

# Two Queue Single Server Model for the DQDB MAN\*

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## Abstract

This paper presents an approximate analytical model to estimate the mean packet waiting times at the stations in the IEEE 802.6 subnetwork of a metropolitan area network. Each station is modeled as a two queue single server system, which serves data packets and requests from downstream stations according to the DQDB protocol. The model estimates the mean waiting time of the requests and, in turn, using the discrete time work conservation law, estimates the mean waiting time for packets. Simulation experiments shows that the model accurately works even under very high traffic loads.

## 1. Introduction

The Distributed Queue Dual Bus (DQDB) protocol is the medium access mechanism for the IEEE 802.6 MAN. There is interest in developing analytical models to predict the performance of the protocol under various network configurations. However, the high degree of interaction between the stations in a network using the DQDB protocol makes performance analysis a challenge.

There is considerable work on the DQDB protocol, and a comprehensive review of the work is provided in Mukherjee and Bisdikian [7]. Most of the work, however, has been devoted to aspects such as fairness issues, and improvement of the protocol. A number of studies have been conducted using simulation. There is relatively little work on analytical modeling of performance. Potter and Zukerman [8] consider a queueing system which involves several distributed local queues and a central server which performs round robin processor sharing and the resulting queue is analyzed under the assumption of a uniform traffic profile across the network. Their model provides only

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network-wide performance measures for each priority level since the model does not identify the individual stations. Moreover, their model does not apply to the system where stations are widely separated since the model ignores the propagation delay.

Mukherjee and Banerjee [6] present a Markov chain based analytical model for the description of the network provided that all the stations are equally spaced. The system state descriptor consists of the req-counter and cd-counter values at the various stations, the status of all of the busy and request bits in transit over the entire network, and the queue lengths at the various stations. The model can predict an individual station's throughput and mean packet delay for known loading patterns although the state space complexity becomes quite large even for a small size system.

Tran-Gia and Stock [9] present an approximate analysis of a DQDB network in which each station in the network is modeled as a system of nested M/G/1 queues. Bisdikian [2] presents the single-buffer DQDB network model in which all upstream (downstream) stations of the tagged station is collapsed down into an aggregate station, called L\_\_NET(R\_\_NET), which generates a busy slot on bus A with probability  $\alpha$  (a request on bus B with probability  $\beta$ ). An imbedded Markov chain is formulated describing the number of requests registered at the instant of a packet arrival. The mean waiting time for a packet at each station in the DQDB network is obtained by considering the statistics of the req-counter and the cd-counter at a station. This model is the only one that explicitly considers the movement of the counters in a DQDB station even though it only considers stations with a single buffer.

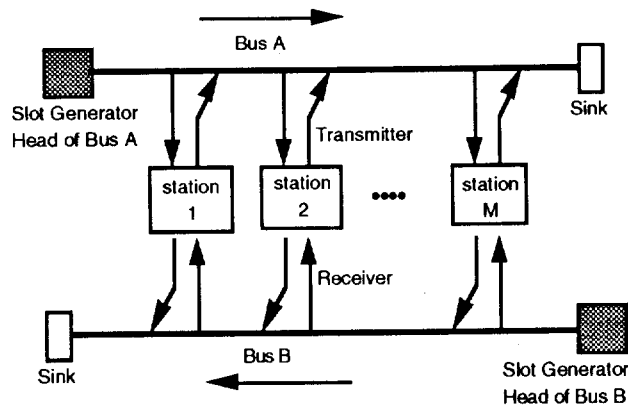
In Bisdikian [4], a priority queueing system with a Quasi-Gated Service Discipline is proposed. The expression for the mean packet waiting time contains two unknown probability terms. The upper and lower bound of the mean waiting time are obtained by bounding these unknown terms.

Except the work of Mukherjee and Banerjee [6], none of the models takes into consideration the distance between stations. Hence, any possible unfairness that are related to the distance between stations cannot be captured. All models reviewed above, except Potter and Zukerman [8], deal with individual station performance measures. Thus, they are able to capture the relative position-dependent characteristics of various stations in a DQDB network.

The organization of the paper is as follows: the DQDB protocol is summarized in the remainder of this section. Section 2 presents the model. Section 3 presents the results of some simulation experiments. Section 4 concludes the paper with a summary.

The DQDB protocol is a medium access control mechanism for forming a global First-Come-First-Served (FCFS) transmission queue among stations. It controls access to the shared medium for the stations that wish to transmit packets. It is based on a dual bus architecture, with transmissions taking place on two buses, bus A and bus B, as shown in Figure 1. The mechanism

Figure 1. Dual Bus Topology



for transmissions on bus A is identical to that bus B. Moreover, transmissions on bus A are independent of transmissions on bus B. Hence, for ease of presentation, we only discuss the transmissions on bus A. For any arbitrary station  $m$  in a network with  $M$  stations, we refer to stations 1 through  $m-1$  (stations  $m+1$  through  $M$ ) as its upstream (downstream) stations.

In the DQDB network, the upstream heads of the two buses (HOBs) continuously generate slots of fixed duration (the same in both buses) that travel along their respective buses. The duration of a slot is equal to the size of a data packet. Each slot contains, in its header, a *busy* bit and three *request* bits. The busy bit indicates whether or not the slot is occupied by a packet, while each request bit is used for sending a request for future packet transmission for each priority level. We consider a single priority level in this paper, and therefore we assume that each slot contains a single request bit. We also consider the queue arbitrated (QA) slot access function only.

Every station maintains two buffers for each bus: the Transmission Buffer (TB) for the "distributed queue" and a local buffer. The local buffer stores the arriving packets according to the FCFS discipline. Only the packet at the head of the local buffer joins the TB. Packets in the individual TBs compete for transmission on the bus. The Distributed Queue is thus a protocol for forming a global distributed queue among ready stations.

Every station has a request counter (req-counter), a countdown counter (cd-counter), and a local request queue counter (q-counter) for packet transmissions on a bus. These counters actually govern the DQDB protocol. The req-counter maintains the number of requests from downstream stations. It is increased by one whenever a request bit set passes on bus B and decreased by one whenever an empty slot passes on bus A until it becomes 0. The cd-counter maintains the number of empty slots that a station should pass on before it transmits its packet. As soon as a packet in a station enters the TB, the station copies the value of req-counter into cd-counter and sets the value of req-counter

to zero. The station will then continuously decrease the cd-counter by one for every empty slot passing by bus A until it becomes zero. It will then set the busy bit in the next empty slot and transmit its packet. The q-counter keeps the number of local packets which arrived at a station and not yet sent a request on bus B. It is increased by one whenever a packet enters the TB and decreased by one whenever the station sets a request bit on bus B until it becomes 0.

## 2. The model

We now consider a system in which all stations are stable. We are interested in determining the mean access time experienced by a packet at an arbitrary station,  $m$ . The access time of a packet is defined as the time interval from the instant the packet appears in the system until transmission starts. In this model, there are two classes of customers denoted as class a and class b customers. Class a customers are the packets for transmission at station  $m$  which arrive according to a Poisson process at rate  $\lambda_m$ . Class b customers are the requests registered at station  $m$ ; these requests are assumed to be registered (arrive) according to a Bernoulli process at rate  $\beta_m$ . The two classes of customers wait in different queues for service, and within each queue service is provided on a FCFS basis. For convenience, in this section we drop the station index  $m$  from the notation. Thus, for instance we will use  $\lambda$  in place of  $\lambda_m$ , and  $\beta$  in place of  $\beta_m$ . Since we are considering a stable system, we can estimate  $\beta$  as

$$\beta = \sum_{k=m+1}^M \lambda_k. \quad (1)$$

Let  $\alpha$  denote the probability that a slot passing station  $m$  on bus A is busy. We can estimate  $\alpha$  as

$$\alpha = \sum_{k=1}^{m-1} \lambda_k. \quad (2)$$

Equations (1) and (2) implicitly assume that each slot on bus A (bus B) is busy (contains a request) with probability  $\alpha$  (with probability  $\beta$ ) independent of the status of other slots. Also note that we assume  $\lambda_k$  is the mean arrival rate of packets at station  $k$ . If we define the service time for a packet (and for a request) as the time interval from the instant it becomes the next one to be served until it is fully served, then the service time (for either a packet or a request) is a geometric random variable with parameter  $\alpha$ . Let  $S$  denote the service time, and let  $E[S]$ , and  $E[S^2]$  denote

the first two moments of  $S$ . We have

$$E[S] = \frac{1}{1-\alpha}, \text{ and } E[S^2] = \frac{1+\alpha}{(1-\alpha)^2}. \tag{3}$$

Figure 2 presents the two queue model. In the Figure,  $Q_p$  and  $Q_r$  denote the queues for the packets and requests, respectively. The server switches from  $Q_p$  to  $Q_r$  after completing service on the packet in the TB. On switching to  $Q_r$ , all the requests present at  $Q_r$  at the switching instant are served. At this point, if the TB is nonempty, the server switches back to  $Q_p$  to serve the packet waiting in the TB. However, if  $Q_p$  is empty, the server remains at  $Q_r$  and continues to serve all the requests in  $Q_r$  that arrive prior to the arrival of the next packet at  $Q_p$ . The switchover time between the two classes is zero. We assume that the packets and requests arriving at a station during a slot interval are registered at the end of the slot. If a packet and a request arrive at an idle station in the same slot interval, we assume that the packet is served first. It is observed that this mechanism exactly represents the DQDB protocol.

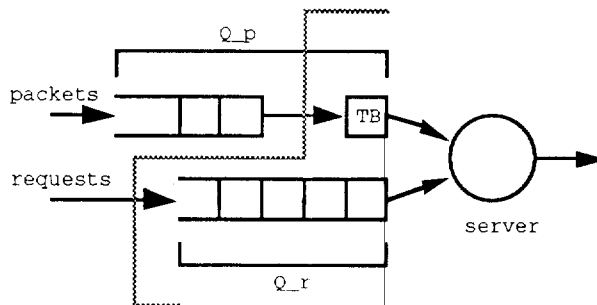


Figure 2. The two-queue single server model

Since only a fraction,  $(1-\alpha)$ , of the slots are available to station  $m$  on bus  $A$ , we observe that station  $m$  is stable only if  $\lambda + \beta < 1 - \alpha$ . This follows since at most one packet or one request is served during a slot. Therefore we have the following condition for stability:

$$\alpha + \beta + \lambda < 1. \tag{4}$$

Let  $\rho_p = \lambda E[S]$  and  $\rho_r = \beta E[S]$  denote the fraction of time the queueing system is utilized by the packets and requests, respectively. For stability, we require  $\rho = \rho_p + \rho_r$  to be less than 1. Note that this stability condition is consistent with equations (3) and (4).

We define the waiting time of a packet (request) as the time interval from the instant that a packet (request) is registered at the station until it is admitted for service. Let  $W_p$  ( $W_r$ ) denote the

waiting time for the packet (request). Since this queuing system is work conserving, we have (Bisdikian [3]):

$$\rho_p E[W_p] + \rho_r E[W_r] = \frac{\rho H + \frac{\rho_p^2}{2} + \rho_p \rho_r}{1 - \rho}, \quad (5a)$$

where

$$H = \frac{\alpha(\beta + \lambda)}{(1 - \alpha)^2}. \quad (5b)$$

Let  $R_p$  denote the access time for a packet, which is the time a packet remains in  $Q_p$  until it begins transmission. By definition,

$$R_p = W_p + S. \quad (6)$$

From equation (5a), we note that if we can compute  $E[W_r]$ , then we can obtain  $E[W_p]$ . We therefore evaluate the mean waiting time for an arbitrary request arriving at station  $m$  (call it the "tagged request"). The tagged request experiences one of the following cases.

- a) The station is idle when the tagged request arrives. The probability of this occurrence is  $1 - \rho$ .

Let  $\omega$  be the probability that at least one packet arrives in a slot. Note that  $\omega$  is also the probability that a packet is served prior to the tagged request. Since the mean service time for either a packer or a request is  $1/(1 - \alpha)$ , the mean waiting time for the tagged request is

$$E[W_r | \text{server idle}] = \frac{\omega}{1 - \alpha}. \quad (7)$$

- b) The tagged request arrives while a packet is being served. The probability of this occurrence is  $\rho_p$ . Since service times are discrete, the mean residual service time for the packet in service

is equal to  $\frac{E[S^2]}{2E[S]} - \frac{1}{2}$  (Cooper[5], and so using equation (3) for  $E[S]$  and  $E[S^2]$ ,

$$E[W_r | \text{serving a packet}] = \frac{\alpha}{1 - \alpha} + E[Q_r | \text{serving a packet}] \frac{1}{1 - \alpha}, \quad (8)$$

where  $E[Q_r | \text{serving a packet}]$  is the conditional mean length of  $Q_r$ .

- c) The tagged request arrives while another request is being served. The probability of this occurrence is  $\rho_r$ . The mean waiting time for the tagged request in this case is the sum of the following components: the mean residual service time for the request being served, the time to serve the requests which arrived ahead of the tagged request, and the time to serve a packet in the transmission buffer. The last term is based on the contents of TB when the

tagged request arrives: if TB is nonempty at this instant then the packet in TB will be served before the request; if TB is empty, then a packet is served before the request if the packet also arrives in the same slot interval in which the request arrives (this occurs with probability  $\omega$ ). Hence,

$$E[W_r | \text{serving a request}] = \frac{\alpha}{1-\alpha} + E[Q_r | \text{serving a request}] \frac{1}{1-\alpha} \\ + \Pr\{Q_p \geq 1 | \text{serving a request}\} \frac{1}{1-\alpha} + \Pr\{Q_p = 0 | \text{serving a request}\} \frac{\omega}{1-\alpha}. \tag{9}$$

From equations (7) through (9), and using Little's law to set  $E[Q_r] = \beta E[W_r]$ , we get

$$E[W_r] = \frac{H}{1-\rho_r} + \frac{(1-\rho_p)\omega + (1-\omega)\xi}{(1-\alpha)(1-\rho_r)}. \tag{10}$$

Where  $\xi = \Pr\{Q_p \geq 1, \text{ and a request is in service}\}$ .

**Remark:** Bisdikian [4] obtains an expression for  $E[W_r]$  similar to that presented in equation (10). Let  $t$  denote the time at which a slot on bus A reaches station  $m$ . The expression obtained by Bisdikian is based on a model wherein a request or a packet, which arrives at station  $m$  during  $(t-1, t]$  is admitted into the system at  $t^+$ . The resulting expression for  $E[W_r]$  in his model involves the above unknown term  $\xi$  and another unknown term  $\sigma = \Pr\{Q_p \geq 2, \text{ and a packet is in service}\}$ . Bisdikian obtains bounds on  $E[W_r]$  by deriving upper and lower bounds on the two unknowns. note that in our model, a request or a packet arriving during  $(t-1, t]$  is admitted into the system at time  $t$ . It is observed that our model represents the actual DQDB protocol more accurately.

From equations (5), (6), and (10), we have

$$E[R_p] = \frac{1}{\rho_p} \left( \frac{\rho H + \frac{\rho_p^2}{2} + \rho_p \rho_r}{1-\rho} - \rho_r E[W_r] \right) + E[S]. \tag{11}$$

The only unknown in equation (10) is  $\xi$ . Let  $q_0$  denote the long run probability that  $Q_p$  is empty; hence  $(1-q_0) = \Pr\{Q_p \geq 1\}$ . Since  $\rho_p = \Pr\{Q_p \geq 1, \text{ and a packet is in service}\}$ ,

$$1 - q_0 = \xi + \rho_p, \text{ or } \xi = 1 - q_0 - \rho_p. \tag{12}$$

Hence we can obtain  $E[R_p]$  from equations (10) through (12), if we can compute  $q_0$ . In the following subsection, we show how  $q_0$  is estimated.

## 2.1 Estimating $q_0$

Note that  $q_0$  is the long run probability that the TB of station  $m$  is empty. To estimate  $q_0$ , we consider an alternate model, Model 2, to capture the queueing mechanism at station  $m$ . In Model 2, the service time for a packet is defined as the time from the instant the packet arrives at the TB until the time the packet is fully transmitted (i.e., when the last bit of the packet is transmitted). The service time by this definition includes the time to clear all the requests which are ahead of the packet.

Let  $B$  denote the service time in Model 2, and let  $E[B]$  denote its mean. If we estimate  $E[B]$ , we can use it to compute  $q_0$  as:

$$q_0 = 1 - \lambda E[B]. \quad (13)$$

The service time,  $B$ , depends upon the number of requests that a packet finds upon its arrival at the TB and this, in turn, depends upon the number of requests that the previous packet left behind when it departed. We now focus on the number of requests that an arbitrary packet leaves behind.

**Definition** A *type b packet* is a packet which finds the server *busy* (the TB occupied by another packet) upon its arrival. A *type e packet* is a packet which finds the server *empty* (the TB empty) upon its arrival.

Type b packets wait for some time in the Local FCFS Queue, and type e packets immediately join the TB upon their arrival. Let  $p(i,j)$  denote the probability that a packet leaves behind  $i$  requests and the very next packet leaves behind  $j$  requests. Let  $p_b(i,j)$  ( $p_e(i,j)$ ) denote the probability that there are  $j$  requests in the system when a type b packet (type e packet) departs, given that there were  $i$  requests in the system when the previous packet departed. Then we can write

$$p(i,j) = (1-q_0)p_b(i,j) + q_0p_e(i,j), \quad 0 \leq i, j. \quad (14)$$

Let  $\Psi(i)$  denote the long run probability that an arbitrary packet leaves behind  $i$  requests. We have

$$\Psi(i) = \sum_{j=0}^{\infty} \Psi(j) p(j,i), \quad i \geq 0, \quad \text{with} \quad \sum_{i=0}^{\infty} \Psi(i) = 1. \quad (15)$$

Let  $B(i)$  denotes the conditional service time for an arbitrary packet given that previous packet left behind  $i$  requests, and let  $E[B(i)]$  denote its mean. Also let  $B_b(i)$  ( $B_e(i)$ ) denote the conditional service time for a type b packet (type e packet) given that there were  $i$  requests in the system when the previous packet departed. Let  $E[B_b(i)]$  ( $E[B_e(i)]$ ) denote the mean of  $B_b(j)$  ( $B_e(i)$ ). Then,

$$E[B(i)] = (1-q_0)E[B_b(i)] + q_0E[B_e(i)], \quad i \geq 0. \quad (16)$$



If we can determine  $E[B(i)]$ ,  $i \geq 0$ , then we obtain the mean service time for an arbitrary packet as

$$E[B] = \sum_{i=0}^{\infty} \psi(i) E[B(i)] \tag{17}$$

2.1.1 Estimating  $p_b(i,j)$  and  $E[B_b(i)]$

The probability  $p_b(i,j)$  is the probability that the request counter has a value  $i$  when service begins on a type  $b$  packet and has a value  $j$  when the service ends. The conditional service time,  $B_b(i)$ , for a type  $b$  packet is an integer number of slots comprising  $i+1$  empty slots and  $B_b(i)-(i+1)$  busy slots. Furthermore, the last of these  $B_b(i)$  slots is always empty since it is the slot in which the packet is transmitted. Since each slot is busy with probability  $\alpha$ , the probability that  $B_b(i)$  takes on the value  $k$ , is the joint probability that  $i$  slots among the first  $k-1$  slots are empty, and the  $k^{\text{th}}$  slot is also empty; i.e.,

$$\Pr\{B_b(i) = k\} = \binom{k-1}{i} (1-\alpha)^{i+1} \alpha^{k-1-i}, \quad k-1 \geq i. \tag{18}$$

Hence

$$\begin{aligned} E[B_b(i)] &= \sum_{k=i+1}^{\infty} k \binom{k-1}{i} (1-\alpha)^{i+1} \alpha^{k-1-i} \\ &= (i+1) \sum_{k=i+1}^{\infty} \binom{k}{i+1} (1-\alpha)^{i+1} \alpha^{k-(i+1)}. \end{aligned}$$

The summation in the above equation is equal to  $1/(1-\alpha)$ . Hence,

$$E[B_b(i)] = \frac{i+1}{1-\alpha}. \tag{19}$$

During a time  $k$ ,  $j$  requests arrive at station  $m$  with probability  $\binom{k}{j} \beta^j (1-\beta)^{k-j}$ . Thus,

$$p_b(i,j) \Big|_{B_b(i)=k} = \binom{k}{j} \beta^j (1-\beta)^{k-j}, \quad k \geq j. \tag{20}$$

From equations (18) and (20),

$$p_b(i,j) = \begin{cases} \sum_{k=i+1}^{\infty} p_b(i,j) \Big|_{B_b(i)=k} \Pr\{B_b(i)=k\}, & i+1 \geq j, \\ \sum_{k=j}^{\infty} p_b(i,j) \Big|_{B_b(i)=k} \Pr\{B_b(i)=k\}, & i+1 < j. \end{cases}$$

$$= \begin{cases} (1-\alpha)^{i+1}\beta^j(1-\beta)^{i+1-j}A, & i+1 \geq j, \\ (1-\alpha)^{i+1}\beta^j\alpha^{j-i-1}B, & i+1 < j. \end{cases} \quad (21)$$

where

$$A = \sum_{k=0}^{\infty} \binom{i+k}{i} \binom{i+1+k}{j} [\alpha(1-\beta)]^k, \text{ and}$$

$$B = \sum_{k=j}^{\infty} \binom{j-1+k}{j} \binom{j+k}{j} [\alpha(1-\beta)]^k.$$

Equation (21) does not result in a closed form solution, and so we evaluate  $p_b(i,j)$  numerically.

### 2.1.2 Estimating $p_e(i,j)$ and $E[B_e(i)]$

The service time for a type  $e$  packet depends on the number of requests it finds on its arrival, which also depends upon the number of requests that previous packet left behind. Let  $t_0$  denote the time at which the previous packet departed from the system, leaving  $Q_p$  empty. We consider probability of the number of requests present at a time  $t_n = t_0 + n$ ,  $n=0, 1, \dots$ , given that there were  $i$  requests at time  $t_0$  and no packet arrived during the time interval  $[t_0, t_n]$ . (Note that TB is therefore idle during  $[t_0, t_n]$ .) Let  $Y_{t_n}$  denote the number of requests present at time  $t_n$ . It is observed that  $Y_{t_n}$ ,  $n=0, 1, \dots$ , forms a time-homogenous Markov chain. Let

$$q^{(n)}(i, i') = \Pr[Y_{t_n} = i' \mid Y_{t_0} = i], n \geq 1. \quad (22)$$

denote the  $n$ -slot-time ( $n$ -step) transition probability for  $Y_{t_n}$ . If we evaluate  $q^{(1)}(i, i')$ , then we can recursively evaluate  $q^{(n)}(i, i')$ ,  $n > 1$ , from

$$q^{(n)}(i, j') = \sum_k q^{(n-1)}(i, k) q^{(1)}(k, j'), n > 1. \quad (23)$$

To evaluate  $q^{(1)}(i, i')$ , we consider two cases:  $i=0$  and  $i > 0$ .

Case a)  $i=0$ : If  $i=0$  at time  $t_0$ , the station will have one outstanding request at time  $t_1$  only if a marked slot passes by the station at  $t_1$ . Otherwise, the req-counter remains 0 at  $t_1$ . Thus,

$$q^{(1)}(0,1) = \beta, \text{ and } q^{(1)}(0,0) = 1 - \beta. \quad (24)$$

Case b)  $i > 0$ : Since an empty slot on bus A decreases the req-counter by 1 and a marked slot on bus B increases the req-counter by 1, it is easily seen that

$$q^{(1)}(i, i-1) = (1-\alpha)(1-\beta), \quad q^{(1)}(i, i) = \alpha(1-\beta) + (1-\alpha)\beta, \text{ and } q^{(1)}(i, i+1) = \alpha\beta. \quad (25)$$

Thus  $q^{(n)}(i, j')$  can be recursively evaluated for  $m > 1$  from equations (23), (24) and (25).

To compute  $p_e(i, j)$ , we consider the case where a type  $e$  packet arrives at the station during the interval  $[t_n, t_{n+1}]$ ,  $n \geq 0$ , given that there were  $i$  requests present at time  $t_0$ . From the memoryless property of the packet interarrival times, this probability is obtained as  $e^{-\lambda n} (1 - e^{-\lambda})$ . At time  $t_n$ , the station has  $i'$  outstanding requests with probability  $q^{(n)}(i, i')$ . If the slot passing station  $m$  at time  $t_n$  is empty (with probability  $1 - \alpha$ ) then  $i' - 1$  outstanding requests (if  $i' > 0$ ) should be cleared before the packet can be transmitted. If the slot passing station  $m$  is busy, then  $i'$  requests must be cleared before the station can transmit the packet. We showed that a packet which finds  $i' > 0$  requests upon its arrival will leave  $j$  requests with probability  $p_b(i', j)$  and will experience an expected service delay of  $\frac{i' + 1}{1 - \alpha}$ . With this result and the above discussion, it follows that

$$p_e(i, j) = \sum_{n=0}^{\infty} e^{-\lambda n} (1 - e^{-\lambda}) \left\{ \sum_{i'=1}^{\infty} q^{(n)}(i, i') \{ \alpha p_b(i', j) + (1 - \alpha) p_b(i' - 1, j) \} + q^{(n)}(i, 0) p_b(0, j) \right\}, \quad (26)$$

$$E[B_e(i)] = \sum_{n=0}^{\infty} e^{-\lambda n} (1 - e^{-\lambda}) \left\{ \sum_{i'=1}^{\infty} q^{(n)}(i, i') \left( \frac{(i' + 1)\alpha}{1 - \alpha} + i' \right) + \frac{q^{(n)}(i, 0)}{1 - \alpha} \right\}. \quad (27)$$

The expressions for  $p_e(i, j)$  and  $E[B_e(i)]$  do not lead to closed form solutions. However, these terms can be evaluated numerically.

### 2.1.3 Solution Procedure for $q_0$

In order to obtain  $E[B]$ , we first need to evaluate  $\Psi(\cdot)$ , the steady state distribution of the number of requests left behind by an arbitrary packet, which is given by the system of equations (15). Although this system of equations has an infinite dimension, we can obtain an accurate numerical estimate of  $E[B]$  by ignoring all  $p(i, j)$  entries for  $i, j > \text{some } N$  where  $N$  is a sufficiently large number. The reduced system of equations is of the form  $A \Psi = y$ , where

$$A(i, j) = \begin{cases} 1 - p(i, j) & 0 \leq i = j \leq N - 1, \\ -p(j, i) & i \neq j, 0 \leq i \leq N - 1, 0 \leq j \leq N, \\ 1 & i = N, 0 \leq j \leq N, \end{cases} \quad (28)$$

with  $\Psi = (\Psi(0), \Psi(1), \Psi(2), \dots, \Psi(N))^T$ , and  $y = (0, 0, \dots, 1)^T$ .

The terms  $p(i, j)$  and  $E[B(i)]$  in equations (14) and (16) include the unknown,  $q_0$ , which is a function of  $E[B]$ . However, all terms in equations (14) through (17) can be calculated through the following iterative procedure.

Step1 : (Initialization) For a sufficiently large number  $N$ , compute  $p_b(i, j)$ ,  $0 \leq i, j \leq N$ , using (21).

For a sufficiently large number  $N$ , compute  $q^{(n)}(i,j)$ ,  $n=2,3, \dots, N$ , using (23), (24), and (25). Also for a sufficiently large number  $N$ , evaluate  $p_e(i,j)$ ,  $0 \leq i,j \leq N$ , using (26). Obtain  $E[B_b(i)]$  and  $E[B_e(i)]$  using (19) and (27).

Step 2 : Initialize  $q_0 = \gamma$ , where  $\gamma$  is a value in  $[0, 1]$ . Set an index  $n=0$ . Initialize  $E[B^{(n)}]=0$ .

Step 3 : Set  $n=n+1$ . Compute  $p(i,j)$ ,  $0 \leq i,j \leq N$ , using equation (14), and  $\Psi(i)$ ,  $0 \leq i \leq N$ , by solving the reduced system of equations given in (28). Compute  $E[B^{(n)}]$  using (17).

Step 4 : Compute  $q_0 = 1 - \lambda E[B^{(n)}]$ . If  $|E[B^{(n)}] - E[B^{(n-1)}]| \leq \epsilon$ , stop. Otherwise, go to Step 3.

### 3. Simulation Experiments

In this section, we present twelve simulation experiments to demonstrate the accuracy of the analytical model in estimating the term  $q_0$  and the mean packet access times using the two queue model. We consider four levels of system traffic loads (0.6, 0.8, 0.93, and 0.99). At each level we conduct three experiments with different  $\alpha$  and  $\beta$  values. For each experiment, 20 independent replications were made. The elapsed time for each replication is one million slot times. The packet arrival rate at the station is normalized to packets/slot-time. Packet access times are scaled to one-slot-time. For all examples, we provide the mean packet access time,  $E[R_p]$ ; and  $q_0$ . We also

Table 1 : Comparison of analytical results with simulation

$\alpha$	$\beta$	$\lambda$	$\alpha + \beta + \lambda$	$q_0$		$E[R_b]$	
				Analysis	Simulation	Analysis	Simulation
0.120	0.360	0.120	0.600	0.823	$0.820 \pm 0.001$	0.692	$0.668 \pm 0.005$
0.240	0.240	0.120		0.801	$0.807 \pm 0.001$	0.974	$0.980 \pm 0.007$
0.360	0.120	0.120		0.781	$0.779 \pm 0.001$	1.198	$1.188 \pm 0.015$
0.150	0.450	0.200	0.800	0.588	$0.582 \pm 0.001$	2.551	$2.527 \pm 0.016$
0.300	0.300	0.200		0.552	$0.559 \pm 0.001$	3.319	$3.365 \pm 0.028$
0.450	0.150	0.200		0.524	$0.527 \pm 0.001$	3.648	$3.665 \pm 0.035$
0.186	0.558	0.186	0.930	0.336	$0.373 \pm 0.003$	11.479	$10.965 \pm 0.177$
0.372	0.372	0.186		0.309	$0.324 \pm 0.002$	15.328	$14.522 \pm 0.193$
0.558	0.186	0.186		0.290	$0.284 \pm 0.002$	15.510	$14.798 \pm 0.383$
0.1725	0.5175	0.300	0.990	0.037	$0.045 \pm 0.005$	109.053	$98.657 \pm 9.524$
0.3450	0.3450	0.300		0.036	$0.041 \pm 0.004$	119.896	$108.376 \pm 10.277$
0.5175	0.1725	0.300		0.034	$0.038 \pm 0.005$	111.545	$100.287 \pm 9.023$

provide the 95% confidence intervals for  $E[R_p]$ , and  $q_0$ .

Note that in the two-queue single-server system, all upstream (downstream) stations of an arbitrary station is collapsed into an aggregate station, which generates geometrically distributed busy slots on bus A at rate  $\alpha$  (geometrically distributed requests on bus B at rate  $\beta$ ). In a real DQDB network, however, the geometric distribution assumption for busy and request slots might be questioned. For this reason, we simulated a DQDB network with six stations. The distance between any pair of two adjacent stations is set to one slot time. The packet arrival rates at all the stations are set in such a way that a specific station can have designed values of  $\alpha$  and  $\beta$ . In Table 1, it is observed that our model accurately estimate the mean packet access time when the total system traffic load is moderate ( $\alpha + \beta + \lambda = 0.6$  and  $0.8$ ). Under high traffic loads ( $\alpha + \beta + \lambda = 0.93$  and  $0.98$ ), our model tends to overestimate the mean access times even though the deviations from the simulations are not quite high. This is probably due to the fact that under high traffic loads, the geometric distribution assumption on busy and requests slots does not accurately hold.

## 4. Conclusion

The main contribution of this paper is the development an analytical model for the performance evaluation of the DQDB network. In the model a general system is considered in which all stations in the network are stable. Only an arbitrary station in the DQDB network is considered. The upstream (downstream) stations of the considered station are collapsed into an aggregate station, which is assumed to geometrically generate busy slots on bus A (requests on bus B). Under this framework, the two queue single server model is developed in which a server (or station) serves packets and requests according to the DQDB protocol. The mean waiting time for requests is derived and in turn, using the discrete time work conservation law, the mean packet waiting time is also derived. An alternate model (called Model 2) is also presented in order to estimate the unknown term in the Model. Simulation experiments show that our analysis highly accurately estimates the behavior of the network even under heavy system traffic loads.

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