Sliding Mode Control for Helicopter Attitude Regulation at Hovering

Hovering에서의 헬리콥터 자세제어를 위한 슬라이딩 모드 제어

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요 약 : 본 논문에서는 약간의 가정하에 리모트 제어용 모형 헬리콥터의 동역학방정식을 유도하였으며, 이를 토대로하여 헬리콥터의 자세안정을 위한 제어 알고리듬을 제안하였다. 제어이론으로서는 파라메타의 변화 및 외란에 강인한 가변구조 제어이론을 활용하였다. 본 제어 알고리듬에서는 헬리콥터의 위치이동 제어에 대하여서는 다루지를 못하였으며, 단지 헬리콥터의 hovering 상태에서의 자세 안정화에만 촛점을 두어 제어 알고리듬을 제안하였다. 컴퓨터 모사를 통하여, 제안된 제어 알고리듬의 타당성을 보였으며, 약 2-3초의 시간이 경과된 이후 자세가 안정화 됨을 볼 수 있었다.

Keywords : variable structure system, orientation of helicopter, attitude of helicopter, robustness

I. Introduction

It has been well recognized that special efforts are required and advanced control techniques are often appropriate for control of large-scale, nonlinear, naturally unstable and highly cross-coupled systems such as helicopters. It is very difficult to control helicopter because its dynamics are highly nonlinear with strongly coupled modes. Many researchers have designed controller for helicopter flight. S. Ahmad [1], J. K. Pieper[2] analyzed the dynamics of helicopter and designed controller based on sliding motion control.

I. Postlethwaite, D. J. Walker[3] adopted H_∞ robust control theory for designing controller. M. Sugeno [4] applied fuzzy control theory to design a controller. In this paper, we derived simplified dynamic equations of electric radio controlled model helicopter under the assumption that we can neglected the speed of wind (helicopter) compared with those of main rotor and tail rotor and blade angles of main rotor and tail rotor deviated from their equilibrium values are very small. We design the robust controller based on the variable structure system for the robustness to parameter variations and external disturbances. In this first research step, we don't consider the translational motion of helicopter and focus only on stabilization of attitude of helicopter. Therefore, the proposed control scheme can be applied to helicopter at hovering. Even though we don't expect that the behaviour of the model helicopter is merely a scaled version of the behaviour of a full scale helicopter, a robust control algorithm for stabilizing of model helicopter may be applied to full scale helicopter. There are two kinds of radio controlled model helicopters and they have different mechanism for activating the rolling and pitching motion of helicopter. In one mechanism, rolling and pitching motion is mainly achieved by changing the lateral tilt angle and longitudinal tilt angle of rotating plane of main rotor with respective to $x-y$ plane of body attached coordinate system by rolling and pitching command. In the other mechanism, rolling and pitching motion is mainly achieved by changing the lateral cyclic pitch angle and longitudinal cyclic pitch angle of main rotor's blade by rolling and pitching command. The model helicopter considered in this paper is former type and it is actuated by PWM servos, one each for lateral tilt angle, logitudinal tilt angle of rotating plane of main rotor, collective pitch angle of main rotor blade, collective pitch angle of tail rotor blade, and engin throttle. Section II and III of this paper, we derive the simplified dynamic equations of model helicopter. In section IV, we suggested a robust controller for stabilizing the attitude of helicopter based on variable structure system. In section V, we verify the validness of proposed controller by computer simulation. Section VI contains the conclusions.

II. Dynamics of Helicopter

In this section, we derive the simplified dynamics of helicopter under the following assumptions.

- We neglect the speed of helicopter compared with the speed of main rotor and tail rotor.($V_\infty = 0$)
- The lift is proportional to pitch angle of main rotor and tail rotor.
- The Pitch angle of main rotor and tail rotor is not twisted.
- The moment of inertia J_b and mass m_b helicopter with respect to body attached coordinate system are fixed.
- Blade angles of main rotor and tail rotor deviated from their equilibrium values are very small.

We set the relationship between the control command and control action as follows.

- $\psi(t)$: lateral tilt angle ($\psi(t) = g_r\, r(t)$)
- $\theta(t)$: logitudinal tilt angle ($\theta(t) = g_p\, p(t)$)

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- $\Omega(t)$: angular velocity of main rotor($\Omega(t) = g_o\, e(t)$)
- $\alpha_1(t)$: angle of attack of main rotor blade ($\alpha_1(t) = g_{\alpha_1} e(t)$)
- $\alpha_2(t)$: angle of attack of tail rotor blade ($\alpha_2(t) = g_{\alpha_2} e(t)$)

Therefore we have four control command inputs $r(t), p(t), e(t), y(t)$ to control the motion of helicopter. We set the world coordinate system and body attached coordinate system as $(x_\omega, y_\omega, z_\omega)$ and (x_b, y_b, z_b) respectively and each coordinate system is shown in Fig. 1.

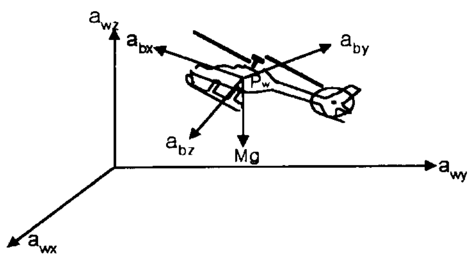

Fig. 1. Coordinate sytems of helicopter.

Then we obtain the following equations concerning dynamics of helicopter under the assumption that $\omega_b(t)$ is very small as follows,

$$J_b\, \dot{\omega}_b(t) + \omega_b(t) \times J_b \omega_b(t) \cong J_b\, \dot{\omega}_b(t) = \tau(t) \tag{1}$$

$$m_b\, \ddot{p}_\omega(t) = f_\omega(t) \tag{2}$$

where $\omega_b(t)$ and $\tau_b(t)$ are angular velocity and total torque of helicopter with respect to body attached coordinate system respectively and $p_\omega(t)$ and $f_\omega(t)$ are position of helicopter's center of gravity and total force with respect to world coordinate system respectively. In general, $f_\omega(t)$ consists of three forces, *i.e.*, f_{mrw} occuring from main rotor, f_{trw} occuring from tail rotor and gravitational force, as follows,

$$f_\omega(t) = f_{mrw}(t) + f_{trw}(t) + f_g \tag{3}$$

where

$$f_{mrw}(t) = {}^wR_b(t)\, f_{mrb}(t) \tag{4}$$

$$f_{trw}(t) = {}^wR_b(t)\, f_{trb}(t) \tag{5}$$

$$f_g = m_b\, g, \tag{6}$$

${}^wR_b(t)$is rotational matrix of body attached coordinate system with respect to world coordinate system and f_{mrb} and f_{trt} are forces generated by main rotor and tail rotor, respectively, and are expressed with respect to body attached coordinate system. Total torque $\tau_b(t)$ consists of two components, i.e. τ_{mrt} occuring from main rotor and τ_{trt} occuring from tail rotor as follows, and they are expressed with respect to body attached coordinate system.

$$\tau_b(t) = \tau_{mrb}(t) + \tau_{trb}(t) \tag{7}$$

Now we analyze each component of forces and torques in the next section.

III. Analysis of forces and torques

For the analysis of forces and torques of helicopter, we set the coordinate system for the rotating plane of main rotor as (x_{mr}, y_{mr}, z_{mr}) and call it as main rotor coordinate system and this coordinate system is generated by a rotation of $\phi(t)$ about a_{bx} followed by a rotation of $\theta(t)$ about a_{by}. The location of main rotor and tail rotor with respect to body attached system are shown in figure 2. The forces and torque derived in this section are based on the momentum theory and blade element theory [6].

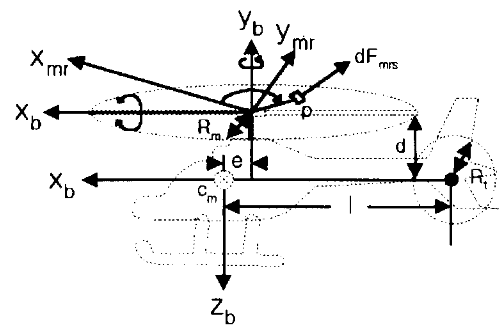

Fig. 2. Coordinate systems of helicopter's body.

A. Analysis of forces

From the assumptions, the differential lift force df_{mr} of differential surface of main rotor $rdrd\phi$ shown in Fig. 2 can be approximated as

$$df_{mr}(t) = -g_{fm}\, \Omega(t)^2 \alpha_1(t)\, r^3 dr d\phi\ \ a_{mrz} \tag{8}$$

where g_{fm} is proportional coefficient of main rotor. So total force of main rotor $f_{mr}(t)$ with respect to the main rotor coordinate system can be expressed as

$$f_{mr}(t) = \int df_{mr} = -g_{fm}\, \Omega(t)^2 \alpha_1(t) \frac{\pi r_1^4}{2}\ a_{mrz} \tag{9}$$

where r_1 is the radius of main rotor. From figure 2, we can see that the rotational matrix between main rotor coordinate system (x_{mr}, y_{mr}, z_{mr}) and body attached coordinate system (x_b, y_b, z_b) can be derived as follow as

$$ {}^bR_{mr}(t) = \begin{bmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi(t) & -\sin\phi(t) \\ 0 & \sin\phi(t) & \cos\phi(t) \end{bmatrix} \tag{10}$$

Therefore

$$f_{mrb}(t) = {}^bR_{mr}(t)\, f_{mr}(t) \tag{11}$$

In the similar method, we can derive the lift force of tail rotor $f_{trb}(t)$ as follows

$$f_{trb}(t) = -g_{ft}\, \Omega(t)^2 \alpha_2(t) \frac{\pi r_2^4}{2}\ a_{by} \tag{12}$$

where r_2 is the radius of tail rotor. If we consider all the components of forces derived above and gravitational force, we can derive the following translational dynamics.

$$m_b \begin{bmatrix} \ddot{p}_{xw} \\ \ddot{p}_{yw} \\ \ddot{p}_{zw} \end{bmatrix} = m_b\, g \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} + {}^wR_b(t)\,[{}^bR_{mr}(t)\, f_{mr}(t) + f_{trb}(t)] \tag{13}$$

where ${}^wR_b(t)$ is the rotational matrix between body attached coordinate system (x_b, y_b, z_b) and world coordinate system (x_w, y_w, z_w).

B. Analysis of torque

We can derive the torque of tail rotor with respect to body attached coordinate system as follows,

$$\tau_{trb}(t) = -l \ a_{bx} \times f_{trb}(t) = g_{ft} \ l \ \Omega(t)^2 \alpha_2(t) \frac{\pi r_2^4}{2} \ a_{bz} \quad (14)$$

where operator $\times$ means the cross product of two vectors. Now we derive the torque of main rotor. At first we derive the position of point p on the rotating plane of main rotor with respect to body attached coordinate system by using Homogeneous coordinate system [5]. The generalized coordinate p_{bg} of point p can be expressed as follows.

$$p_{bg} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \\ 0 \\ 1 \end{bmatrix} \quad (15)$$

From the above equation, p_b, coordinate of point p with respect to body attached coordinate system, and $df_{mrb}(t)$ are expressed as follows,

$$p_{bg} = \begin{bmatrix} \cos\theta \ r\cos\phi + \sin\theta \ \sin\psi \ r\sin\phi - e \\ \cos\psi \ r\sin\phi \\ -\sin\theta \ r\cos\phi + \cos\theta \ \sin\psi \ r\sin\phi - d \\ 1 \end{bmatrix} = \begin{bmatrix} p_b \\ 1 \end{bmatrix} \quad (16)$$

$$df_{mrb}(t) = \begin{bmatrix} -\sin\theta\cos\psi \\ \sin\psi \\ -\cos\theta\cos\psi \end{bmatrix} g_{fm} \Omega(t)^2 \alpha_1(t) \ r^3 dr d\phi \quad (17)$$

and we can derive differential torque $d\tau_{mrb}(t)$ as

$$d\tau_{mrb}(t) = p_b \times df_{mrb}(t). \quad (18)$$

From the mathematical manipulations, we obtain

$$\tau_{mrb}(t) = \int d\tau_{mrb}(t) = \begin{bmatrix} -e \\ 0 \\ -d \end{bmatrix} \times \begin{bmatrix} -\sin\theta\cos\psi \\ \sin\psi \\ -\cos\theta\cos\psi \end{bmatrix} g_{fm} \Omega(t)^2 \alpha_1(t) \frac{\pi r_1^4}{2} \quad (19)$$

By using above torques of main rotor and tail rotor, we can derive rotational dynamics approximately with respect to body attached coordinate system as follows,

$$J_b \begin{bmatrix} \dot{\omega}_{bx} \\ \dot{\omega}_{by} \\ \dot{\omega}_{bz} \end{bmatrix} = \tau_{trb}(t) + \tau_{mrb}(t). \quad (20)$$

C. Relationship between coordinate systems

Let $\omega_w(t)$ and $\omega_b(t)$ be the angular velocity of helicopter with respect to world coordinate system and body attached coordinate systems respectively.

The relation between $\omega_w(t)$ and $\omega_b(t)$ becomes

$$\omega_w(t) = {}^wR_b(t) \ \omega_b(t) \quad (21)$$

where

$$\omega_w(t) = \begin{bmatrix} \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{bmatrix}, \quad \omega_{b(t)} = \begin{bmatrix} \omega_{bx}(t) \\ \omega_{by}(t) \\ \omega_{bz}(t) \end{bmatrix} \quad (22)$$

Also we can see that the following relation is satisfied

$$ {}^wR_b(t) \ {}^wR_b(t)^{-1} = I. \quad (23)$$

If we differentiate above equation with respect to time, we have

$$ {}^w\dot{R}_b(t) \ {}^wR_b(t)^{-1} + {}^wR_b(t) \ {}^w\dot{R}_b(t)^{-1} = 0. \quad (24)$$

If we define matrix $S(\omega_w)$ as follows,

$$S(\omega_w) = \begin{bmatrix} 0 & -\omega_z(t) & \omega_y(t) \\ \omega_z(t) & 0 & -\omega_x(t) \\ -\omega_y(t) & \omega_x(t) & 0 \end{bmatrix} \quad (25)$$

then the following equation is established [5].

$$ {}^w\dot{R}_b(t) = S(\omega_w) \ {}^wR_b(t) \quad (26)$$

We obtain following equation by applying above equation to (20).

$$\begin{aligned} & J_b \frac{d}{dt}({}^wR_b(t)^{-1} \ \omega_w(t)) \\ &= J_b ({}^w\dot{R}_b(t)^{-1}\omega_w(t) + {}^wR_b(t)^{-1} \ \dot{\omega}_w(t)) \\ &= J_b (- {}^wR_b(t)^{-1} S(\omega_w) \ \omega_w(t) + {}^wR_b(t)^{-1} \ \dot{\omega}_w(t)) \\ &= \tau_{trb}(t) + \tau_{mrb}(t) \end{aligned} \quad (27)$$

If we use the fact that $S(\omega)\omega = 0$ for all ω, we obtain the following rotational dynamic equation as

$$\dot{\omega}_w(t) = {}^wR_b(t) \ J_b^{-1}(\tau_{trb}(t) + \tau_{mrb}(t)) \quad (28)$$

D. Summary of dynamics of helicopter

In this subsection we summarize dynamic equations of helicopter.

$$m_b \ \ddot{p}_w = m_b g + {}^wR_b(t)({}^bR_{mr}(t) \ f_{mr}(t) + f_{trb}(t)) \quad (29)$$

$$ {}^w\dot{R}_b(t) = S(\omega_w) \ {}^wR_b(t) \quad (30)$$

$$\dot{\omega}_w(t) = {}^wR_b(t) \ J_b^{-1}(\tau_{trb}(t) + \tau_{mrb}(t)) \quad (31)$$

where

$$f_{mr}(t) = -g_{fm} \ \Omega(t)^2 \alpha_1(t) \frac{\pi r_1^4}{2} \ a_{mrz} \quad (32)$$

$$f_{trb}(t) = -g_{ft} \ \Omega(t)^2 \alpha_2(t) \frac{\pi r_2^4}{2} \ a_{by} \quad (33)$$

$$\tau_{trb}(t) = -g_{ft} \ l \ \Omega(t)^2 \alpha_2(t) \frac{\pi r_2^4}{2} \ a_{bz} \quad (34)$$

$$\tau_{mrb}(t) = \begin{bmatrix} -e \\ 0 \\ -d \end{bmatrix} \times \begin{bmatrix} -\sin\theta\cos\psi \\ \sin\psi \\ -\cos\theta\cos\psi \end{bmatrix} g_{fm} \Omega(t)^2 \alpha_1(t) \frac{\pi r_1^4}{2} \quad (35)$$

IV. Design of controller

In this section, we propose a control scheme based on variable structure system. In this proposed control method, we focus only on stabilization of attitude of helicopter without considering the translational motion. Let us consider the following dynamic equations concerning the attitude of helicopter,

$$ {}^w\dot{R}_b(t) = S(\omega_w(t)) \ {}^wR_b(t) \quad (36)$$

$$\dot{\omega}_w(t) = {}^wR_b(t) J_b^{-1} \tau(t) \quad (37)$$

where

$$\tau(t) = \tau_{mrb}(t) + \tau_{trb}(t). \quad (38)$$

The control object is to find the control laws such that ${}^wR_b(t)$ converges to identity matrix I as fast as possible, i.e.,

$$\lim_{t \to \infty} {}^wR_b(t) = I, \quad (39)$$

As you can see, if rotational matrix ${}^wR_b(t)$ converges to identity matrix I, the attitude of helicopter is stabilized. We propose control scheme in 2 steps.

Step 1 : Find $\omega_w(t)$ which guarantee that ${}^wR_b(t)$ converges to identity matrix I.

Let us define the following performance index J as

$$J = tr[({}^wR_b(t) - I)({}^wR_b(t) - I)^T]. \tag{40}$$

The time derivative of J can be expressed as

$$\begin{aligned}\dot{J} &= tr[\frac{d}{dt}[({}^wR_b(t) - I)({}^wR_b(t) - I)^T]] \\ &= -2\ tr[S(\omega_\omega)\ {}^wR_b(t)].\end{aligned} \tag{41}$$

If we define rotational matrix ${}^wR_b(t)$ as follows

$${}^wR_b(t) = \begin{bmatrix} R_{11}(t) & R_{12}(t) & R_{13}(t) \\ R_{21}(t) & R_{22}(t) & R_{23}(t) \\ R_{31}(t) & R_{32}(t) & R_{33}(t) \end{bmatrix} \tag{42}$$

we obtain

$$\begin{aligned} tr[S(\omega_\omega)\ {}^wR_b(t)] &= \omega_x(t)(R_{23}(t) - R_{32}(t)) \\ &+ \omega_y(t)(R_{31}(t) - R_{13}(t)) + \omega_z(t)(R_{12}(t) - R_{21}(t)). \end{aligned} \tag{43}$$

Therefore we choose $\omega_x(t)$, $\omega_y(t)$, and $\omega_z(t)$ as follows in order to make the time derivative of J be less than zero

$$\begin{aligned} \omega_x(t) &= k_x(R_{23}(t) - R_{32}(t)) \\ \omega_y(t) &= k_y(R_{31}(t) - R_{13}(t)) \\ \omega_x(t) &= k_z(R_{12}(t) - R_{21}(t)), \end{aligned} \tag{44}$$

where k_x, k_y and k_z can be any positive constants.

Step 2 : Find $\tau(t)$ such that equation (44) can be satisfied. At first, we set the sliding surfaces $s(t)$ as follows

$$s(t) = \begin{bmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{bmatrix} = \begin{bmatrix} \omega_x(t) - k_x(R_{23}(t) - R_{32}(t)) \\ \omega_y(t) - k_y(R_{31}(t) - R_{13}(t)) \\ \omega_z(t) - k_z(R_{12}(t) - R_{21}(t)) \end{bmatrix} \tag{45}$$

The control object is to find $\tau(t)$ such that the state trajectory hit the sliding surfaces within a finite time and stay in sliding surface thereafter. For the mathematical simplicity, we define new sliding surfaces as follows

$$\sigma(t) = \begin{bmatrix} \sigma_x(t) \\ \sigma_y(t) \\ \sigma_z(t) \end{bmatrix} = J_b\ {}^wR_b(t)^T \begin{bmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{bmatrix} \tag{46}$$

As you can see, if the trajectory hit the new sliding surfaces defined in equation (53) within a finite time, then it also hit sliding surfaces defined in equation (52) because matrix J_b and ${}^wR_b(t)$ are nonsingular for all time t. The time derivative of the new sliding surface $\sigma(t)$ can be expressed as

$$\begin{aligned} \dot{\sigma}(t) &= J_b\ {}^w\dot{R}_b(t)^T \begin{bmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{bmatrix} + J_b\ {}^wR_b(t)^T \begin{bmatrix} \dot{s}_x(t) \\ \dot{s}_y(t) \\ \dot{s}_z(t) \end{bmatrix} \\ &= J_b\ {}^wR_b(t)^T S(\omega(t))^T \begin{bmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{bmatrix} \\ &+ J_b\ {}^wR_b(t)^T \begin{bmatrix} \dot{\omega}_x(t) - k_x(\dot{R}_{23} - \dot{R}_{32}) \\ \dot{\omega}_y(t) - k_y(\dot{R}_{31} - \dot{R}_{13}) \\ \dot{\omega}_z(t) - k_z(\dot{R}_{12} - \dot{R}_{21}) \end{bmatrix} \end{aligned} \tag{47}$$

Therefore we choose the control law as follows which guarantee that the state trajectory hits the sliding surface within a finite time and then holds on it.

$$\begin{aligned} \tau(t) &= J_b\ {}^wR_b(t)^T \begin{bmatrix} k_x(\dot{R}_{23} - \dot{R}_{32}) \\ k_y(\dot{R}_{31} - \dot{R}_{13}) \\ k_z(\dot{R}_{12} - \dot{R}_{21}) \end{bmatrix} \\ &- J_b\ {}^wR_b(t)^T S(\omega(t))^T \begin{bmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{bmatrix} - \begin{bmatrix} k_1 sgn(\sigma_x(t)) \\ k_2 sgn(\sigma_y(t)) \\ k_3 sgn(\sigma_z(t)) \end{bmatrix} \end{aligned} \tag{48}$$

where k_1, k_2, and k_3 can be any positive constants. In real situation the control inputs are $\theta(t)$, $\psi(t)$ $\alpha_1(t)$ $\Omega(t)$, and $\alpha_2(t)$ so we must find relation between $\tau(t), \theta(t), \psi(t)$, $\Omega(t)$, $\alpha_1(t)$ and $\alpha_2(t)$ from the equation (34) and (35). Let us assume that the above relation can be expressed as

$$\tau(t) = F(\theta(t), \psi(t), \Omega(t), \alpha_1(t), \alpha_2(t)). \tag{49}$$

Using equation (48) and (49), we obtain real control input as follows

$$\begin{bmatrix} \theta(t) \\ \psi(t) \\ \Omega(t) \\ \alpha_1(t) \\ \alpha_2(t) \end{bmatrix} = F^{-1}(\begin{bmatrix} \tau_x(t) \\ \tau_y(t) \\ \tau_z(t) \end{bmatrix}) = F^{-1}(\tau(t)). \tag{50}$$

The over all control systems are shown in Fig. 3.

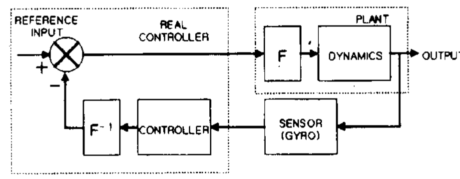

Fig. 3. Block diagram of control system.

V. Simulation

In this section, we set the parameter values of helicopter as shown in table 1 and simulate to illustrate the effectiveness of proposed control algorithm. The moment of inertia of helicopter J_b is set as follows so that we can not neglect the term $\omega_b(t) \times J_b\ \omega_b(t)$ in the dynamics if the angular velocity of helicopter $\omega_b(t)$ is not small.

$$J_b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4.1 & 0 \\ 0 & 0 & 4.1 \end{bmatrix} \tag{51}$$

The initial value of rotational matrix of helicopter is given by

$${}^wR_b(0) = \begin{bmatrix} 0.951 & -0.076 & 0.299 \\ 0.168 & 0.940 & -0.294 \\ -0.259 & 0.330 & 0.908 \end{bmatrix} \tag{52}$$

This means that initial attitude of helicopter is rotated through 10 degree about x_ω axis, 15 degree about y_ω axis, and 20 degree about z_ω axis. As you can see in Table 1, the value of g_{fm} is 25 times the value of

Table 1. the parameter values of model helicopter.

parameter	value	parameter	value
g_{fm}	$0.1(kg/m^3)$	e	$0.01(m)$
g_{ft}	$2.5(kg/m^3)$	d	$0.22(m)$
r_1	$0.78(m)$	l	$0.96(m)$
r_2	$0.135(m)$	Ω	$2400(rpm)$

y_ω axis, and 20 degree about z_ω axis. As you can see in table 1, the value of g_{fm} is 25 times the value of g_{ft} because the gear ratio between main rotor and tail rotor is 5. The proposed control law is chosen with the design parameters k_x=3, k_y=3, k_z=3, k_1=3, k_2=10 and k_3=6. The sampling period is set to 0.01 second.

Even though we derive the control law under the assumption that $\omega_b(t)$ is very small, in simulation we consider the term $\omega_b(t) \times J_b \omega_b(t)$ in dynamics of rotational motion ,*i.e.*, we apply proposed control law to real dynamics such as

$$J_b \dot{\omega}_b(t) + \omega_b(t) \times J_b \omega_b(t) = \tau_b(t) \qquad (53)$$

Because of the robustness of sliding mode control, we can obtain good results even though we consider simplified model to derive the control law. The simulation results are shown in Figs. 4, 5 and 6.

From Fig. 4, it can be seen that rotational matrix ${}^wR_b(t)$ tracks identity matrix I within 1.5 seconds. From Fig. 5, it can be seen that the state trajectory hit the sliding surfaces within 1 second. As you can see in Fig. 6, there occur chattering in control input signals. In real implemetation of control law, we should consider the chattering problems. This problem can be solved simply if we use tangent-hyperbolic function instead of sgn function or use low pass filter in the controller.

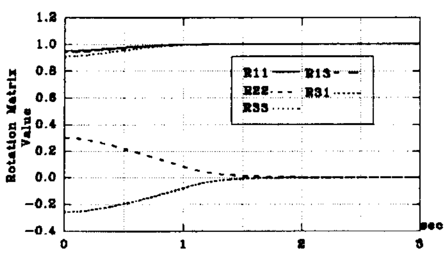

Fig. 4. Rotational matrix ${}^wR_b(t)$.

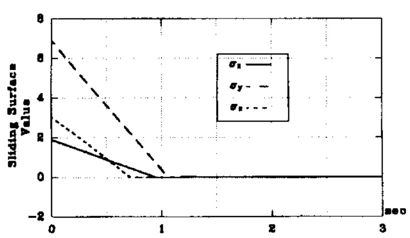

Fig. 5. Sliding surfaces ($\sigma_x, \sigma_y, \sigma_z$).

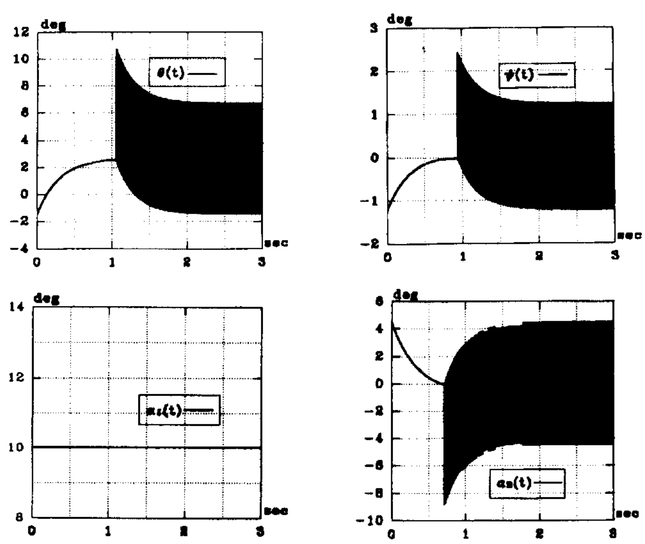

Fig. 6. Control input signals ($\phi(t), \theta(t), \alpha_1(t), \alpha_2(t)$).

VI. Conclusion

In this paper, we derived the simplified dynamic equations of model helicopter under the assumption that the angular velocity of helicopter is very small and proposed robust controller for stabilizing the attitude of helicopter by using the variable structure system. Even though we used the simplified dynamic model for deriving the control law, we obtain good simulation results because of the robustness of sliding mode control. From this research, we know the fact that the design of controller is more easy if the model helicopter is designed so that the moment of inertia is close to identity matrix. Even though the model helicopter is not designed so that the moment of inertia is close to identity matrix, the robustness of sliding mode control can cover this kind of control problem. The values of g_{fm} and g_{ft} must be known to implement the proposed control algorithm and therefore some algorithm to estimate these values from the raw data of model helicopter must be developed. The remaining research work is to invent some kinds of sensors such as gyroscope to obtain the information of ${}^wR_b(t)$ and ω_w to implement proposed control law. We hope that the proposed control algorithm can be modified and be applied to real helicopter. Furthermore the chattering problem must be consider to implement proposed controller and the simple remedy of it is that we use tangent-hyperbolic function instead of sgn function. From the computer simulation results, we verify the effectiveness of proposed control algorithm.

Nomenclature

$\phi(t)$	: lateral tilt angle
$\theta(t)$	: longitudinal tilt angle
$a_1(t)$	: angle of attack of main rotor
$a_2(t)$	: angle of attack of tail rotor
${}^wR_b(t)$	: rotational matrix of body attached coordinate system with respect to world coordinate system
${}^bR_{mr}(t)$	: rotational matrix of main rotor coordinate system with respect to body attached coordinate system
r_1	: radius of main rotor
r_2	: radius of tail rotor
J_b	: moment of inertia of helicopter
m_b	: mass of helicopter
$\Omega(t)$	: angular velocity of main rotor
$\tau(t)$	: total torque of helicopter
$f_\omega(t)$	: total force of helicopter
$p_w(t)$	: position of helicopter's center of gravity

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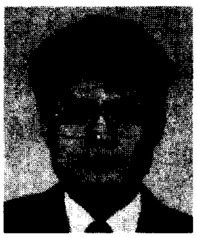

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