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On Transition Procedure Using an Optimal Quantile Estimator under Uncertainty

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Abstract

This paper deals with the perishable inventory models with uncertainties of demand functions. The traditional perishable inventory costs of holding and stockout are incorporated into the cost function. The average expected cost will be minimized to find the optimal quantile estimator. After three candidate estimators are proposed on the basis of order statistics, they will be evaluated by the simulation results and statistical analysis. Then the transition procedure algorithm using this estimator will be proposed to make the optimal decision under uncertainty.

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1. Introduction

In this study primarily we are concerned with a perishable inventory model with uncertainties of demand functions. The classic form of this kind of the model was described in Morse and Kimball's early operations text[8]. Many manufacturers or suppliers must decide how many commodities to be prepared from period to period. Thus, a fixed periodical demand quantity for automobiles or appliances fluctuate randomly. So does the number of spares or the amount of ammunition to acquire along with a new system or a mission.

In advance of period's demand, the manufacturer must produce and stock his products to meet the customer's need. If he prepares more products than customers will purchase from him, he will have products left at the end of the period. Then he will incur a loss on each, since old products will have little salvage value. On the other hand, if he prepares too few products, he will sell out early and his later customers will not be able to buy from him. Thus, he will incur a cost which is called lost profit for each product that he could have sold, but could't.

This paper presents the expected cost solutions for the model with either cases of continuous and discrete random demand. In order to make the optimal decision as to the inventory level, the inventory holding and stockout costs will be incorporated into the cost function. This study will consider primarily the problem of minimizing the expected cost. The study including statistical analysis will be extended to the more general demand distribution in addition to the simulation study[2]. And then the study will propose the transition procedure for the model from uncertainty.

In section 2, the expected cost function will be derived and then an optimal solution for the function will be found for either cases of continuous and discrete models. Three candidate estimators using order statistics will be proposed and compared with each other for the optimal inventory decision in section 3. In section 4, after analyzing the input data, those proposed estimators will be evaluated by the simulation results and then a transition procedure algorithm from uncertainty will be proposed. Finally, conclusions will be given in the final section.

2. The Expected Cost Function and the Optimal Solution

In this section a perishable inventory model is examined under uncertainty conditions. We will first structure the general form of the problem and then show how optimal solution can be obtained when the estimates of demand distributions are available. First a cost function will be set up and then a derived expected cost function will be minimized to find the optimal solution.

2.1 The Expected Cost Function

Consider the perishable inventory model in which the cycle length is fixed and there is only one opportunity to order. Demand is a random variable (RV), X with probability density function $f(x)$ if continuous and probability mass function $p(x)$ if discrete. Assume that the relevent costs are holding cost and shortage or stockout cost only.

Let X be a continuous RV and let the cost of a perishable-goods be C and the selling price be F, provided that $F > C$. Any goods are not sold at the end of the period are sold for scrap at a value of S, provided that $C > S$. Then the cost function of Q is defined by the followings ;

$$C(Q) = \begin{cases} (C-S)(Q-X), & \text{if } 0 < X < Q, \\ (P-C)(X-Q), & \text{elsewhere} . \end{cases}$$

Hence, the expected total cost is a function of Q, which is given by ;

$$E[C(Q)] = (C-S)\int_0^Q (Q-x)f(x)\,dx + (P-C)\int_Q^\infty (x-Q)f(x)\,dx. \qquad (2.1.1)$$

where Q is the inventory purchased at the beginning of that period. The first of the right hand side is the expected holding cost and the second term is the expected shortage or outage cost. Analogically the expected total cost for the discrete RV, X is given by the followings ;

$$E[C(Q)] = (C-S)\sum_{x=0}^{Q}(Q-x)p(x) + (P-C)\sum_{x=Q+1}^{\infty}(x-Q)p(x). \qquad (2.1.2)$$

2.2 The optimal solution for the Expected Cost

Consider the first case where the RV is continuous. To find the optimal solution of Q, Q^* which minimizes the expected total cost, differntiate the equation (2.1.1) with respect to Q, set the result equal to zero and then solve for Q as follows ;

$$\frac{dE[C(Q)]}{dQ} = (C-S)\int_0^Q f(x)dx - (P-C)\int_Q^\infty f(x)dx = 0. \qquad (2.2.1)$$

Since $\int_Q^\infty f(x)dx = 1 - \int_0^Q f(x)dx$, the following is obtained from (2.2.1) ;

$$\int_0^{Q^*} f(x)dx = \frac{P-C}{P-S}. \qquad (2.2.2)$$

Consequently, we have $F(Q^*) = \frac{P-C}{P-S}$ where Q^* is the optimal quantity of Q.

Also differentiate the left hand side of the equation (2.2.1) with respect to Q, then the second derivative of $E[C(Q)]$ evaluated at $Q=Q^*$ is given by ;

$$\left[\frac{d^2E[C(Q)]}{dQ^2}\right]_{Q=Q^*} = \left[(C-S)\frac{d}{dQ}\int_0^Q f(x)dx\right]_{Q=Q^*} - \left[(P-C)\frac{d}{dQ}\int_Q^\infty f(x)dx\right]_{Q=Q^*}$$

$$= (P-S)f(Q^*) > 0 ,$$

because $P > S$ and $f(Q^*)$, being a density, is always positive. Therefore let p, pth

quantile, be $\frac{(P-C)}{(P-S)}$, then $Q^* = F^{-1}(p)$ which is the optimal(minimum) solution of the problem.

Let X be a discrete RV, then to find the optimal soluton Q^* defined for integer value of its argument, necessary conditions in terms of the expected total cost function can be described as the function of Q^* as follows ;

$$E[C(Q^*)] < E[C(Q^*+1)] \text{ and } E[C(Q^*)] > E[C(Q^*-1)]. \qquad (2.2.3)$$

Expanding two inequalities in equation (2.2.3) with the expected cost expression (2.1.2), and simplifying with the discrete distribution function $F(x) = \sum_{Q=0}^{x} f(Q)$ yields the following two inequalities ;

$$F(Q^*-1) < p \text{ and } F(Q^*) > p.$$

These two inequalities may be combined to give the following optimal solution ;

$$F^{-1}(p) < Q^* < F^{-1}(p)+1.$$

3. Proposition and Effect of the Quantile Estimators

If a decision maker is able to predict the form of the demand distributions, then a parametric approach can be more efficient and have a higher convergence rate to the true value than a nonparametric equivalent. Various considerations relating to the choice of the well-fitting demand distributions and about inventory demand predictions can be given in detail such as suggesting the Poisson distribution when the lead time is relatively low, or normal distribution when it is not relatively low.

But since it is not easy to specify the form of the demand distributions using insufficient data, for the early periods, a nonparametric estimation method can be considered. A nonparametric approach using the empirical distribution function is reasonably applicable, unless we are forced to operate the model at the tail of the distribution.

It is assumed that the first 33 periods when the available data is inadequate is critical. The decision maker operate the system well during that critical periods, i.e., at transition phase a nonparametric approach can be applied. Three different estimators of X_p, using an order statistics will be developed in the followings.

3.1 The First Candidate Estimator for the Optimum ; $\overline{X}_p$

The first estimator for the optimum inventory level will be proposed as follows ; Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be a set of order statistics from an unknown distribution where

$X_{(r)}$ is the rth largest value, i. e., rth order statistic.

Let r be defined by the followings ;

$$r = \begin{cases} np & \text{if } np \text{ is an integer} \\ [np+1] & \text{if } np \text{ is not an integer} \end{cases}$$

where p is the pth saple quantile, n is the number of periods so far and $[z]$ denotes the integeral part of z[3]. For large n, $\overline{X}_p = X_{(r)}$ which is a consistent and unbiased point estimator of X_p. In Figure 1, the rth order statistics, $X_{(r)}$ is represented graphically by the distribution function $F_D(X)$ and the empirical distribution function $G_D(X)$.

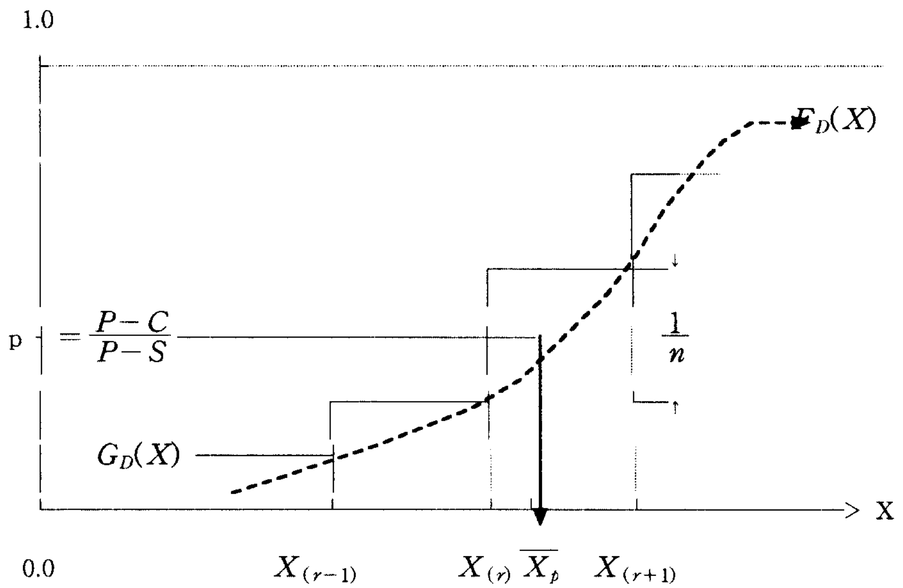

Figure 1. Cummulative Distribution Function $F_D(X)$ and

Empirical Distribution Function $G_D(X)$

Known results of this estimator include the followings for a uniform (0,1) distribution ;

$$E[X_{(r)}] = E[U_{(r)}] = r/(n+1),$$

$$V(X_{(r)}) = V(U_{(r)}) = r(n-r+1)/(n+1)^2(n+2) \text{ and}$$

$$Cov[X_{(r)}, X_{(s)}] = Cov[U_{(r)}, U_{(s)}] = r(n-s+1)/(n+1)^2(n+2) \quad \text{for } r < s.$$

Similarly, for any other distrinutions,

$$E[X_{(r)}] = F_D^{-1}\left(\frac{r}{n+1}\right),$$

$$V(X_{(r)}) = \frac{r(n-r+1)}{(n+1)^2(n+2)}[f_D(F_D^{-1}(\frac{r}{n+1}))]^{-2} \quad \text{and}$$

$$Cov[X_{(r)}, X_{(s)}] = \frac{r(n-s+1)}{(n+1)^2(n+2)} \frac{1}{f_D[F_D^{-1}(\frac{r}{n+1})].f_D[F_D^{-1}(\frac{s}{n+1})]}$$

provided $r < s$ [7].

Later these results will be used to evaluate the efficiency of the estimator of X_p .

3.2 The Second Candidate Estimator for the Optimum ; $\hat{X}_p$

As a second estimator of X_p, $\hat{X}_p$ is offered to smooth the empirical distribution function by a linear interpolation. The area between empirical distribution function $G_D(X)$ and the actual distribution function $F_D(X)$ might be considered to be a measure of deviation. A new empirical distribution function $\overline{E}$ can be obtained by interpolation which was done at the midrange. A possible realization for $\hat{X}_p$ is shown in Figure 2.

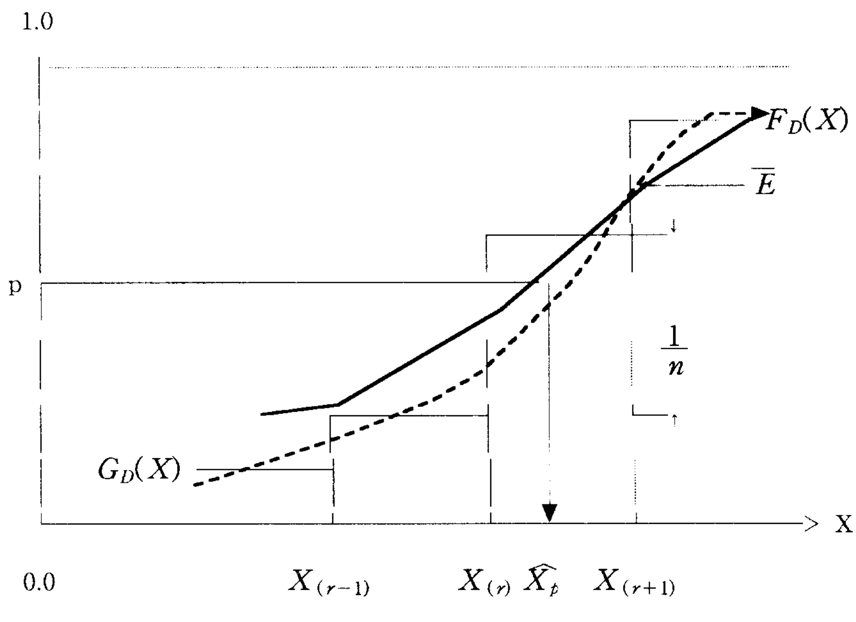

Figure 2. The Second Estimator ; $\hat{X}_{(r)}$

The area between $\overline{E}$ and $F_D(X)$) would possibly be smaller than that between $G_D(X)$ and $F_D(X)$.The second estimator can be derived and proposed as follows ;

$$\hat{X}_p = X_{(r-1)} + 1/2(2np-2r+3)[X_{(r)} - X_{(r-1)}].$$

3.3 The Third Candidate Estimator for the Optimum ; $\hat{X}_p$

As a third estimator of X_p, a pooled order statistics, combining other order statistics,

is proposed. The problem is to compare the variance of the rth order statistics. To examine this subject, it is reasonable to begin with the uniform $(0,1)$ distribution, since the expectations, variances and etc. of it's order statistics are simple and exact. Let $\hat{X}_p$ be a pooled estimator in the form of a linear combination of the order statistics, $X_{(r-1)}$, $X_{(r)}$ and $X_{(r+1)}$. Here the third estimator is of the form as ;

$$\hat{X}_p = \alpha X_{(r-1)} + \beta X_{(r)} + \gamma X_{(r+1)} \text{ , where } \alpha + \beta + \gamma = 1.$$

If demand has a uniform distribution, then the coefficients have to be $\alpha = \beta = 1/2$ and $\beta = 0$ to minimize the variance of $\hat{X}_p$[2]. This implies that a third reasonable estimator of X_p can be proposed as follows ;

$$\hat{X}_p = 1/2[X_{(r-1)} + X_{(r+1)}].$$

This is an unbiased estimator since the expectation of $\hat{X}_p$, $E[\hat{X}_p] = r/(n+1)$, yielding minimum variance,

$$V[\hat{X}_p] = \frac{2r(n-r+1)-(n+1)}{2\,(n+1)^2(n+2)}.$$

If the ratio of two proposed estimators, $\overline{X}_p$ and $\hat{X}_p$, is taken, the ratio should

always be;
$$\frac{V[\overline{X}_p]}{V[\widehat{X}_p]} = \frac{2r(n-r+1)}{2r(n-r+1)-(n+1)} > 1 .$$

This implies that $V[\overline{X}_p] > V[\widehat{X}_p]$ and consequently, it can be said that $\widehat{X}_p$ is better estimator than $\overline{X}_p$ in the case of uniform distribution but it is hard to conclude in general. Hence, a simulatin method will be considered to evaluate the proposed estimators in the followings. The variances of $\overline{X}_p$ and $\widehat{X}_p$ are drawn for $n=20$ in Figure 3. When r is close to bounds (either close to 1 or n), the variance difference becomes larger.

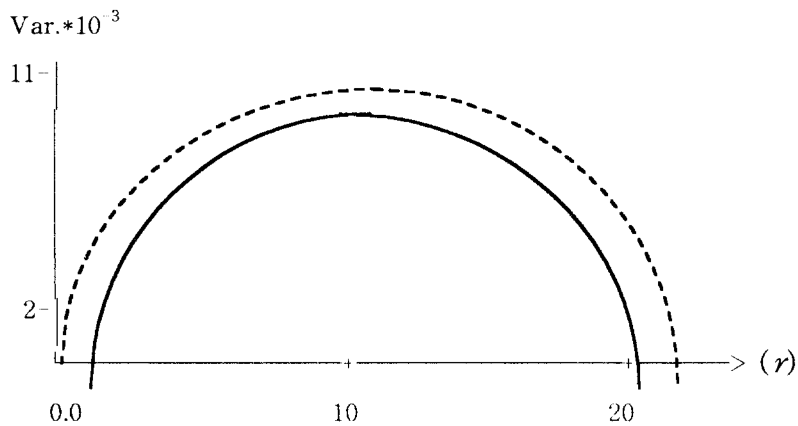

Figure 3. A Plot of Comparative Variances of $\overline{X}_p$(Dotted Line) and $\widehat{X}_p$.

4.Evaluation of the Proposed Estimators for Transition

The performances of the proposed estimators will be examined for several typical demand distributions by the simulation method in this section. After explaining the reasons for using simulation, input data analysis will be followed. The structure of the simulation program will be described and then the results of the simulation will be discussed to evaluate those proposed estimators and to find the transition procedures using an optimal estimator.

4.1 Rationale for Simulation

The purpose of the study is to find a transition procedure i.e., which estimatorof X_p to use and when, during the transition from uncertainty. The performance of an alternative transition procedure depends directly on the performance of the proposed estimator of X_p applied. A primary reason as for simulation , of course, was to study the question of how long one should continue using the minimax procedure under uncertainty before changing to a transition using a quantile estimator. Additionally, we wish further examine the question of which of the proposed estimators to use.

In summary, the reasons for the simulation will be given as follows ;

1. To find the proper period to begin the transition by the use of one of the proposed estimators leaving the uncertainty case and the minimax decision rule.
2. To provide information to supplement incomplete analytical knowledgs of order statistics regarding population quantiles.
3. To evaluate the impact of the approximations made in developing the optimal proposed estimator over various demand distributions.

In this simulation, a variety of possible forms of distributions will be considered for generality. They include a Uniform (0,20) as a standard demand distribution, a Poisson $(\lambda = 10)$ for retail demand distribution, a Normal $(35, 10^2)$ for demand distribution at factory
level, Triangular(a=0, b=10, c=20), Triangular(a=0, b=12, c=18) and Triangular(a=0, b=14, c=16)
distributions as left-skewed distributions and right-skewed distributions with long tails such as Weibull(ν=0, α=1, β=0.5), Weibull(ν=0, α=1, β=1) and Weibull(ν=0, α=1, β =2) for some retail and wholesale situations[1] where three parameters are ν, which

is the location ; $\alpha(>0)$, which is the scale ; and $\beta(>0)$, which is the shape parameter.

4.2 Input Data Analysis for the Simulation

Input data is one of the most important factor in a simulation study. There should be a great emphasis on procedures that are statistically valid rather than using the procedures that requires least amount of data[4]. Therefore these input data should be tested for the validation of the model before using them in the simulation. In this study, input data include the demand distributions such as Uniform, Normal, Poisson, Weibull and Triangular distributions mentioned previously. The statistical goodness of fit test can be useful for testing of the assumed model.

Now consider only input data generated by a Weibull(ν =0, α =1, β =0.5) distribution. The assumption of Weibull distribution will be tested using these data by the method of χ^2 goodness of fit test. The other cases can be easily proved since they are quite simple cases.

Suppose an arbitrary random sample of size 50, X_1, X_2, $\cdots$,X_{50} , that are spanned by the generator of Weibull random variates as follows [5];

$$X_i = \alpha[-\ln(1-R_i)]^{1/\beta} \text{ , where } R_i \text{ is from } Uniform(0,1).$$

A data set of Weibull random variates will be given in Appendix A-1. These observations are assumed to come from the Weibull distribution with pdf given by ;

$$f(x) = (\beta/\alpha)(x/\alpha)^{\beta-1}\exp[-(x/\alpha)^{\beta}], x \geq 0.$$

From this, the likelihood function can be shown as follows ;

$$L(\alpha,\beta) = (\frac{\beta^n}{\alpha^{\beta n}})[\prod x_i^{(\beta-1)}]\exp[-\sum(\frac{x^i}{\alpha})^{\beta}] \text{ .}$$

To find the maximum likelihood estimates of α and β , after taking the partial derivatives with respect to α and β and setting each zero, we will solve the following nonlinear equation as ;

$$f(\beta) = \frac{n}{\beta} + \sum \ln x_i - (n\sum x_i^{\beta}\ln x_i)/(\sum x_i^{\beta}) = 0 \text{ . Then we have } \alpha = (\frac{1}{n}\sum x_i)^{1/\beta} \text{ [6].}$$

A numerical analytic technique is necessary to find an approximate numerical solution in the above nonlinear equation. The Newton Raphson method will be applied as follows ;

$$\beta_j = \beta_{j-1} - f(\beta_{j-1})/f'(\beta_{j-1}) \text{ with } \beta_0 = \frac{\overline{X}}{s} \text{ and sttoping condition,}$$

$$|f(\beta_j)| < \varepsilon = 0.001 .$$

For these data, n=50, $\overline{X}$ = 1.908, s = 3.513, so that β = 0.543 . After four iterations, β_4 = 0.510 $\cong$ 0.5 and α = 1.012 $\cong$ 1.0 are the approximate solutions. Table 1 contains the needed values to complete each iteration that is executed by the numerical method which is written in FORTRAN . The detailed computer program can be accessed from the author.

Table 1. Iterative Estimation of Parameters for the Data

j	β_j	$f(\beta_j)$	$f'(\beta_j)$	β_{j+1}
0	.543	-9.673	-282.718	.509
1	.509	.489	-312.121	.510
2	.510	.001	-310.662	.510
3	.510	.000	-310.559	.510

Now the hypothesis to be tested are as follows ;

H_0 : X_i is Weibull (ν = 0, α =1, β = .5) distributed versus H_1 : not H_0 .

To test the data as Weibull distributed, let k=8 class intervals, so that each interval will have the equal probability, say, p=.125. The endpoints of the class intervals, denoted by a_i, can be found by a_i = α [-ln(1-ip)] for i=1, ··· ,7. They are given as follows ; .018, .083, .221, .480, .962, 1.922, and 4.324. The first interval is [0., .018) where the observation frequency(Oi) is 6 and expected frequency(Ei) is 6.25, so that the contribution to the χ^2 statistic value, $(O_i-E_i)^2$ /E_i , is .01. The χ^2 statistic value is .24. Since the test statistic is less than $\chi^2_{5,.05}$ = 11.1 , H_0 can not be rejected. Therefore, it can be concluded that the Weibull assumption is valid and the Weibull generator can be used to generate the Weibull input data for the simulation.

4.3 Discussion of the Simulation Results

In every simulation run, inventory stocking decisions are made successively using one particular demand distribution and one particular value of the quantile, $p=(P-C)/(P-S)$, say, demand generated from a normal parent quantiles for quantile 0.1,… etc. Each run consisted of 33 successive decision periods starting with no demand information other than the range estimate, and replicated 50 times. The computer simulation package can be accessed from the author. In this thesis, the algorithm for the simulation program can be only described in the following ways ;

1). Define a demand distribution having maximum demand and the quantile p given by $(P-C)/(P-S)$ where F is the selling price, C is the cost of goods and S is the scrap value.

2). Compute the minimax cost for the case of uncertainty and ideal cost for the case of risk.

3). Generate a random demand variates from the specified distributions.

4). Find the single-period cost according to each of estimators such as *Minimax*, *Ideal*, $\overline{X}_p$, $\hat{X}_p$ and $\tilde{X}_p$ by comparing their inventory levels with the demand generated for this period. Record the cost.

5). Repeat step 3) and 4) for 33 periodss.

6). Again beginning from the first period, repeat step 5) for 50 replications and at the end of 50 replications find the expected cost.

7). Compute the average expected cost and then tabulate the results.

The simulation results will be given and evaluated in the followings. The quantile value will be identified by p in presenting these simulation results. Table 2 in Appendix A-2 shows that the average expected cost with the normalization cost given by $(R-ID)/ID$ in the paranthesis where R means a proposed estimator cost and ID is ideal case cost. As a special case, if we pursue the Uniform demand distribution in Table 2, the minimax cost and Ideal cost would match each other because the ideal solution is always same as the minimax cost solution given by $[(P-C)/(P-S)]D_{\max}$ whenever the maximum demand $D_{\max}$ is determined exactly.

For a distribution from Table 2, the normalization costs over all quantile p will be calculated for every R_i where R_i is $\overline{R}$, $\hat{R}$, or $\tilde{R}$. Repeat this procedure for every distribution and then they will be tabulated in Table 3. Table 3 shows the mean normalization costs and its confidence intervals for every distribution.

Table 3. Mean Normalization Costs and Confidence Intervals

Est.\ Dist.	Uniform	Normal	Weibull	Triangular	Poisson	Overall
$\overline{R}$	.1161(.0343,.1979)	.1073(.0284,.1863)	.0518(.0232,.0803)	.1304(.0411,.2198)	.0836(.0162,.1509)	.4892
$\hat{R}$	.1201(.0941,.1461)	.0966(.0670,.1261)	.0674(.0264,.1085)	.1244(.1072,.1417)	.0746(.0370,.0901)	.4831
$\tilde{R}$	.1058(.0712,.1403)	.0982(.0708,.1256)	.0416(.0134,.0697)	.1224(.0922,.1527)	.0636(.0370,.0901)	.4316

A statistical test for the null hypothesis H_0 : $R_i = 0$ for all i i.e., there is no difference among those proposed estimators, should be performed at first. For the data in Table 3, proc anova for two-way factorial design is carried out. Since the null hypothesis is rejected under $\alpha = 0.05$ (p value = 0.0467) as being shown in Table 4, it can be concluded that there is significant difference among those estimators. Futhermore, in the case of distribution factor there should be significant difference as to the distribution form.

Table 4. *ANOVA* Table for the Normalization Data

Factor	DF	Anova SS	Mean Square	F Value	pr > F
Estimator	2	0.00040048	0.00020024	4.60	0.0467
Distribution	4	0.01045770	0.00261442	60.10	0.0001
Error	8	0.00034801	0.00004350		
Total	14	0.01120619			

On the bais of the simulation results and the statistical analysis, the relatively favorable transition procedure is to apply $\hat{R}$ for Normal and $\tilde{R}$ for Uniform, Weibull, Triangular and Poisson distributions as being shown in Table 3. The mean normalization costs and confidence intervals using the proposed estimators shown in Figure 4 was plotted by the proc plot of *SAS*.

Referring to the Table 3 and Figure 4, the third estimator, $\tilde{R} = \tilde{X}_p$ give the minimum normalization cost solution which corresponds to the minimum expected cost solution and also the confidence interval of this estimator is less than that of any other estimator in most cases of considered distributions. Therefore, the estimator, $\tilde{X}_p$ is recommended as the best(optimal) estimator. Also it has been shown that the optimal transition period is the seventh period for overall quantiles in a previous study[2].

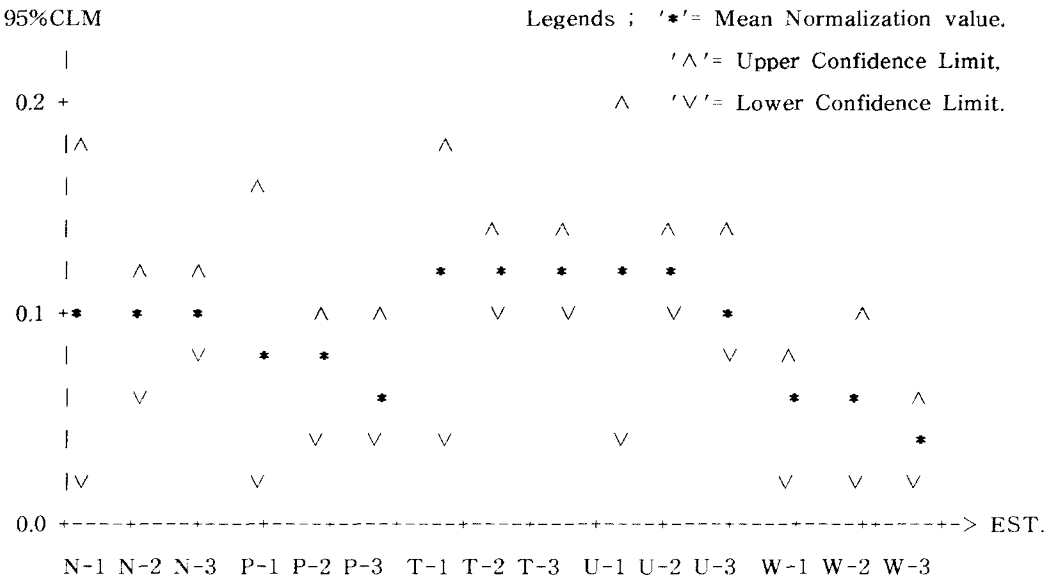

Figure 4. Plot of Mean and Confidence Intervals Using the Proposed Estimators Where N-1 stands for Applying $\bar{R}$ to Normal and P-2 stands for applying $\hat{R}$ to Poisson and W-3 stands for applying $\tilde{R}$ to Weibull Distribution, etc.

To see the effect on the very right tail of a distribution on transition procedure, $p \geq 0.95$ can be used for all distributions. At this quantile, the ***Minimax*** cost is below $\overline{R}$, $\hat{R}$ and $\tilde{R}$ costs except the Poisson distribution. Hence a transition procedure algorithm using the optimal estimator, $\tilde{X}_p$ can be proposed in the following ways ;

a). Approximate the range of a specified demand distribution.

b). Set $p=(P-C)/(P-S)$.

c). If $p \geq 0.95$ go to step e), otherwise go to step d).

d). Apply the ***Minimax*** for the first 6 periods, otherwise apply $\tilde{R}$ for periods such that $7 \leq n \leq 33$, and go to step f).

e). Apply the ***Minimax*** for all periods.

f). Analyze data i. e., may fit a probability distribution for the data and begin to make a decision under risk.

5. Conclusions

For the perishable inventory model, a nonparametric decision procedure that allows transition from uncertainty has been developed. Three candidate estimators using the order statistics were proposed and applied to the model for finding the minimum expected cost solutions. Testing these estimators for several cases which represents the demand distributions in general, it was found that excluding cases such as the very tail quantiles, the pooled estimator, $\tilde{R}= \tilde{X}_p$ provided essentially the better estimates than any other estimators over quntiles, p = 0.1,0.2,···, 0.9 even though $\overline{R}$ and $\hat{R}$ worked well. Especially, at the very right tail quantiles such that $p \geq 0.95$, the ***Minimax*** estimator gives better solutions than any other estimators.

Since one of the most important factor in a study using the simulation method is an analysis of the input data, statistical tests should establish the basis for any generation of random variates concerning the assumption of a demand distribution before simulation. Consequently the distributional assumption with the associated parameter estimates were tested to be valid by χ^2 goodness of fit test.

The statistical test using *ANOVA* asserts the fact that there is significant difference among the estimators under the significance level, α = 0.05. To find the optimal estimator, the minimum expected cost, its normalization cost to the ideal cost and confidence interval for the mean normalization cost were considered as performance measures for evaluating the proposed estimators. On the basis of the evaluation results, the estimator $\tilde{R}$ is recommended as the optimal estimator. Also a transition procedure algorithm for the perishable inventory model has been finally proposed to make the optimal decision from uncertainty.

For further study, some work can be afforded to improve the nonparametric approaches and expected to cover all possble kinds of demand distributions.

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Appendix A-1. A Data set of Weibull($\nu=0, \alpha=1, \beta=0.5$) Random Variates.

.379	6.342	.227	3.331	.126	2.016	.060	1.281	.022	.826
.003	.528	13.949	.328	5.012	.193	2.846	.103	1.757	.047
1.125	.014	.725	.001	.460	9.251	.282	4.215	.162	2.451
.083	1.536	.035	.988	.009	.635	.0001	.400	6.949	.241
3.543	.135	2.124	.066	1.346	.025	.867	.005	.555	17.684

Appendix A-2. Table 2 ; Average Expected Costs and Their Normalization Costs to the Ideal Cost, $(R-ID)/ID$, over 33 Periods and 50 Replications, where p is Quantile, $M=Minimax$, $\overline{R}=\overline{X}_p$, $\hat{R}=\hat{X}_p$, $\tilde{R}=\tilde{X}_p$, and ID = Ideal Cost.

p	M	ID	$\overline{R}$	$\hat{R}$	$\tilde{R}$
* Uniform(0, 20) Distribution ;					
0.1	3.581(.000)	3.581(.000)	3.963(.107)	4.015(.121)	3.911(.092)
0.2	6.334(.000)	6.334(.000)	6.782(.071)	6.883(.087)	6.769(.069)
0.3	8.335(.000)	8.335(.000)	8.866(.064)	9.125(.095)	9.060(.087)
0.4	9.771(.000)	9.771(.000)	10.342(.058)	10.694(.094)	10.462(.071)
0.5	10.225(.000)	10.225(.000)	10.747(.051)	11.177(.093)	10.967(.073)
0.6	9.449(.000)	9.449(.000)	10.331(.091)	10.624(.124)	10.551(.117)
0.7	8.441(.000)	8.441(.000)	9.007(.067)	9.443(.119)	9.175(.087)
0.8	6.289(.000)	6.289(.000)	7.218(.148)	7.327(.165)	7.553(.201)
0.9	3.609(.000)	3.609(.000)	5.009(.388)	4.269(.183)	4.167(.155)
* Normal(35, 10^2) Distribution ;					
0.1	11.900(.694)	7.024(.000)	7.969(.135)	8.026(.143)	8.143(.159)
0.2	19.073(.562)	12.214(.000)	12.705(.040)	12.745(.043)	12.927(.058)
0.3	22.768(.637)	13.911(.000)	14.791(.063)	15.232(.095)	15.129(.088)
0.4	22.344(.409)	15.863(.000)	16.485(.039)	16.701(.053)	16.925(.067)
0.5	19.601(.285)	15.250(.000)	16.231(.064)	16.630(.090)	16.571(.087)
0.6	17.577(.073)	16.375(.000)	16.971(.036)	17.505(.069)	17.609(.075)
0.7	13.922(.023)	13.609(.000)	14,761(.185)	15.059(.107)	15.047(.106)
0.8	11.047(.046)	11.576(.000)	12.250(.058)	12.856(.111)	12.630(.091)
0.9	7.180(.009)	7.117(.000)	9.580(.346)	8.244(.158)	8.205(.153)
* Weibull($\nu=0$, $\alpha=1$, $\beta=0.5$) Distribution ;					
0.1	1.073(.299)	.826(.000)	.853(.032)	.852(.032)	.843(.021)
0.2	2.036(.358)	1.499(.000)	1.546(.032)	1.549(.034)	1.517(.012)
0.3	2.977(.248)	2.385(.000)	2.445(.025)	2.461(.031)	2.420(.015)

0.4	3.961(.169)	3.387(.000)	3.461(.022)	3.489(.030)	3.439(.015)
0.5	4.012(.134)	3.538(.000)	3.655(.033)	3.713(.050)	3.653(.033)
0.6	4.274(.084)	3.944(.000)	4.089(.037)	4.165(.056)	4.085(.036)
0.7	4.639(.031)	4.501(.000)	4.782(.062)	4.819(.071)	4.671(.038)
0.8	4.669(.003)	4.657(.000)	5.080(.091)	5.180(.112)	5.051(.085)
0.9	4.340(.088)	3.990(.000)	4.515(.132)	4.751(.191)	4.465(.119)

* Triangular(a=0, b= 12, c=18) Distribution ;

0.1	3.306(.195)	2.766(.000)	3.056(.114)	3.117(.137)	3.096(.129)
0.2	5.280(.162)	4..546(.000)	4.867(.085)	4.944(.102)	4.965(.107)
0.3	6.496(.184)	5.486(.000)	5.778(.078)	5.962(.113)	5.990(.118)
0.4	6.943(.119)	6.206(.000)	6.433(.072)	6.664(.110)	6.573(.090)
0.5	6.589(.048)	6.290(.000)	6.388(.058)	6.609(.094)	6.539(.083)
0.6	5.752(.011)	5.690(.000)	5.963(.106)	6.115(.134)	6.126(.136)
0.7	5.108(.008)	5.067(.000)	5.017(.073)	5.233(.119)	5.115(.094)
0.8	3.999(.035)	3.864(.000)	3.895(.158)	3.903(.160)	4.083(.214)
0.9	2.579(.106)	2.332(.000)	2.766(.430)	2.225(.151)	2.187(.131)

* Poisson(λ = 10) Distribution ;

0.1	3.094(.415)	2.186(.000)	2.321(.062)	2.340(.070)	2.376(.087)
0.2	4.556(.097)	4.148(.000)	3.628(.125)	3.737(.099)	3.739(.099)
0.3	4.833(.062)	4.553(.000)	4.575(.005)	4.702(.033)	4..698(.032)
0.4	4.773(.161)	5.689(.000)	5.005(.120)	5.150(.095)	5.119(.100)
0.5	5.333(.040)	5.131(.000)	5.403(.053)	5.472(.067)	5.465(.065)
0.6	6.321(.120)	5.645(.000)	5.409(.042)	5.700(.010)	5.593(.009)
0.7	6.798(.504)	4.518(.000)	4.758(.053)	5.085(.126)	4..877(.080)
0.8	6.252(.480)	4.223(.000)	4.205(.004)	4.375(.036)	4..307(.020)
0.9	3.975(.578)	2.519(.000)	3.244(.288)	2.860(.135)	2.721(.080)