

# Three-Step Input Control Scheme for Minimization of Robot's Vibration

Wan-Shik Jang\*, Kyoung-Suk Kim\*\*, Ho-Yun Kim\*\*\*

로봇의 진동 최소화를 위한 3계단 입력 제어 기법

장완식\*, 김경석\*\*, 김호윤\*\*\*

## Abstract

This paper provides a practical control scheme called three-step input method in order to minimize both robot response time and the resulting residual vibration when the robot manipulator reaches its defined end point. This work is concerned with defining a simple practical method to utilize step inputs to achieve optimum response. The optimum response is achieved by using a self-adjusting input command function that is obtained during a real time processing. The practicality of this control scheme is demonstrated by using an analog computer to simulate a simple flexible robot and conventional servo controller. The experiments focus on point-to-point movement. Also, this method requires little computational effort through the intelligent use of conventional servo control technology and the robot's vibration characteristics.

## 1. Introduction

### 1.1 Background

Positioning is a fundamental function of robot manipulators. In order to achieve high speed and accurate positioning, it is necessary to control the robot's vibratory response in a cost effective manner. The faster the motion, the larger the vibration energies that must be controlled so that

both the robot's response time and residual vibration are minimized at the same time. This residual vibration results from inherent compliance of the robot's structural elements. These structural elements tend to be lightly damped so that any residual vibration requires considerable additional settling time before the robot is considered to have completed its task.

In recent years, a number of methods have been attempted to improve the robot's response

\* Cho-Sun University, Dept. of Mechanical Engineering, (FA(POV))

\*\* Cho-Sun University, Dept. of Mechanical Design Engineering

\*\*\* Cho-Sun Graduate School, Dept. of Mechanical Engineering

times and to control the residual vibration. These methods are computationally intensive; and hence are not cost effective. These techniques can be divided into two broad categories, dependent upon one's viewpoint. One category is based on closed-loop feedback control techniques<sup>(1)~(3)</sup>, which increases damping by a limited amount. If the inherent damping is very low, this increase may be insufficient to adequately improve the response. The other category is based on input command shaping techniques<sup>(4)~(5)</sup>, which has examined the transient vibration of robot manipulators in terms of frequency content of the system inputs and outputs. This approach inherently assumes that the system inputs are not actually transient, but are one cycle of a repeating wave form.

Thus, the objective of this paper is to develop a practical control scheme called three-step input method<sup>(7)</sup> whereby a flexible robot arm is moved from one position to another in the least amount of time with a minimum of residual vibration present when the arm reaches its defined end point.

### 1.2 Problem Definition

The basic idea of this study comes from the response of an undamped single degree of freedom (S.D.F) system to the step input as shown Fig.1. The system wants to ring about its static deflection when subjected to a single step input as shown Fig.1(a). However, if a second equal step is applied when the displacement is a maximum as shown in Fig.1(b), the system remains at rest at twice the initial static deflection, and there is no residual vibration. Thus, the system has moved from its final position in a minimum amount of time of one-half natural period, and there is no residual vibration. This is an ideal situation and defines the physical response limit.

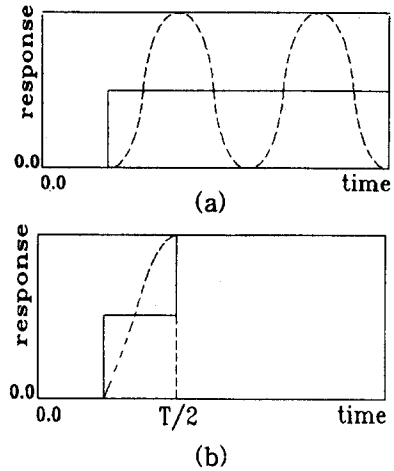


Fig. 1 Basic response of an undamped S.D.F system

a) Single step, ringing. b) Dual step, no ringing

However, real structural systems have small amounts of damping, so that it is physically impossible to eliminate residual vibrations when the defined end point is achieved using this dual step input. Thus a differently shaped input command function is required to eliminate the residual vibration when responding in the least amount of time.

## 2. Robot Model

The robot manipulator has major mechanical and electrical components. These two components are modeled separately and then combined for the system.

### 2.1 Mechanical Model

To study the problem of reducing residual vibration for point-to-point motion, the robot structure is modeled as lumped masses ( $m_1$  and  $m_2$ ) and spring( $k$ ) with damping( $c$ ), representative of a simple cartesian manipulator as shown in Fig.2. In this model, motion  $x_1$  represents the robot's base response and  $x_2$  represents the robot's

end point response. The dynamic equations for this system are

$$m_1 \ddot{x}_1 + c \dot{x}_1 + kx_1 - c\dot{x}_2 - kx_2 = F_1(t) \quad (1)$$

$$m_2 \ddot{x}_2 + c \dot{x}_2 + kx_2 - c\dot{x}_1 - kx_1 = F_2(t)$$

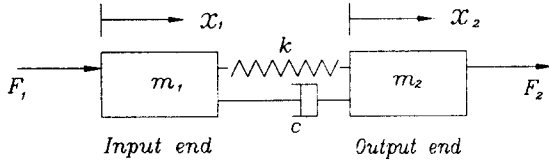


Fig.2 A simple flexible manipulator model

### 2.2 Controller Model

A conventional controller model is implemented in this scheme. This means that the force or torque generated by the servo motor (actuator) is proportional to the current passing through its armature. In addition, a back emf. is generated that is proportional to the robot's input velocity,  $\dot{x}_1$ , so that the input force  $F_1$  becomes

$$F_1 = GK_1 E - K_2 \dot{x}_1 \quad (2)$$

where,  $G$  is the system's forward gain,  $K_1$  is the force constant of the servo motor,  $K_2$  is the back emf. term dependent on the robot's base motion response  $x_1$ , and  $E$  is the input error that is given by

$$E = x_{in} - x_1 \quad (3)$$

where,  $x_{in}$  is the input command function.

### 2.3 Robot Manipulator Model

Now, if Eq.(1), (2), and (3) are combined when  $F_2 = 0$ , we obtain the equations of motion for a servo controlled robot manipulator as

$$\begin{aligned} m_1 \ddot{x}_1 + (c + K_2) \dot{x}_1 + (k + GK_1)x_1 - c\dot{x}_2 - kx_2 &= GK_1 x_{in} \\ m_2 \ddot{x}_2 + c \dot{x}_2 + kx_2 - c\dot{x}_1 - kx_1 &= F_2(t) = 0 \end{aligned} \quad (4)$$

Eq.(4) allows us to construct the block diagram of the manipulator control system as shown in Fig.3.

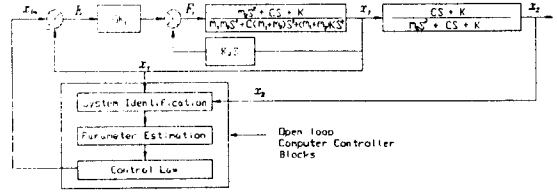


Fig.3 Block diagram of the flexible manipulator control system.

If a robot is used to pick up an object with mass  $m$ (or inertia), the robot's dynamic characteristics are modified. The force on mass  $m$  is  $F_2$  and is given by

$$F_2 = -m \ddot{x}_2 \quad (5)$$

Substitution of Eq.(5) into Eq.(4) changes the robot manipulator's dynamic equation to

$$\begin{aligned} m_1 \ddot{x}_1 + (c + K_2) \dot{x}_1 + (k + GK_1)x_1 - c\dot{x}_2 - kx_2 &= GK_1 x_{in} \\ m_2 \ddot{x}_2 + c \dot{x}_2 + kx_2 - c\dot{x}_1 - kx_1 &= F_2(t) = -m \ddot{x}_2 \end{aligned} \quad (6)$$

Eq.(6) clearly shows the effects of the object's mass on the robot arm's dynamic performance. Any control scheme must be able to sense the effect of this inertia change on the robot's structural behavior.

### 3. Control Scheme

The proposed control scheme began with trying to satisfy three major objectives when dealing with a flexible robot arm. First, it is desired to move a prescribed distance in the least amount of time. Second, there is to be no residual vibration at the end of the motion. Third, conventional servo system components are to be used along with minimum computational effort to generate the required system input command function  $x_{in}(t)$  shown in Fig.4. Before the three step control law is implemented, a fundamental decision is required as to whether the required total motion can be achieved through a single

three step process as shown in Fig.4. These choices are made based on  $x_{final}$  and  $x_{max}$  where  $x_{final}$  is the required final displacement and  $x_{max}$  is the maximum motion consistent with torque motor limits or structural stress limits.

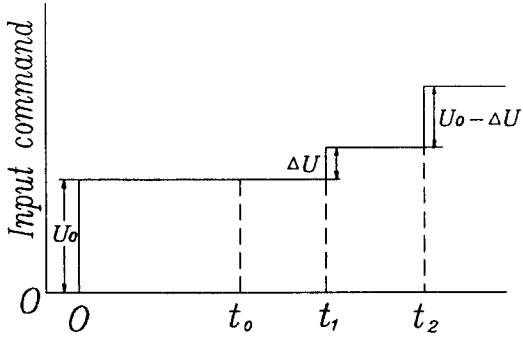


Fig.4 Three-step input command function.

### 3.1 Three Step Control Law for Small Motion

If  $x_{final} \leq x_{max}$ , then the three step control law for small motion is shown in Fig.4.

This control law has three step inputs determined by the following method :

1. Apply a step input that corresponds to half of the distance the robot is to move,

$$U_0 = x_{final} / 2$$

2. Measure the time,  $t_0$  it takes the system to achieve this half distance. This time corresponds to approximately one-fourth of the system's natural period,  $T_n$ . The reason this is approximately the quarter period is due to the fact that the system is responding with its damped natural frequency instead of its undamped natural frequency.
3. Based on time  $t_0$ , calculate two natural period of the mechanical system  $T_n$ .
4. With this information, calculate when the second step( $\Delta U$ ) input needs to be in order to overcome the robot's damping behavior.
5. Issue the second step input command( $\Delta U$ )

at the appropriate time,  $t_1$ .

6. Issue the third step input command( $U_0 - \Delta U$ ) at the appropriate time,  $t_2$ .

Note : Step 3 and step 4~6 is explained in section 3.2 and 3.3, respectively

### 3.2 System Identification

System identification is concerned with the determination of either the unknown or the known mechanical system natural period,  $T_n$ . For this study, the manipulator end point response,  $x_2$ , is used for monitoring variable to identify when the position of the response has reached halfway to the defined end point at time  $t=t_0$ . When time  $t_0$  is obtained, the natural frequency  $\omega_n$  of the unknown (or known) mechanical system is computed from a typical step input response of the damped spring mass system as follow<sup>(6),(7)</sup>

$$\omega_n = \frac{1}{t_0 \sqrt{1 - \zeta^2}} \left( \frac{\pi}{2} + \phi \right) \quad (7)$$

$$T_n = \frac{2\pi}{\omega_n} \quad \text{where, } \tan \phi = \zeta / \sqrt{1 - \zeta^2}$$

### 3.3 Parameter Estimation

Damping causes a significant change in the robot's behavior to step inputs since the high velocity causes an impulse to be imparted to the output end through the damping mechanism. Hence, one cannot use the simple two step approach outlined above for the undamped case, but must resort to at least one additional intermediate step  $\Delta U$  as shown in Fig.4. The command input function shown in Fig.4 has five unknown parameters involving  $U_0$ ,  $\Delta U$ ,  $t_1$ ,  $U_1$  and  $t_2$ .  $U_0$  is defined to be half of the defined end point, while  $U_1$  is expressed as  $(U_0 - \Delta U)$ . Then,  $t_1$  is selected at three-eighths of the mechanical

system natural period  $T_n$  so that there is time for both system identification and parameter estimation to be done. Thus, among five unknown parameters, only two parameters,  $\Delta U$  and  $t_2$ , remain as unknowns. Proper choice of these parameters are important factors in making the three-step input method work.

Even though the structural damping of most real mechanical systems is in the range of 1% ~ 6%, the simple functional relationship for  $\Delta U/U_0$  and  $t_2/T_n$ <sup>(5)-(6)</sup> is developed in the range of 1% ~ 25% to perform the broad analog simulation. For  $\Delta U/U_0$  and  $t_2/T_n$  vs  $\zeta$ , the results are

$$\Delta U/U_0 = \begin{cases} \ln(1 + 10.08\zeta - 33.48\zeta^2) & ; (0 < \zeta < 0.11) \\ 0.7719 + 0.8498\zeta & ; (0.11 < \zeta < 0.25) \end{cases} \quad (8)$$

$$t_2/T_n = \begin{cases} 0.5012 + 1.145\zeta & ; (0 < \zeta < 0.11) \\ 0.5353 + 0.8498\zeta & ; (0.11 < \zeta < 0.25) \end{cases} \quad (9)$$

Since a number of decisions need to be made dependent on the robot's dynamic characteristics, the time corresponding to  $T_n/4$  is measured as the output and achieves the designated input step displacement that corresponds to  $\Delta U$ . The robot's actual damping can be estimated in an iterative manner<sup>(6)</sup> from the measured time  $t_0$ . Thus, input command function  $x_{in}(t)$  shown in Fig.4 is expressed as

$$x_{in}(t) = U_0 u(t) + \Delta U u(t - t_1) + (U_0 - \Delta U) u(t - t_2) \quad (10)$$

## 4. Analog Simulation

### 4.1 Analog Simulation Model

The TR-10 analog computer is used to acquire the behavior of the dynamic control system involving the one degree of freedom flexible manipulator shown in Fig.2. The mathematical model on the damped spring-mass system needs

to construct the corresponding electrical model. Thus, Eq.(5) may be alternatively rearranged as

$$\begin{aligned} \dot{x}_1 &= - \int [a_1 \dot{x}_1 + a_2 x_1 - a_3 \dot{x}_2 - a_4 x_2 - a_5 x_{in}(t)] dt \\ \dot{x}_2 &= - \int [b_1 \dot{x}_2 + b_2 x_2 - b_1 \dot{x}_1 - b_2 x_1] dt \end{aligned} \quad (11)$$

To avoid the Op-Amp output becoming so large that the Op-Amp overloads or the problem variable becomes so small that the Op-Amp noise is the predominant output signal, magnitude scaling is needed the displacement variables,  $x_1$  and  $x_2$ , and the velocities,  $\dot{x}_1$  and  $\dot{x}_2$ . These displacement and velocity scaling factors,  $S_d$  and  $S_v$ , allow Eq.(11) to be expressed as<sup>(6)</sup>

$$\begin{aligned} S_{v_1} \dot{x}_1 &= - \int [D_1(S_{v_1} \dot{x}_1) + D_2(S_{d_1} x_1) - D_3(S_{v_1} x_2) \\ &\quad - D_4(S_{d_1} x_2) - D_5(S_{d_1} x_{in}(t))] dt \\ S_{v_2} \dot{x}_2 &= - \int [D_6(S_{v_2} \dot{x}_2) + D_7(S_{d_2} x_2) \\ &\quad - D_8(S_{v_1} x_1) - D_9(S_{d_1} x_1)] dt \end{aligned} \quad (12)$$

where,

$$D_1 = a_1 = 2\omega_1 \zeta_1, \quad D_2 = \frac{S_{v_1} a_2}{S_{d_1}} = 2\pi f_1,$$

$$D_3 = \frac{S_{v_1} a_2}{S_{v_2}} = \frac{4\pi m_2 f_2^2 \zeta_2}{m_1 f_1},$$

$$D_4 = \frac{S_{v_1} a_4}{S_{d_1}} = \frac{2\pi f_2^2}{f_1}$$

$$D_5 = \frac{S_{v_1} a_5}{S_{d_1}} = \frac{1}{\omega_1} \left( \omega_1^2 - \frac{m_2 \omega_2^2}{m_1} \right),$$

$$D_6 = b_1 = 2\omega_2 \zeta_2, \quad D_7 = \frac{S_{v_2} b_2}{S_{d_1}} = 2\pi f_2,$$

$$D_8 = \frac{S_{v_2} b_1}{S_{v_1}} = 2\omega_1 \zeta_2, \quad D_9 = \frac{S_{v_2} b_2}{S_{d_1}} = 2\pi f_2$$

, and  $S_{v_i} \dot{x}_i$ ,  $S_{d_i} x_i$  and  $S_{d_i} x_{in}(t)$  are new variable expressed in volts, and  $f_1 (= \omega_1/2\pi)$  and  $f_2 (= \omega_2/2\pi)$  represent the servo system and mechanical system natural frequency, respectively. Also,  $\zeta_1$  and  $\zeta_2$  represent the mechanical and servo system damping ratio, respectively.

The standard analog circuits employed in this simulation is shown in Fig.5.

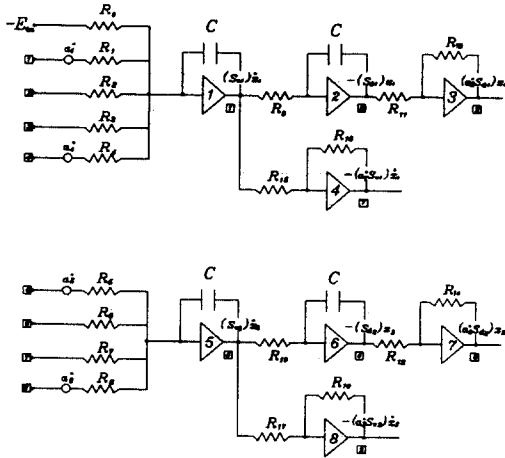


Fig.5 The simplified electric circuit for the analog simulation

For real time control, the general procedure for the experimental analog simulation is shown in Fig.6. Fig.6 shows that the analog simulation is performed through the host computer, and this controls the TR-10 analog computer and the DT2801-A I/O board, and the Pro 380 computer and ADM are used to measure the overall system performance signals, and the Tektronix 2201 Digital Storage Oscilloscope is used for visual observation of the overall system performance.

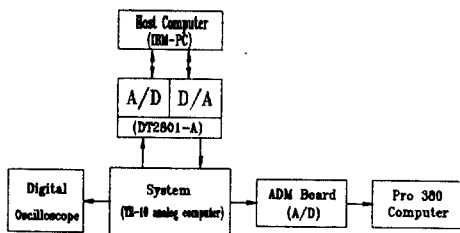


Fig.6 The schematic diagram of overall experimental analog simulation

## 4.2 Results

The purpose of study focuses on point-to-point movement of the manipulator with small distance. The servo system damping ratio  $\zeta_1$  is

fixed at 70 % for all experiment. Because the structural damping of most real mechanical systems is in the range of 1%~6%, the experiments are performed the mechanical damping ratio in the 1%~10%, to show the robustness of three-step input method. The experiments are classified as three cases according to the mechanical damping values of  $\zeta_2 = 1, 5, \text{ and } 10\%$ . The servo natural frequency  $f_1$  is fixed at 10 Hz while the mechanical natural frequency is varied from 1 to 5 Hz so that the natural frequency ratio  $n$  defined by  $f_1/f_2$  varies from 2 to 10. The values of  $f_2$  and  $n$  used are shown in Table 1. The experimental data results are acquired using a Pro 380 ADM A/D converter operating at a sampling frequency of 250 Hz.

Table 1. The values of  $f_2$  and  $n$  used for analog simulation

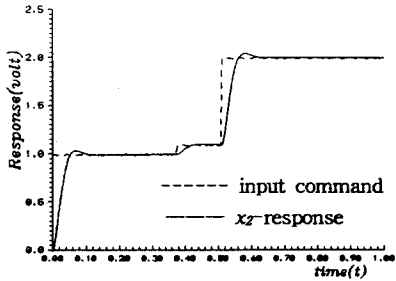
$f_2$	$n$
5 Hz	2
2 Hz	5
1 Hz	10

### 4.2.1 Case 1

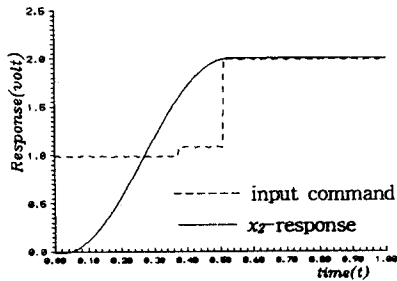
In Case 1, a lightly damped mechanical system with  $\zeta_2 = 1\%$  is used.

Fig.7 shows the results when  $n = 10$ , where Fig.7a is the base response  $x_1$ , and Fig.7b is the manipulator end point response  $x_2$ . Fig.7b shows that the end point residual vibration has been nearly eliminated. The reason is that even though  $x_1$  is not a step input like the input command, the change is nearly a step as far as its end position is concerned, and hence the manipulator end point response,  $x_2$ , is very close to that which occurs for a true step input. The maximum error is about 0.254 %, and time,  $t_2$ , is 0.512 sec., which is approximately a half period of the mechanical system.

Fig.8 shows the results with  $n = 5$ . In this case, the residual vibration of the end point at the defined end point has also been eliminated as shown in Fig.8b because the base response,  $x_1$ , still appears to be nearly a step input to the mechanical system as shown in Fig.8a. These results show the maximum error is 0.325 %, and



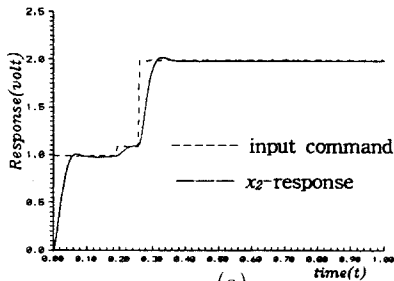
(a)



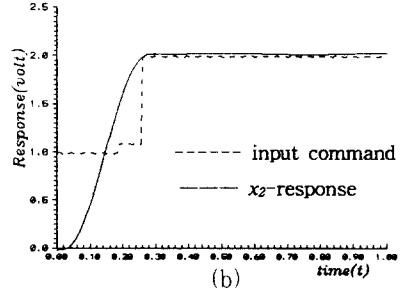
(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.7 In the case of 1% damping ratio, system responses with  $n = 10$



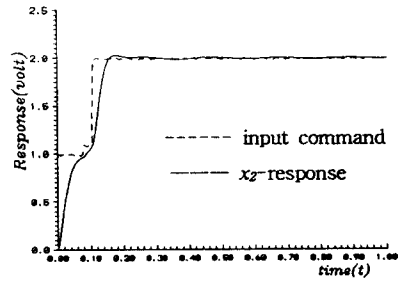
(a)



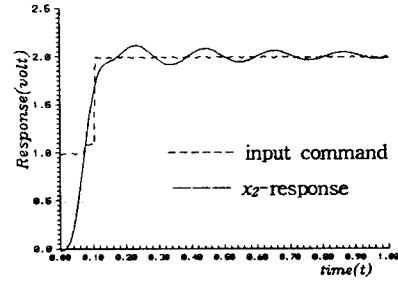
(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.8 In the case of 1% damping ratio, system responses with  $n = 5$



(a)



(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.9 In the case of 1% damping ratio, system responses with  $n = 2$

time,  $t_2$ , is about 0.526 sec., which is approximately a half period of the mechanical system.

Fig.9 shows the results when  $n = 2$ . As expected, the end point residual vibration has not been eliminated as shown in Fig.9b because the base response,  $x_1$ , cannot follow the input

command as required. These results show that the maximum error is about 5.35 %, and response,  $x_2$ , reaches the defined end point at a time considerably beyond the desired time,  $t_2$ , of 0.102 sec, which is calculated using Eq.(9)

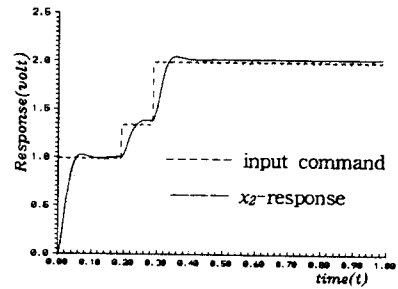
#### 4.2.2 Case 2

In Case 2, the mechanical system damping ratio  $\zeta_2$  is assumed to be 5 %.

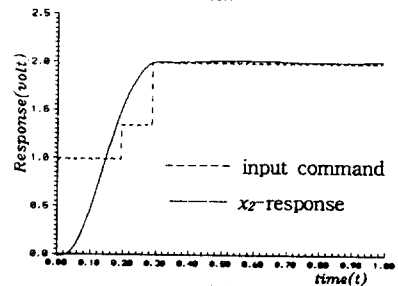
Fig.10 shows the results with  $n = 10$ . As shown in Fig.10b, the end point residual vibration has been nearly eliminated because base response,  $x_1$ , follows the input command closely enough to appear as a step input to the mechanical system as shown in Fig.10a. Also, these results are identical conditions. The maximum error is about 0.484 %, and time,  $t_2$ , is about 0.559 sec., which is about 12 % longer than a half period of the mechanical system.

Fig.11 shows the results with  $n = 5$ . The end point residual vibration at the defined end point has been eliminated as shown in Fig.11b because the base response,  $x_1$ , still appears to be nearly a step input to the mechanical system as shown in Fig.11a. These results show that the maximum error is 0.538 %, and time,  $t_2$ , is 0.28 sec., which is about 12 % longer than the half period of the mechanical system.

Fig.12 shows the results when  $n = 2$ . As expected, the end point residual vibration has not been eliminated as shown in Fig.12b because the base response,  $x_1$ , cannot follow the input command as shown in Fig.12a. These results show that the maximum error is about 4.08 %, and the end point response,  $x_2$ , initially reaches the defined end point at a time considerably beyond the desired time,  $t_2$ , of 0.112 sec calculated by Eq.(9) as shown in Fig.12b. It is also seen that the residual vibration damps out much more quickly than the 1 % damping case.



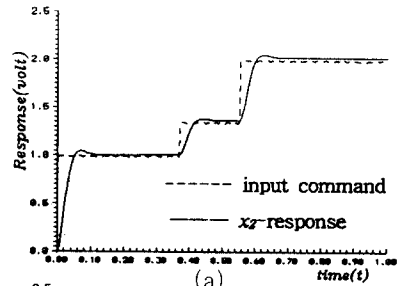
(a)



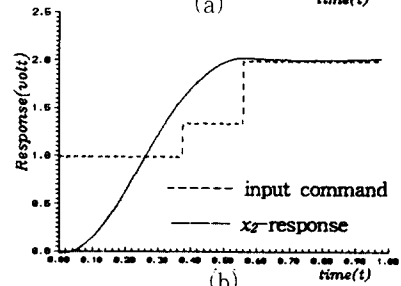
(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.10 In the case of 5% damping ratio, system responses with  $n = 10$



(a)

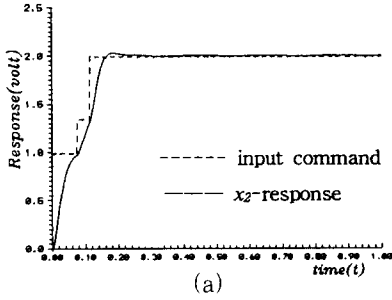


(b)

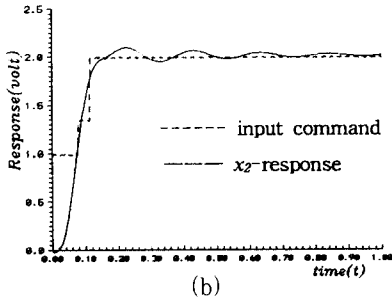
a) Base response  $x_1$ , b) End point response  $x_2$

Fig.11 In the case of 5% damping ratio, system responses with  $n = 5$





(a)



(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.12 In the case of 5% damping ratio, system responds with  $n = 2$

### 4.2.3 Case 3

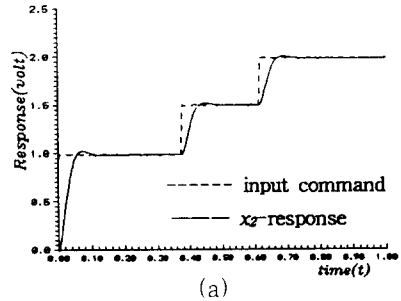
In Case 3, the mechanical system damping ratio  $\zeta_2$  is increased to 10 %.

Fig.13 shows the results when  $n = 10$ . As shown in Fig.13b, the end point residual vibration has been nearly eliminated because the base response,  $x_1$ , can follow the input command closely enough to appear as nearly a step input to the mechanical system as shown in Fig.13a. The maximum error is about 0.28 %, and time,  $t_2$ , is 0.615 sec, which is about 23 % longer than the mechanical system half period of 0.5 sec.

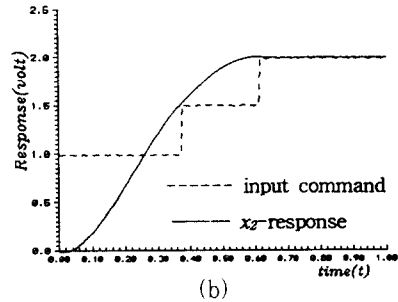
Fig.14 shows the results when  $n = 5$ . The end point residual vibration at the defined end point has also been eliminated as shown in Fig.14b, because the base response,  $x_1$ , still appears to be a step input to the mechanical system as shown in Fig.14a. These results show that the maximum error is 0.348 %, and time,  $t_2$ , is

0.307 sec, which is about 23 % longer than the mechanical system half period of 0.25 sec.

Fig.15 shows the results with  $n = 2$ . As expected, the end point residual vibration has not been eliminated as shown in Fig.15b because the base response,  $x_1$ , cannot adequately follow the input command. These results show that the maximum error is about 2.38 %, and response,  $x_2$ , reaches initially the defined end point beyond the calculated time,  $t_2$ , of 0.123 sec. with a considerable time delay.



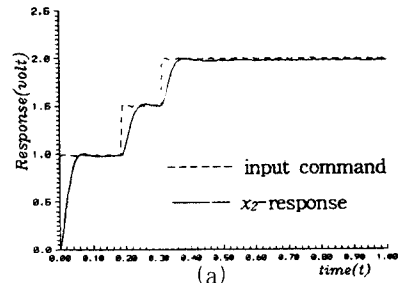
(a)



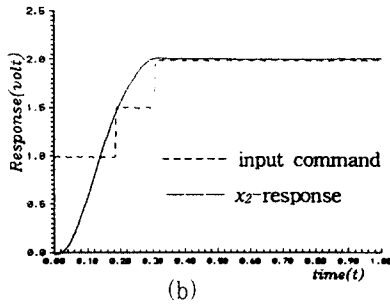
(b)

a) Base response  $x_1$ , b) End point response  $x_2$

Fig.13 In the case of 10% damping ratio, system responds with  $n = 10$

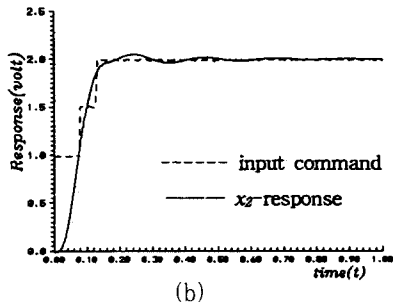
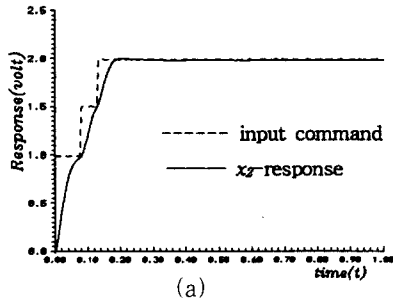


(a)



a) Base response  $x_1$ , b) End point response  $x_2$

Fig.14 In the case of 10% damping ratio, system responses with  $n = 5$



a) Base response  $x_1$ , b) End point response  $x_2$

Fig.15 In the case of 10% damping ratio, system responses with  $n = 2$

## 5. Conclusion

The use of three-step input method for commanding the computer controlled simple flexible manipulator showed that significant residual vibration reduction along with the minimized

response time can be achieved.

The development of the parameter estimation model used for the three-step input method shows that two parameters,  $\Delta U$  and  $t_2$ , must be known in order to apply the correct input at the proper times. It has known that  $\Delta U$  values are dependent both on the mechanical damping,  $\zeta_2$  and half-step size  $U_0$ , while  $t_2$  depends on both the mechanical damping,  $\zeta_2$ , and natural frequency,  $f_2$ . It is also found that this parameter estimation model reduces computation complexity. Based on parameter estimation model, the experimental analog simulation study in point-to-point movement shows that an effective frequency ratio  $n$  must be greater than 5 in order to nearly eliminate residual vibration along with the minimum response time when the mechanical damping varies from 1 % to 10 %. This research work places great emphasis on the importance of the natural frequency ratio  $n$  in designing a suitable control system to implement the three-step input method.

## Acknowledgments

This study was supported by Factory Automation Center for Parts of Vehicles (FACPOV) in Chosun University, Kwangju, Korea(1996). FACPOV is designated as a regional research center of Korea Science and Engineering Foundation (KOSEF) and operated by Chosun University.

## Reference

1. D.M. Aspinwall, "Acceleration Profiles for Minimizing Residual Response." ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 102, No. 1, pp. 3-6, 1980.
2. J.A. Breakwell, "Optimal Feedback Slewing of Flexible Spacecraft." AIAA Journal of Guidance

- and Control, Vol 4, No. 5, pp. 472-479, 1981.
3. S.E. Bruke and J.E. Hubbard. "Active Vibration Control of a Simply Supported Beam Using a Spatially Distributed Actuator." IEEE Control System Magazine, Vol 7, No. 4, pp. 25-30, 1987.
  4. R.H. Cannon and E. Schmitz. "Initial Experiments on the End-Point Control of a Flexible One-Link Robot." The International Journal of Robotics Research, Vol 3, No. 3, pp. 62-75, 1984.
  5. P.H. Meckl and W.P. Seering. "Minimizing Residual Vibration for Point-to-Point Motion." ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol 107, No. 4, pp. 378-382, 1985.
  6. W.S. Jang. "An Open Loop Control Scheme to Minimize Flexible Robot Response Time While Minimizing Residual Vibrations." Ph.D. Thesis, Iowa State University, Ames, Iowa, 1991.
  7. K.G. McConnel and W.S. Jang. "Robot Control Scheme." United States Patent #5,594,309, Jan.14, 1997.