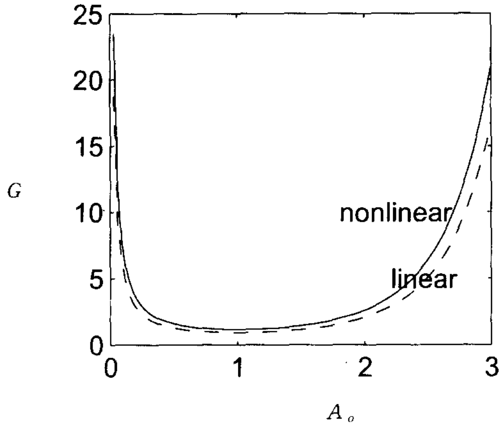

Fig. 5. Mean Number of Waves in a Group with a Linear Counterpart for $\xi=0.03$ and $\zeta_o=1.17$ (solid line: nonlinear, dashed line:linear)

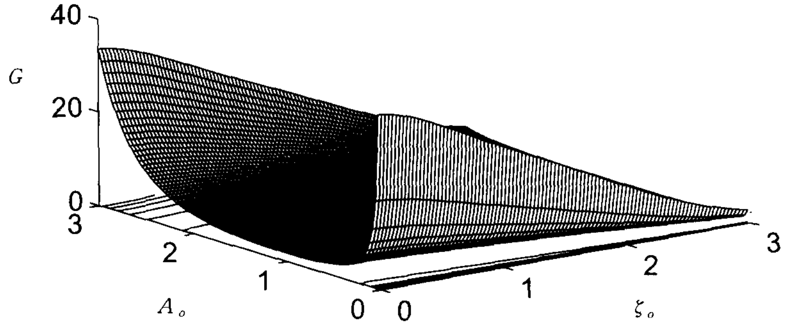

Fig. 4. 3-D and Contour Plot of Mean Number of Waves in a Group for Varying ζ_o and A_o with $\xi=0.03$

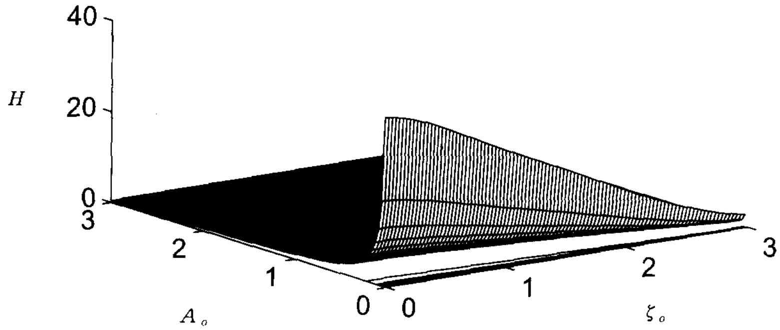

Fig. 6. 3-D and Contour Plot of Mean Number of Waves in a High Run for Varying ζ_o and A_o with $\xi=0.03$

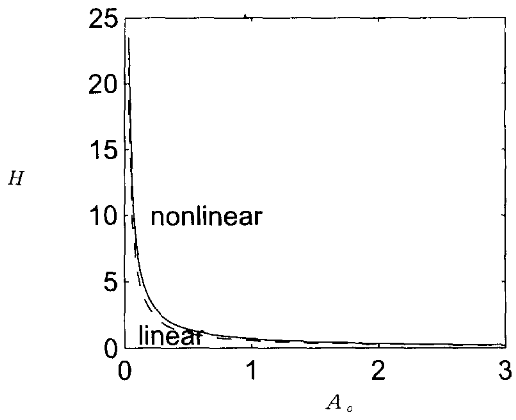

Fig. 7. Mean Number of Waves in a High Run with a Linear Counterpart for $\xi = 0.03$ and $\zeta_o = 1.17$ (solid line: nonlinear, dashed line: linear)

and the linear counterpart obtainable by setting $\varepsilon = 0$ is also included in Fig. 5 for thecomparison. The mean number of waves in a high run H with a linear counterpart is plotted in Figs. 6 and 7. It is noted that for random process of moderate bandwidth, the average number of waves in a group and the mean number of waves in a high run differ from the linear counterpart. The general character of this difference is that G and H is increased over the entire range of wave envelope.

7. Conclusions

After the Gaussian model was first developed by Rice (1944), there was a great deal of progress on the theory of nonlinear waves. But these progresses was not extended to the nonlinear statistical properties of wave groups due to the complicated form of nonlinear random waves which deserve much attention in the context of probable damage to coastal defences or offshore structures. In a case when the underlying frequency spectrum is narrow, the stochastic representation of a nonlinear sea surface is reduced to a familiar form in which each realization is an amplitude modulated second order Stokes wave. In contrast the intricate complexity of the expression of nonlinear waves of finite bandwidth, such an approximation constitutes a simpler formulation to study numerically or analytically the nonlinear effects on the statistical description of wave properties. But considering the side band instability of Stokes waves, the narrow band assumption at the site away from the generating area is no longer valid. For waves of finite bandwidth, an approximate wave model proposed by Tung et al. (1989) is promising alternative from which the joint distribution of nonlinear wave elevation and its first derivative can be obtained and the structure of which is simple enough for statistical properties of such non-linear waves to be obtainable. Based on this wave model, overdue task of investigating non-linear effects on the statistical properties of wave groups in terms of the average number of waves in a group and the mean number of waves in a high run was resumed in this study utilizing the complex envelop and the total phase function, random variable transformation technique and perturbation method.

It turns out that phase distribution is modified significantly by nonlinearities, and it shows systematic excess of values near the mean phase and corresponding deficiency on both sides away from the mean. For the case of threshold crossing rate, it is noted that threshold crossing rate reaches its maxima just below mean water level, and considerable amount of probability mass is shifted toward the larger values of water surface elevation as nonlinearity is getting profound. Further-more, the mean waves in a high run associated with nonlinear waves are shown to have larger values than the linear coun- terpart. Similar trend can also be found in the average number of waves in a group.

References

Abramowitz, M., and Stegun, I.A. (1968). Handbook of *mathematical functions.* Dover, Mineola, NY.

Cho, Y.J. and Yoon, T.H. (1992). "Peak distribution of nonlinear random waves of finite bandwidth." *Proceedings of the Sixth IAHR International Symposium on Stochastic Hydraulics.* Taipei.

Goda, Y. (1983). "Analysis of wave grouping and spectra of long travelld swell." *Rept.*, Port and Harbour Research Inst., Vol. 22., No. 1.

Huang, N.E., Long, S.R., Tung, C.C., Yuan,Y., and Bliven, L.F. (1981). "A unified two-parameter wave spectral model for general sea state." *J. Fluid Mech.*, Vol. 112, pp. 203-224

Huang, N.E., Long, S.R., Tung, C.C., Yuan, Y., and Bliven, L.F. (1983). "A non-Gauss ian statistical model for surface elevation of nonlinear random wave fields." *J. Geophys. Res.*, Vol. 88, pp. 7597-7607.

Lin, Y.K. (1967). *Probabilistic theory of structural dynamics.* McGraw-Hill, New York, NY.

Longuet-Higgins, M.S. (1957). "Statistical analysis of a random, moving surface." *Phil. Trans. R. Soc. Lond.* A 966, Vol. 249, pp. 321-387.

Longuet-Higgins, M.S. (1963). "The effect of nonlinearities on statistical distribution in the theory of sea waves." *J. Fluid Mech.*, Vol. 21, pp. 459-480.

Longuet-Higgins, M.S. (1984). "Statistical properties of wave groups in a random sea state." *Phil. Trans. R. Soc. Lond.* A 1512, Vol. 312, pp. 219-250.

Papoulis, A. (1965). *Probability, random variables and stochastic processes.* McGraw-Hil, New York, NY.

Rice, S.O. (1944). "Mathematical analysis of random noise." reprinted in selected papers on noise and stochastic processes. Dover Pub. Inc, New York, pp. 133-294

Tayfun, M.A. (1980). "Narrow-band nonlinear sea waves." *J. Geophys. Res.*, Vol. 85, pp. 1548-1552.

Tayfun, M.A. (1986). "Narrow-band representation of ocean waves 1, theory." *J. Geoghys. Res.*, Vol. 91, pp. 7743-7752.

Tung, C.C., Huang, N.E., Yuan, Y., and Long, S.R. (1989). "Probability function of breaking limited surface elevation." *J. Geophys. Res.*, Vol. 94, No. C1, pp. 967-972.

〈최종본 도착일 : 1997년 5월 24일〉