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Optimal Design of Irrigation Pipe Network with Multiple Sources

Lyu, Heui Jeong* · **Ahn, Tae Jin***

**Prof. of Civil Engrg., Ansung National Univ., Ansung, Kyonggi, Korea, 456-749*

Abstract □ This paper presents a heuristic method for optimal design of water distribution system with multiple sources and potential links. In multiple source pipe network, supply rate at each source node affects the total cost of the system because supply rates are not uniquely determined. The Linear Minimum Cost Flow (LMCF) model may be used to a large scale pipe network with multiple sources to determine supply rate at each source node. In this study the heuristic method based on the LMCF is suggested to determine supply rate at each source node and then to optimize the given layout. The heuristic method in turn perturbs links in the longest path of the network to obtain the supply rates which make the optimal design of the pipe network. Once the best tree network is obtained, the frequency count of reconnecting links by considering link failure is in turn applied to form loop to enhance the reliability of the best tree network. A sample pipe network is employed to test the proposed method. The results show that the proposed method can yield a lower cost design than the LMCF alone and that the proposed method can be efficiently used to design irrigation systems or rural water distribution systems.

Keywords □ optimal design, heuristic method, pipe network, multiple sources.

I. Introduction

The classical steady state approach to designing water distribution systems involves the following subproblems: (i) planning, (ii) design, (iii) analysis. The planning aspect involves the determination of the water requirements of the system. The design problem involves the selection of an optimal topological layout as well as the sizes of the distribution system components. The analysis step involves the evaluation of the flows and pressures in the system, based on the given layout and sizes of the system components.

Supply rates and pipe flows affect the total cost of the pipe network. The global optimum can be obtained through the linear programming in tree pipe network with a single source, because the supply rate and pipe flows are uniquely determined. However, since supply rates or pipe flows are not uniquely determined in tree pipe network with multiple sources or looped pipe network, it is not easy to get the global optimum for the networks. Quindry et al. (1981) presented the algorithms that would yield an optimal pipe flow distribution. The Linear Minimum Cost Flow (LMCF) proposed by Rowell (1979) gets only a local minimum because the model assumes a constant value for the energy gradient over all links. Loganathan et al. (1990) adopted the LMCF model to find the optimal tree pipe network, which was used the initial pipe network to search the global optimum of potential network. The LMCF model find the short path tree network from potential network by identifying a source connected demand nodes.

Rowell and Barnes (1982) and Rowell (1979) proposed the Nonlinear Minimum Cost Flow model for selecting the optimal supply rate at each source node and the optimal layout for a water distribution system. Alperovits and Shamir (1977) presented a two level algorithm that was called the linear programming gradient (LPG) method. The LPG begins with known flow rates for the model and the optimal dual variables obtained from the linear programming are then used to determine the steepest descent direction with regard to a perturbation vector of flows. If the cost of the system decrease, a new LP problem is set up with the updated flow rates and the iterative procedure is repeated until no improvement in cost can be obtained. Kessler and Shamir (1989) introduced the projection of the gradient of objective function onto the constraints surface to improve the search procedure and obtained a local optimal solution. Fujiwara and Khang (1990) and Kessler and Shamir (1991) proposed a two-phase procedure. These methods attempt to find the near global optimum. For the chosen flows based on continuity, the first phase cost minimization problem has a nonlinear convex objective in terms of head loss with a linear constraint region. Then in the spirit of the LPG procedure, an improving direction is generated with the aid of the Lagrange multipliers of the Phase I constraints called NLPG direction.

For irrigation conveyance systems, Labye et al. (1988) presented a method for determining a layout of a tree network. The method consists of three distinct stages which are the proximity layout, the 120o layout, and the least cost layout. However, this method does not consider topographic constraints in determining layout. Since the method is based on a geometric scheme, the method would be inefficient and would converge slowly for a large network.

In this study a heuristic method based on the LMCF model is suggested to determine supply rates at source nodes and then to optimize the given pipe layout. An existing irrigation pipe network is considered to examine the applicability of the proposed method. The optimal layout, the supply rate at each source node, and the optimal pipe diameters are determined.

II. Model Formulation

The following mathematical programming formulation model 1 (M1) for multiple sources is adopted for potential pipe network of upland irrigation system.

(M1) : Minimize

$$\sum_{(i,j)}\sum_{m}C_{(i,j)m}x_{(i,j)m}+\sum_{i\in s}C_{1i}HS_i+\sum_{i\in S}C_{2i}$$

$$Hp_i+\sum_{i\in S}C_{3i}\frac{\gamma T_i}{\eta_i}Qp_iHp_i \quad (1)$$

Subject to

$$-\sum_{K:(i,k)\in A}Q_{(i,k)}+\sum_{K:(k,i)\in A}Q_{(k,i)}=q_i$$

$$\text{for all } i\in N \quad (2)$$

$$-\sum_{(i,k)\in\gamma(k)}\pm\sum_{m}[J_{1(i,j)m}+J_{2(i,j)m}]x_{(i,j)m}+[Hs_i+Hs_i^{ele}]$$

$$+[Hp_i+Hp_i^{ele}]\geq H_k^{min} \text{ for all } i\in s,\ k\in N,$$

$$\text{and } i\in S \quad (3)$$

$$\sum_{(i,j)\in p}\pm\sum_{m}[J_{1(i,j)m}+J_{2(i,j)m}]x_{(i,j)m}=$$

$$b_p \text{ for all } p\in B \quad (4)$$

$$\sum_{(i,j)\in p}\pm\sum_{m}[J_{1(i,j)m}+J_{2(i,j)m}]x_{(i,j)m}=b_p$$

$$\text{for all } (i,j)\in A \quad (5)$$

$x(i,j)m\geq 0$, $Hsi\geq 0$, $Hpi\geq 0$

where :

$C_{(i,j)m}$ =unit cost for the m th new diameter segment in a link(i,j)
$x_{(i,j)m}$ =length of the m th new diameter segment in a link(i,j)
C_{1i} =unit cost of tank at node i
C_{2i} =pump capital cost per pumping head at node i
C_{3i} =unit energy cost
H_{si} =additional head for tank node i
H_{pi} =operating head of a pump at node i
H_i^{ele} = elevation of tank at node i
HP_i^{ele} =elevation of pump at node I
γ =specific weight of water
T_i =pumping period for pump node i
η_i =pump efficiency for pump at node i
Qp_i =pumping rate of pump at node i
s =set of tank nodes
S =set of pump nodes
$Q_{(i,j)}$ =link flow
q_i =consumptive use or demand at node i
r(k) =a path through the network connecting a source node and demand node k
H_k^{min} =minimum pressure head required at node k
p =path connecting source head nodes
B =set of paths connecting source nodes
b_p =zero in the loop and is head difference between the source heads
A =number of links in the network
$L_{(i,j)}$ =length of a link (i,j)
N =number of nodes in the network except source nodes.

The hydraulic gradient due to the frictional head loss is calculated from the Hazen-Williams equation with the SI system of unit : $J_{1(i,j)}=k_{1(i,j)}Q_{(i,j)}^{1.852}D_{(i,j)}^{-4.87}$, for each pipe in which $k_{1(i,j)}=10.7/c^{1.852}$, C is the Hazen-Williams coefficient for new pipe, $Q_{(i,j)}$ is the given flow in link (i,j), and $D_{(i,j)}$ is the new diameter of m th segment in link (i,j). The hydraulic gradient due to the minor head loss is given by the following formula : $J_{2(i,j)} = k_{m(i,j)} Q^{2}_{(i,j)} D^{-4}_{(i,j)} L^{-1}_{(i,j)}$ where $k_{m(i,j)}M$ is the coefficient of minor head loss.

The objective function of Model1 (M1) is the summation of the pipe network cost, the storage tank cost, the pump capital cost, the operating cost for source pump, respectively. Constraint (2) represents the flow continuity equations, constraint (3) represents the hydraulic head requirement at each node, constraint (4) represents the sum of head losses in a path, and constraint (5) represents the length constraints.

III. Heuristic Method for Pipe Network Optimization

For multiple source networks, the supply rate at each source node is unknown and the amount of the supply rate depends on the number of links and demand nodes. Since supply rate of each source node affects the total cost of the system, the following equation is given : Cost$=f(SQi)$, $i=1,\cdots,S$ or Cost $=f(Q_{(i,j)})$, $(i,j)\in A$ in which Cost is the total cost of the system, SQ_i are supply rates at source nodes, $Q_{(i,j)}$ are the link flows, S is the set of source nodes, and A is the set of links.

The following Model2 is suggested for the multiple source pipe network with potential links to determine the optimal supply rates and diameters of pipes.

$$\text{Model2} : \min_{SQ_i} \left[\min_{x\in X} f(x)\right] \tag{6}$$

in which SQ_i are supply rates at source nodes, X is the feasible region made up of constraints (2)~(5), and f(x) is the objective function of the Model1 for link flows and supply rates.

It is observed that the inner Model3 of Model2 is given by

$$\text{Model3} : \min_{x\in X} f(x) \quad \text{for supply rates} \tag{7}$$

The model3 is a nonlinear, discrete, nonconvex programming problem, which may have several local minima. For known flows the model3 is a linear program which can be solved efficiently by using commercially available codes. The model3 is equivalent to the model1 if the link flows are known.

To choose the supply rates in the outer Model2 a heuristic method based on the LMCF model is adopted.

LMCF model : $\min \sum_{(i,k)\in L} L_{(i,j)} \mid Q_{(i,j)} \mid$
subject to

$$\sum_{Vk=k_i} Q_{(k,i)} + Q_i = 0$$

To make the search efficient in the outer Model2, first the LMCF model is used to determine the initial supply rates for potential tree network with multiple sources. However, the LMCF model gets only a local minimum because the model assumes a constant value for energy gradient over all links. Thus, the links in the longest path are then perturbed with the Model3 to determine the optimal supply rate at each source node while attempting to minimize the overall design cost. Since the optimal tree pipe network is divided into S-1 number of sub-tree network, S-1 number of connecting links appear. A minimum commercial diameter is adopted for all the connecting links and the resulting network is analyzed using the hydraulic simulator, TDH.

Once the best tree network is obtained from the potential tree network, the frequency count of reconnecting links by considering link failure is in turn applied to form loop to enhance the reliability of the optimal tree network in the spirit of Loganathan (1990). For example in the optimal tree pipe network with two sources, one link is assumed to be link failure and the network is then divided into two sub-tree networks. To connect the two sub-tree to one tree network, one needs the potential link which is called the reconnecting link. It should be noted that one reconnecting link may become a reconnecting link for several other link failures. For each link failure, the reconnecting link with the highest rank is chosen. If two or more links have the same rank then the link with the smaller length is chosen.

The proposed procedure is summarized in the following.

Step 1. Determine the supply rates and connecting links by the LMCF model from potential tree pipe network.

Step 2. Apply the Model3 to compute the total cost and the connecting link is put as the minimum commercial diameter.

Step 3. Hydraulic simulator is adopted to review hydraulic requirements of the network.

Step 4. Perturb the links the longest path by performing step 2 and step 3. Determine the optimal tree pipe network.

Step 5. Determine the reconnecting link to form loop and review the network hydraulically.

Determine the looped pipe network.

IV. Analysis of Example Network

The network selected from the Chungu irrigation project (RDC, 1994) is revised as shown in Figure 1. The sample network has two sources, twenty eight junction nodes, and thirty seven potential links. The two storage tanks are connected to the pumping stations in reality. Since the location of each storage tank is fixed, the pumps and the tanks are not considered. The link lengths, the design head, and demands are given Table 1. The Hazen Williams friction coefficient is 140 for all links and exponents for discharge and diameter are 1.852 and -4.87 respectively. Polyethylene (PE) pipe are adopted in the study area. To obtain the annual cost of the system, the initial costs of pipe component are converted into annual capital recovery cost by introducing the annual capital recovery factor. The annual capital recovery factor is given by : $R = i(1+i)^n/[(1+i)^n - 1] = 0.1(1+0.1)^{30}/((1+0.1)^{30}-1) = 0.106$, in which i is an interest rate per year, and n is a life span of system. The annual payment equivalent to a present sum of each component of system for the life span of the system at an interest rate i is computed : $c = C \cdot R$ where c is the annual capital recovery cost of a component, and C is capital cost of the component. Table 2 shows commercially available pipe sizes, their capital cost, and annual costs.

The design of the sample pipe network in-

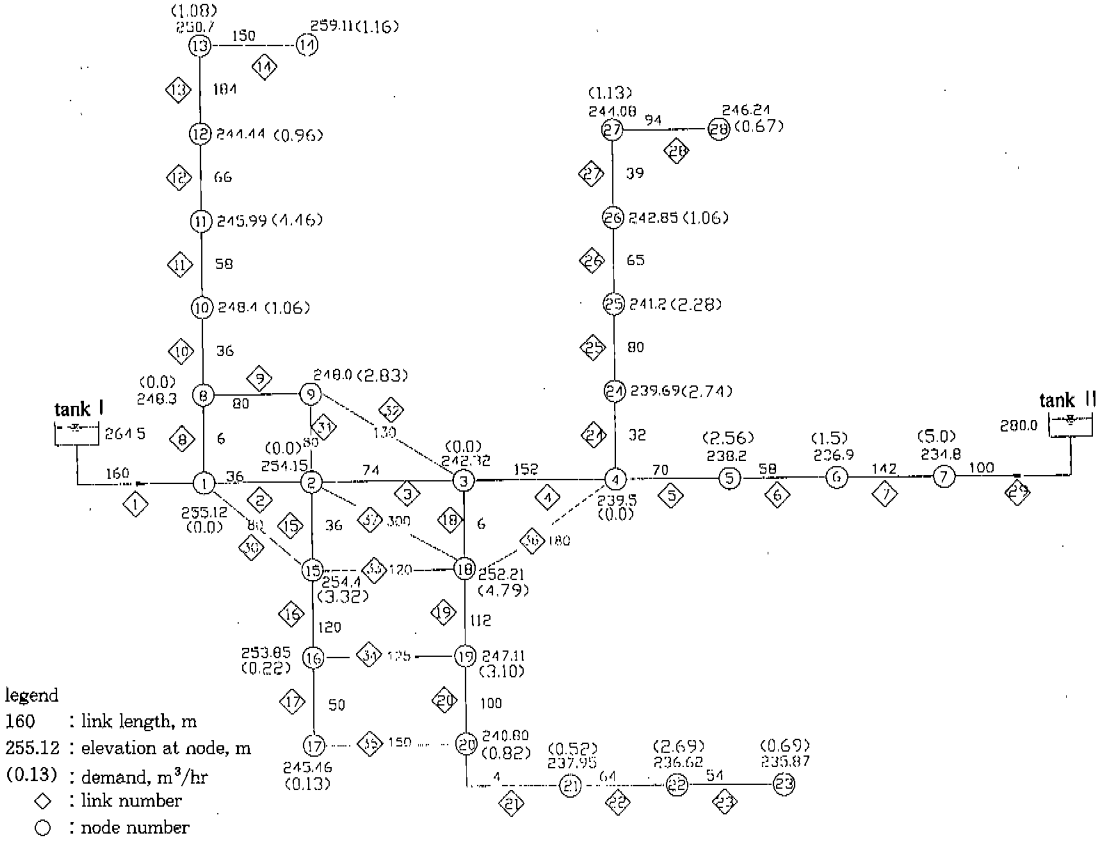

Fig. 1. Sample potential pipe network

volves mainly three decisions : the selection of the tree layout from the given set of potential links, the selection of diameters of links, and the selection of supply rates at source nodes. Table 3 shows the optimal solution obtained by the LMCF model, the model 1 (M1), and TDH. The LMCF model is first used to determine the supply rate at each tank : consequently, the tank I supplies the node numbers 1, 2, 3, 8, 9,⋯,23 and the tank II supplies the node number 4, 5, 6, 7, 24, 25, 26, 27, 28. Thus, the tree network is divided into two sub-tree networks and the link number 4 is the connecting link to make the tree network. The supply rates at the sources obtained by the LMCF model are : 16.94m^3/hr(4.7 l/s) for tank I, 28.83 m^3/hr (8.01 l/s) for tank II. Once the link flows are determined based on the supply rates by the LMCF model, the model 1 (M1) is performed to compute the annual cost. The minimum diameter of 30 mm is assumed for the connecting link 4 and the network has an annual cost of 1,837,416 Won. The resulting tree network was analyzed using the hydraulic simulator, the TDH. The supply rate at each source obtained by TDH is : 7.96 l/s for tank I, 4.76 l/s for tank II.

The results of applying the heuristic method are : the tank I supplies the node number

Table 1. Design requirements for sample network

Node data					Link data	
Link data	Demand (m^3/hr)	Elevation (El. m)	Minimum head, m	Design head(El. m)	Link number	Length (m)
1	0.0	255.12	5.0	260.12	1	160
2	0.0	245.15	5.0	259.15	2	36
3	0.0	245.32	10.0	252.32	3	74
4	0.0	269.50	10.0	249.50	4	152
5	2.56	238.20	10.0	248.20	5	70
6	1.50	236.60	10.0	246.90	6	58
7	5.0	234.80	10.0	244.80	7	142
8	0.0	248.30	10.0	258.30	8	6
9	2.83	248.00	10.0	258.00	9	80
10	1.06	248.40	10.0	258.40	10	36
11	4.46	245.99	10.0	255.99	11	58
12	0.96	244.44	10.0	254.44	12	66
13	1.08	250.70	10.0	260.70	13	184
14	1.16	259.11	3.0	262.11	14	150
15	3.32	254.40	5.0	259.40	15	36
16	0.22	253.85	5.0	258.85	16	120
17	1.13	245.46	10.0	255.46	17	36
18	4.79	252.21	5.0	257.21	18	6
19	3.10	247.11	10.0	257.11	19	112
20	0.82	240.80	10.0	250.80	20	100
21	0.52	237.95	10.0	247.95	21	4
22	2.69	236.62	10.0	246.62	22	64
23	0.69	235.87	10.0	245.87	23	54
24	2.74	239.89	10.0	249.69	24	32
25	2.28	241.20	10.0	251.20	25	80
26	1.06	244.85	10.0	252.85	26	65
27	1.13	244.08	10.0	254.08	27	39
28	0.67	246.24	10.0	256.24	28	94
Tank I		N.W.L. 264.5			29	100
Tank II		N.W.L. 280.0			30	80
					31	80
					32	130
					33	120
					34	125
					35	150
					36	180
					37	300

1, 2, 8, 9,⋯, 17 and the tank II supplies the node number 3, 4, 5, 6, 7, 18, 19,⋯,28. By perturbing link in the longest path of the

Table 2. Diameter and cost data for sample network

Diameter(mm)	Capital cost (Won/m)	Annual cost (Won/m/year)
30	3,565	377
40	4,66	494
50	6,378	676
65	8,238	873
75	10,233	1,084
100	14,829	1,571
125	19,940	2,113
150	23,988	2,542

Table 3. Optimal solution by LMCF model

Link number	Diameter (mm)	Length (m)	Node number	Optimal head(El. m)	Head by TDH (El. m)
1	125	160	1	263.89	263.95
2	75	36	2	263.22	263.37
3	75	94.06	3	258.81	259.33
	65	179.94	4	258.81	259.38
4	30	152	5	260.81	261.38
5	50	70	6	263.66	264.14
6	50	58	7	272.43	272.76
7	50	142	8	263.84	263.93
8	100	5.43	9	263.55	263.60
	75	0.57	10	263.70	263.77
9	50	80	11	263.27	263.35
10	75	36	12	262.88	263.01
11	65	58	13	262.42	262.52
12	50	66	14	262.10	262.17
13	50	184	15	262.81	263.00
14	40	150	16	261.38	261.49
15	50	36	17	260.95	261.04
16	30	120	18	258.64.25	259.22
17	30	50	19	7.11	257.76
18	65	6	20	256.05	256.70
19	65	78.7	21	256.00	256.67
	50	33.3	2	255.63	256.30
20	50	100	23	255.45	256.11
21	50	4	24	258.54	259.14
22	50	64	25	257.53	258.14
23	50	54	26	256.73	257.34
24	30	32	27	256.53	257.13
25	50	80	28	256.23	256.81
26	40	65		Supply rate	
27	40	39		Tank Ⅰ : 8.01 l/s	
28	30	94		Tank Ⅱ : 4.7 l/s	
29	65	51.87			
	50	48.13			

Table 4. Optimal solution by heuristic method

Link number	Diameter (mm)	Length (m)	Node number	Optimal head(El. m)	Head by TDH (El. m)
1	100	160	1	263.90	263.96
2	50	36	2	263.51	263.65
3	30	274	3	263.51	264.14
4	65	152	4	266.40	267.10
5	75	51.12	5	268.58	269.23
	65	18.88	6	270.38	270.90
6	75	58	7	275.16	275.48
7	75	142	8	163.87	263.94
8	100	4.88	9	263.56	263.61
	75	1.12	10	263.70	263.77
9	50	80	11	263.27	263.35
10	75	36	12	262.88	263.01
11	65	58	13	262.42	262.52
12	50	66	14	262.10	262.17
13	50	184	15	263.10	263.28
14	40	150	16	261.67	261.77
15	50	36	17	261.24	261.32
16	30	120	18	262.93	263.75
17	30	50	19	259.82	260.73
18	50	6	20	258.76	259.67
19	50	112	21	258.71	259.64
20	50	100	22	258.34	259.27
21	50	4	23	258.16	259.08
22	50	64	24	265.45	266.23
23	30	54	25	264.44	265.24
24	50	32	26	261.19	261.96
25	50	80	27	260.37	261.13
26	30	65	28	260.07	260.80
27	30	39	Supply rate		
28	30	94	Tank I : 4.5 l/s,		
29	75	100	Tank II : 8.2 l/s		

sample potential tree network, the supply rates at the sources are : 16.22 m^3/hr (4.5 l/s) for tank I, 29.55 m^3/hr (8.2 l/s) for tank II. The link number 3 is the connecting link. By applying the above procedure, the heuristic method yields an annual cost of 1,756,983 Won. The supply rate at each source due to the TDH is : 4.37 l/s for tank I, 8.34 l/s for tank II. Table 4 shows the optimal solution by the heuristic method.

Table 5. Frequency count of reconnecting links

Failure link	Reconnecting link	Reconnecting link	Rank
1	Even if the link is failed, all nodes are supplied from the sources	30	1
2		31	2
3		32	2
4		33	2
5		34	4
6		35	6
7		36	1
29		37	1
8	31, 32		
9	31, 32		
15	30, 33, 34, 35		
16	34, 35		
17	35		
18	33, 34, 35, 36, 37		
19	34, 35		
20	35,		
other link	no reconnecting link		

To form one loop in the optimal tree network, an attempt was made to fail a link and found reconnecting links to maintain the tree network. Table 5 shows the result of the frequency count of reconnecting links and the link 35 is used 6 times. Consequently, the link number 35 whose diameter is 30 mm is adopted to make the optimal loop network as shown in Figure 2 and the annual cost of the network is 1,813,533 Won. The supply rate obtained by TDH is : 4.37 l/s for tank I, 8.34 l/s for tank II.

V. Summary and Conclusions

A heuristic method for designing a looped pipe network has been described. Since the model 1 (M1) has several local minima, each perturbation of link in the longest path moves towards a better local optimum. The various locally optimal designs obtained by

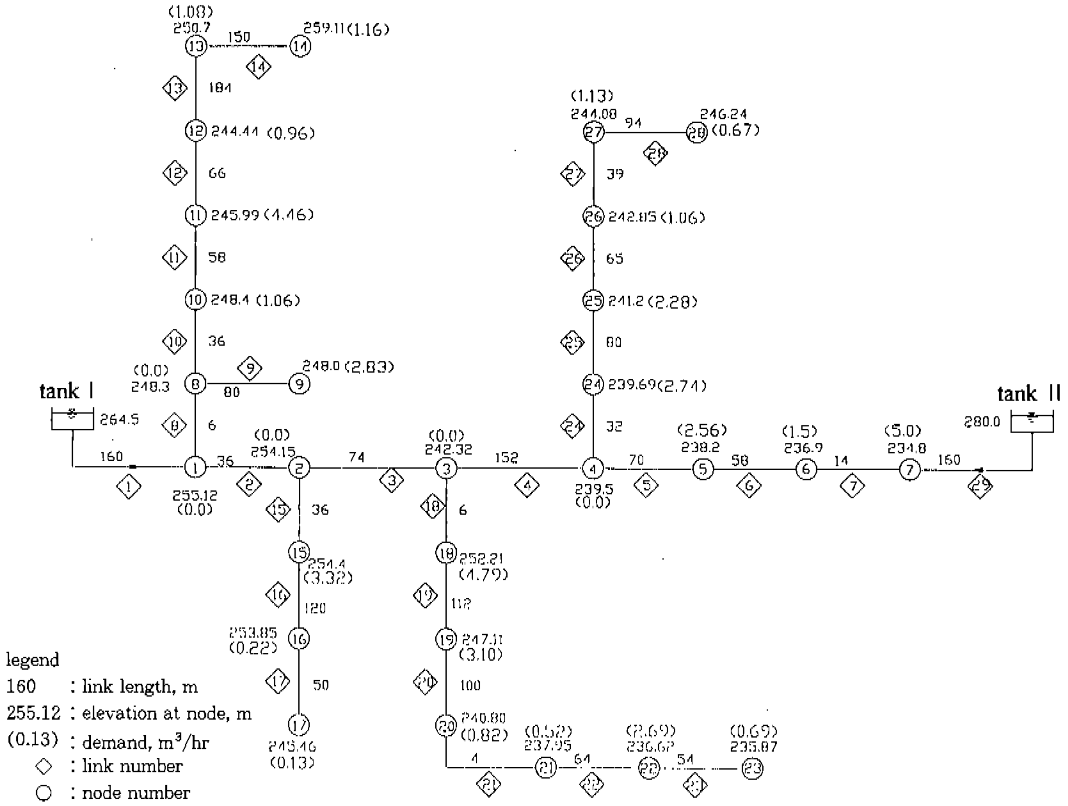

Fig. 2. Optimal looped pipe network

the heuristic method, greatly aid in understanding the feasible region in terms of the objective function surface. As a practical matter, the methodology helps the designer in understanding how close the various designs are in terms of cost. The design of the sample pipe network involves mainly three decisions : the selection of the tree layout from the given set of potential links, the selection of diameters of links, and the selection of supply rates at source nodes. In this study the heuristic method based on the LMCF model is used to determine supply rate at each source node and then to optimize the given tree layout. The heuristic method in turn perturbs links in the longest path of the network to obtain the supply rates for the optimal design of the pipe network. Once the best tree network is obtained, the frequency count of reconnecting link by considering link failure is in turn applied to form a loop to enhance the reliability of the best tree network.

A sample pipe network is employed to test the heuristic method. Two sets of the supply rates are suggested by the heuristic method and the hydraulic simulator respectively. Since both sets of the supply rates are all hydraulically feasible, one can choose either set of supply rates. If the construction costs of water source may be considered, the supply rate at each source node may be uniquely determined. The results show that the proposed method can yield a lower cost design than the LMCF model alone. In conclusion, the proposed method can be efficiently used to design pipe network from potential pipe layout.

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