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Economic Order Quantity and Discount Pricing Policy for the Monopsony Related to the Weapon System Acquisition.

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Abstract

In this paper, we consider an economic order quantity(EOQ) and an optimal discount pricing policy for the monopsony related to the weapon system acquisition. In the monopsony case, a buyer wishes to maximize the profit. However, a seller wants to minimize the total inventory related cost since a buyer can determine the purchase price for the product. We develop a generalized version of EOQ model for the monopsony, including one seller-one buyer model and two seller-one buyer model. A model of buyer reaction to any given pricing scheme is developed to show that there exits a unified pricing policy which motivates the buyer to increase its ordering quantity per order, thereby reducing the joint(buyer and seller) ordering and holding costs in the system.

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1. Introduction

Many economic order quantity(EOQ) models and optimal discount pricing models are developed. The models in the literature have been suggested for determining economic order quantity in case of one seller-one buyer or one seller-multiple buyer groups. In this paper we study an economic order quantity model and an optimal discount pricing model for the monopsony which is the case of multiple seller groups-one buyer. The behavior of a monopsonist is analogous to that of a monopolist in terms of the profit maximization. The monopsonist wants to maximize the profit by purchasing the product at the minimum cost, while the monopolist maximize the profit by selling the product at the maximum price.

Therefore, the model of maximizing profit per unit time over the order quantity can be used for the buyer who is the monopsonist. From the viewpoint of seller(s), however, the cost minimization model can be used.

The contribution of our paper is that the buyer's behavior or the size of order quantity before any discount is offered can be realized in the monopsony case. Additionally we extend the model to allow the joint costs of the system to depend on the discount pricing scheme under the monopsony related to the weapon system acquisition.

Assumptions and Notation

Three assumptions made by Dolan (1978), Lal and Staelin (1984), and Dada and Srikanth (1987) are retained in the analysis. The first assumption is that the inventory policies of seller(s) and buyer can be described by a simple EOQ model based on deterministic demand, no stock outs and deterministic lead times. The second one is that the buyer's annual demand does not increase in response to a quantity discount. The third one is that both the buyer and the seller(s) know their own and each other's holding and ordering costs. In addition to these assumptions, we assume that the purchase price per unit for the buyer is equal to the unit variable cost of production for the seller(s). The production capacity of each seller is large enough for the amount of product that the buyer will require and the product is produced in equal quantity, Q at a time.

For the model of the monopsony related to the weapon system acquisition define the following variables.

B(Q) : Buyer's annual cost excluding the cost of the product associated with purchases from the seller in lot size Q for a discount pricing model.

S(Q) : Seller's annual cost excluding the cost of the product associated with purchases by the buyer in lot size Q for a discount pricing model.

J(Q) : B(Q)+S(Q), the joint cost function for both agents.

H_b : Buyer's cost of holding a unit in inventory for one year.

H_s : Seller's cost of capital per one year.

A_b : Buyer's fixed cost to place an order of any size.

A_s : Seller's fixed cost to process an order of any size.

Q : Size of orders placed by the buyer.

Q_i : Size of quantity obtained from Seller i.

P_o : Selling price for buyer's product.

P_1 : Purchase price per unit to the buyer.

D : Buyer's annual demand rate for the product sold by the seller(s).

2. One Seller - One Buyer Model

EOQ Model with One Seller-One Buyer

In this chapter we assume that the buyer follows an EOQ policy before any discount is offered. In general a buyer and a seller want to minimize the total inventory related cost during the unit time period or one year. In the monopsony, however, a buyer wishes to maximize the profit because the buyer can determine the purchase price per unit, P_1, for the product. The buyer's profit function is given by the revenue minus the total inventory related cost. In the absence of quantity discount, the buyer's annual profit function is

$$\text{Max} \quad DP_0 - (DP_1 + \frac{DA_b}{Q} + \frac{H_bQ}{2}).$$

In this case the optimal order quantity Q^* is given by the EOQ formula.

$$Q^* = \sqrt{\frac{2DA_b}{H_b}}.$$

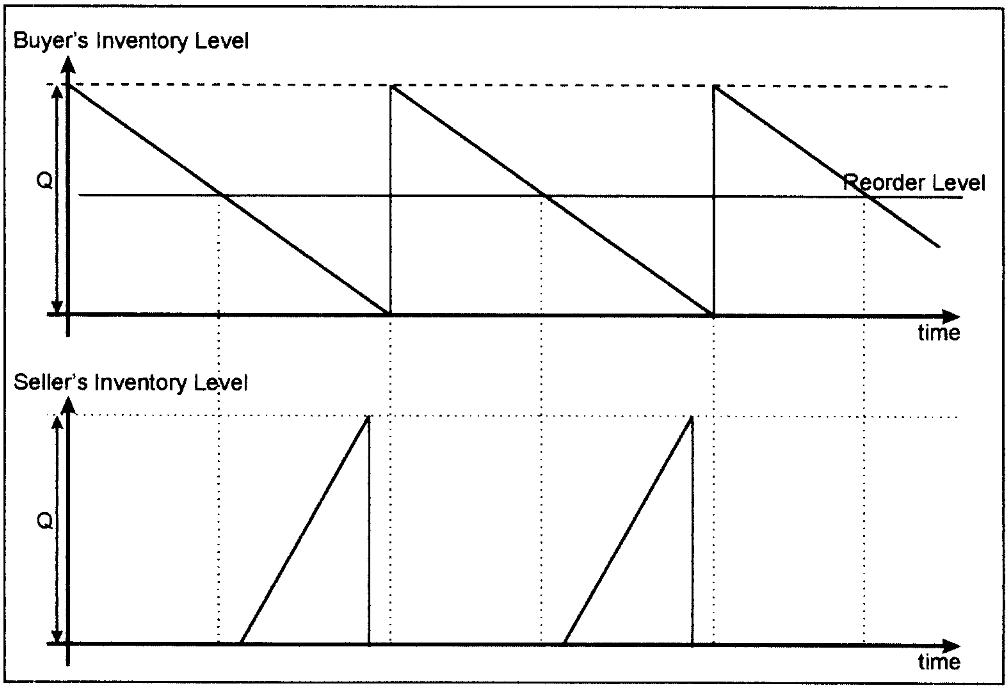
A seller needs to minimize the total cost for the product, including inventory carrying cost. The seller's total annual inventory carrying cost function is

$$\text{Min} \quad DP_1 + \frac{DA_s}{Q} + \frac{H_sQ}{2}.$$

The optimal quantity Q for the seller should be the same as the optimal quantity Q^* for the buyer because there exists only a single seller and a single buyer in this system.

The inventory system characterized by this chapter is described graphically in Figure 1.

Figure 1 Inventory Time Plots of One Seller-One Buyer



Discount Pricing Model with One Seller-One Buyer

We first solve the one seller-one buyer problem with ordering and holding costs assumed to be constant. The buyer's cost function per unit time, excluding the cost of the product, as a function of the quantity ordered is

$$B(Q) = \frac{DA_b}{Q} + \frac{H_b Q}{2}.$$

Similarly the seller's cost function per unit time is

$$S(Q) = \frac{DA_s}{Q} - \frac{H_s Q}{2}.$$

Note that the seller's costs per unit time or one year can decrease in capital costs because the buyer purchases the quantity of the product from the seller. It follows that the joint cost function $J(Q) = B(Q) + S(Q)$ is

$$J(Q) = \frac{D(A_b + A_s)}{Q} + \frac{Q(H_b - H_s)}{2}.$$

Based on the assumption that the buyer uses an EOQ rule to determine its optimal order quantity, the buyer would order Q^* which can be seen above. However, this ordering quantity (Q^*) does not minimize the joint cost function. To see this, differentiate the joint cost function and solve for Q^*_1, the order quantity that minimizes the joint cost function of the one seller and one buyer model. This yields

$$Q^*_1 = \sqrt{\frac{2D(A_b + A_s)}{(H_b - H_s)}}.$$

Since $2(A_b+A_s)D$ is greater than $(2A_bD)$ and (H_b-H_s) is less than H_b, the optimal order quantity Q^*_1 of the joint cost function for the one seller-one buyer model is greater than the optimal order quantity Q^* from the EOQ rule.

3. Two Sellers-One Buyer Model

EOQ Model with Two Sellers-One Buyer

In this chapter we assume that the production costs of each seller are the same and the production capacity of each seller is large enough for the amount of product that the buyer will require. We also assume that the buyer can obtain the required quantity from two sellers in this system. That is, the buyer's optimal order quantity Q^* is equal to the sum of the quantity(Q_{s1}) from Seller 1 and that(Q_{s2}) from Seller 2. ($Q^* = Q_{s1} + Q_{s2}$) The buyer's annual profit function and the optimal order quantity Q^* are the same as those in a single seller and a single buyer model, because the buyer wants to maximize the profit by using the same EOQ rule.

Each seller can produce and sell a part or all of the product. That depends on the seller's total inventory related cost. As illustrated in Figure 2, the total inventory related cost function of Seller 1 can be defined as follows:

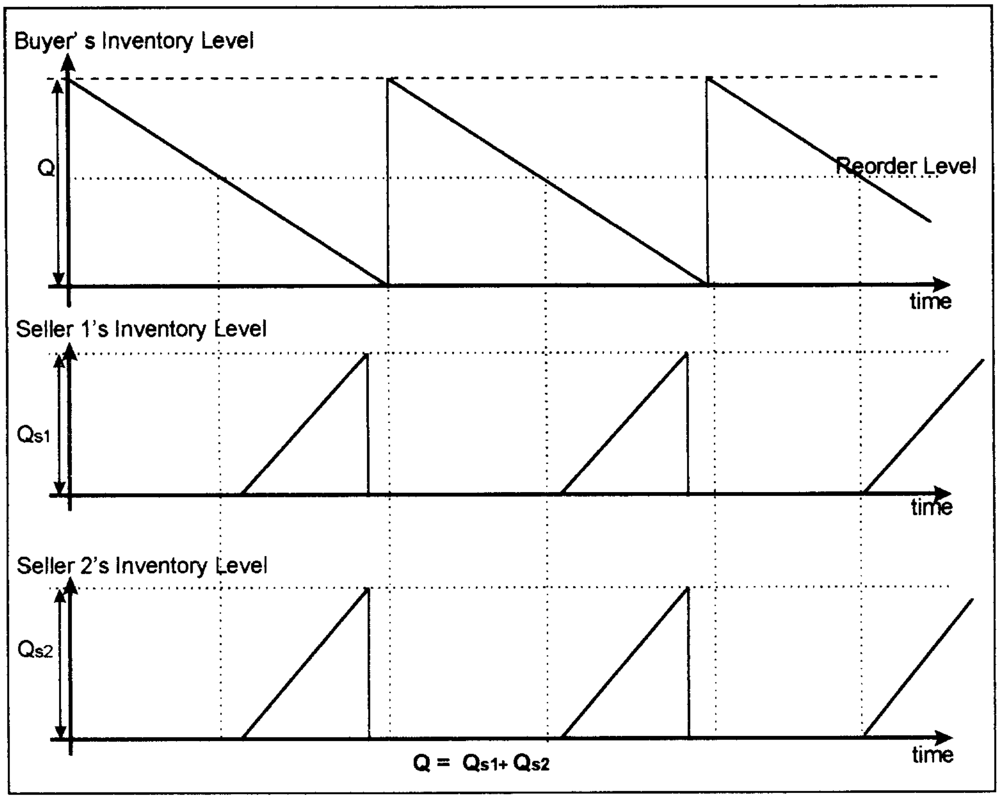
$$\text{Min} \;\; DP_1 + \frac{DA_{s1}}{Q_{s1}} + \frac{H_{s1}Q_{s1}}{2}.$$

Then the optimal quantity for Seller 1 is

$$Q_{s1} = \sqrt{\frac{2DA_{s1}}{H_{s1}}}.$$

This optimal quantity(Q_{s1}) can be determined by the ordering cost and inventory holding cost of Seller 1, and the optimal quantity(Q_{s2}) for Seller 2 can be determined by the ordering and inventory holding cost of Seller 2. Note that they are the different values of the cost function between Seller 1 and Seller 2. The inventory system for the two seller-one buyer model is described graphically in Figure 2.

Figure 2 Inventory Time Plots of Two Sellers-One Buyer



If the production costs of each seller are different, the problem can be another problem rather than the inventory problem. Suppose that quality of the product is the same. Using the above assumption, if the purchase price, P_{s1} for Seller 1 is less than the purchase price, P_{s2} for Seller 2, then the buyer will purchase all of the product which is required for the buyer from Seller 1 ($Q = Q_{s1}$), and vice versa. Therefore, this model follows the Bertrand equilibrium. Then Seller 1 and Seller 2 will have the same minimum cost of the production, as long as they survive in this system.

Discount Pricing Model with Two Sellers-One Buyer

We now extend the model to handle the case of two sellers of different sizes of quantity to sell. We assume that seller's holding costs, ordering costs and demands are the same for two sellers although these costs can differ from each other. The buyer's cost function per unit time for this two sellers-one buyer model is the same as that for the one seller-one buyer model. The seller's cost function per unit time can be defined as follows:

$$S(Q_1) = \frac{DA_{s1}}{Q_{s1}} - \frac{H_{s1}Q_{s1}}{2},$$

$$S(Q_2) = \frac{DA_{s2}}{Q_{s2}} - \frac{H_{s2}Q_{s2}}{2}.$$

where $S(Q_1)$, $S(Q_2)$ are the cost functions for Seller 1 and Seller 2 respectively. The seller's total cost function per unit time or one year is the sum of each seller's annual cost function.

$$\begin{aligned} S(Q) &= S(Q_1) + S(Q_2) \\ &= \frac{DA_s}{Q_{s1}} + \frac{DA_s}{Q_{s2}} - \frac{H_s(Q_{s1} + Q_{s2})}{2} \\ &= \frac{DA_sQ}{Q_{s1}Q_{s2}} - \frac{H_sQ}{2}. \end{aligned}$$

since $Q = Q_{S1} + Q_{S2}$. Then the above equation can be rewritten as:

$$S(Q) = \frac{K_sA_sD}{Q} - \frac{H_sQ}{2},$$

where $$K_2 = \frac{Q^2}{(Q_{s1}\ Q_{s2})}.$$

The joint cost function for the two sellers-one buyer model, $J(Q) = B(Q) + S(Q)$ is

$$J(Q) = \frac{D}{Q}(A_b + K_2\ A_s) + \frac{Q}{2}(H_b - H_s).$$

In order to obtain the optimal order quantity that minimizes the joint cost function of the two sellers-one buyer model, differentiate the joint cost function and solve for Q^*_2. This yields

$$Q^*_2 = \sqrt{\frac{2D(A_b + K_2 A_s)}{(H_b - H_s)}}.$$

Since K_2 is greater than one, the optimal order quantity Q^*_2 for the two sellers-one buyer model is greater than the optimal order quantity Q^*_1 for the one seller-one buyer model.

Note that the optimal order quantity Q^* which is determined by using EOQ rule for this two sellers-one buyer model does not minimize the joint cost function for this two sellers-one buyer model.

4. Multiple Sellers-One Buyer Model

EOQ Model with Multiple Sellers-One Buyer

Assume now that there are a number of sellers and only one buyer for this typical monopsony case and the production costs of each seller are the same and the production capacity of each seller is large enough for the amount of product that the buyer will purchase. Assume also that the buyer's optimal order quantity Q^* is equal to the sum of the order quantity from all the sellers in the system. That is, $Q^* = \sum_{i=1}^{n} Q_{si}$.

The buyer's annual profit function and the optimal order quantity $Q*$ are the same as before, because the buyer use the same EOQ rule to determine the optimal order quantity. The total inventory related cost function per year of Seller i is

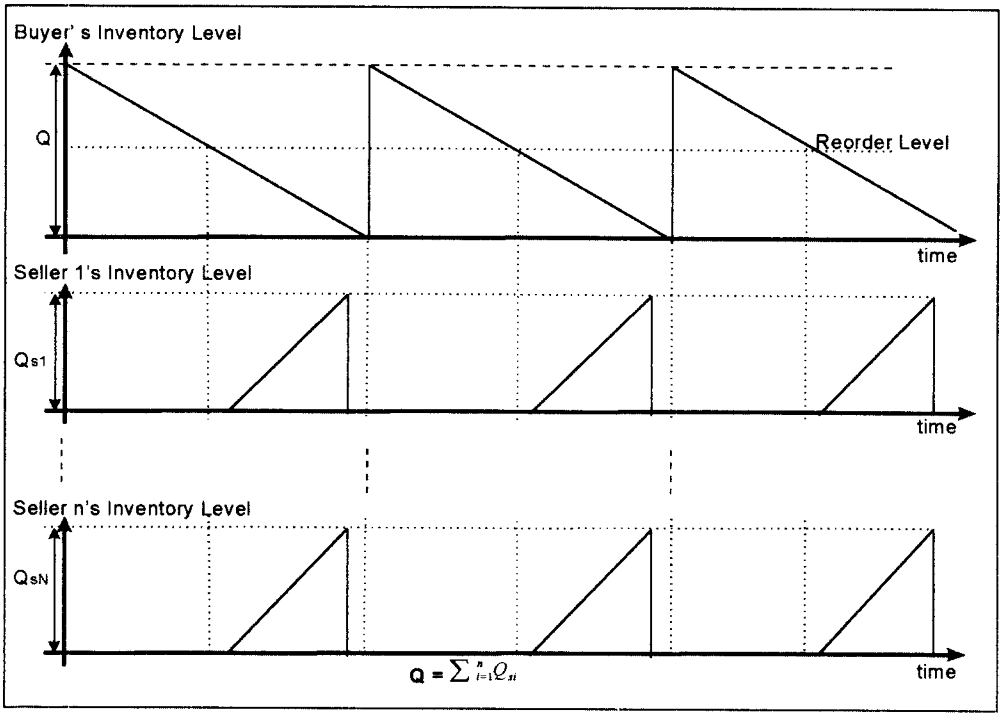
$$\text{Min} \quad DP_1 + \frac{DA_{si}}{Q_{si}} + \frac{H_{si}Q_{si}}{2}.$$

Therefore, the optimal order quantity for Seller i is

$$Q_{si} = \sqrt{\frac{2DA_{si}}{H_{si}}}.$$

The inventory system for the N sellers-one buyer model is described graphically in Figure 3.

Figure 3 Inventory Time Plots of N Sellers-One Buyer



Discount Pricing Model with Multiple Sellers-One Buyer

We now extend the model to handle the case of N sellers of different sizes of quantity to sell. In other words, we have the generalized version of the model for the multiple sellers-one buyer case. Let there be 1,2,.........,n sellers in this system. We assume that the seller's holding costs, ordering costs and demands are the same for N sellers although these costs can differ. The buyer's cost function per unit time for this N sellers-one buyer model is the same as that for one seller-one buyer model.

Now we have N number of cost functions for the sellers, because we have N number of sellers in this system. The cost functions per unit time or one year for each seller become

$$S(Q_1) = \frac{DA_{s1}}{Q_{s1}} - \frac{H_{s1}Q_{s1}}{2},$$

$$S(Q_2) = \frac{DA_{s2}}{Q_{s2}} - \frac{H_{s2}Q_{s2}}{2},$$

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$$S(QN-1) = \frac{DA_{sN-1}}{Q_{sN-1}} - \frac{H_{sN-1}Q_{sN-1}}{2},$$

$$S(QN) = \frac{DA_{sN}}{Q_{sN}} - \frac{H_{sN}Q_{sN}}{2}.$$

The total seller's cost function per unit time is the sum of each seller's cost function per the unit time period. That is, $S(Q) = \sum_{i=1}^{n} S(Q_i)$.

$$S(Q) = \frac{DA_s}{Q_{s1}} + \frac{DA_s}{Q_{s2}} + \ldots + \frac{DA_s}{Q_{sN}} - \frac{H_s}{2}\sum_{i=0}^{n} Q_{si}$$

$$= DA_s \sum_{i=1}^{n}\frac{1}{Q_{si}} - \frac{H_sQ}{2}.$$

since $Q = \sum_{i=1}^{n} Q_{si}$. Then the total seller's cost function per unit time or one year can be rewritten as:

$$S(Q) = \frac{K_nA_sD}{Q} - \frac{H_sQ}{2},$$

where $$K_n = Q(\sum_{i=1}^{n}\frac{1}{Q_{si}}).$$

The joint cost function for the N sellers-one buyer model, $J(Q) = B(Q) + S(Q)$ is

$$J(Q) = \frac{D}{Q}(A_b + K_n A_s) + \frac{Q}{2}(H_b - H_s).$$

Setting the first derivative of above equation with respect to Q equal to zero, we obtain the joint economic lot size Q^*_n is

$$Q^*_n = \sqrt{\frac{2D(A_b+K_nA_s)}{(H_b-H_s)}}.$$

Since K_n for the N sellers-one buyer model is greater than K_2 for the two sellers-one buyer model, the optimal order quantity Q^*_n for the N sellers-one buyer case is greater than the optimal order quantity Q^*_2 for the two seller-one buyer case.

Notice that the optimal order quantity Q^*_i for the i seller-one buyer case is greater and greater when the number of seller is larger and larger.

5. Conclusion

In this paper an EOQ model and an optimal discount pricing model for the monopsony are studied for a unit time period. In the monopsony, the seller's and buyer's behavior can be realized before any discount is offered. The optimal order quantity Q* of the buyer in three cases which are mentioned in this paper, is obtained from the well-known Wilson lot sizing formula(Harris, 1915) with no price discounts available.

Additionally we develop the generalized version of a discount pricing model for the N sellers-one buyer case. The optimal order quantity that minimizes the joint cost function of the one seller-one buyer model is the same as the order quantity from Lal and Staelin's one seller-one group of homogeneous buyers model. However, Lal and Staelin do not have the generalized version of the model for the multiple sellers-one buyer case.

Finally, the model developed is as abstraction of reality. In our case, the seller's holding costs, ordering costs and demands are assumed to be the same each other in the system. This assumption is abstraction of reality which can be relaxed in future studies.

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