On the Adaptive Pre-Processing Technique for the Linearization of Weakly Nonlinear Volterra Systems

볼테라 시스템 선형화를 위한 적응 선행처리 기법

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요 약 : 본 논문에서는 볼테라 비선형 시스템의 선형화를 위한 새로운 적응 선행처리 기법을 제시한다. 특히, 제안된 적응 선행처리 기법은 (i) 순수 비선형 왜곡 보상을 위한 부분(pure nonlinear distortion compensator: PNDC)과, (ii) 선형 왜곡 보상을 위한 선형 역필터(linear inverse filter: LIF)의 두 부분으로 구성된다. 본 논문의 선형화 기법의 장점으로는 기존의 P차 역(Pth-order inverse) 기법에 비하여 계산량이 상당히 감소되며, 적응 알고리듬이 보다 빠르고 안정된 수렴 특성을 나타낸다. 끝으로, 모의실험을 통하여, 제안된 선행처리 기법의 성능 및 실제 적용 가능성을 살펴본다.

Keywords : weakly nanlinear Volterra system, linearization, adaptive signal processing, preprocessing, pth-order inverse

I. Introduction

There have been many signal processing applications where undesirable nonlinearities might degrade the overall system performance [1-4]. Examples of such applications include A/D converter, QAM radio system, digital satellite channel, loudspeaker, etc. To design a compensator for such undesirable nonlinear effects, the system modelling or identification procedure has been required. For that purpose, Volterra series(a power series with memory) and neural networks have been employed to model nonlinear systems in many science and engineering. Some mildly (or weakly) nonlinear systems can be modeled by Volterra series up to a finite order, and some weakly or strongly nonlinear systems may be also modelled by neural networks. However, as the order of system nonlinearities increases, more parameters or weights are required in modelling the corresponding nonlinear system, which makes the computational process more complicated and requires much more computational burden. Thus, many researches related to nonlinear system modeling or identification have been done to relieve such computational burden.

Once the system to be linearized is modeled, several nonlinear compensation methods can be utilized to eliminate or minimize the undesirable nonlinear effects. Depending on the position of compensators with respect to the system being linearized, they can be classified into two categories: a pre-processor (or predistorter) and a post-processor (or equalizer). If the compensator is located before the system, it is called a predistorter, and, if located after the system, it is called a post-processor. In some cases, the pre-processor might be preferred: For example, (i) the sound wave distorted by a loud-speaker cannot be compensated for after it reaches a spatial point, where a post-processor cannot be applied, and (ii) in telecommunication systems, the noise enhancement effects can be avoided or minimized if the nonlinear distortion effects can be in advance compensated for before transmitting signals through the channel.

This paper deals with a new pre-processing technique for the linearization of a weakly nonlinear Volterra system (which is defined as a nonlinear system to be modeled by Volterra series up to third order : nonlinear terms with an order higher than three are small enough to be ignored in the total system response). When the linear part of the Volterra filter has a stable linear inverse, the commonly-used Pth-order inverse[5] can be applied for the nonlinearity compensation. However, as the order of system nonlinearities or the size of the system memory increases, the structure of the corresponding Pth-order inverse system becomes highly complicated and it is very difficult to implement the inverse system. Recently, one approach to solve such problems in the design of nonlinear compensators was proposed in [6], where only the lowest-order nonlinearity can be removed by the linearization method, and still remaining nonlinearities higher than the lowest one might have significant effects on the system performance. Thus, in this paper, a new adaptive pre-processing technique is presented, where the linearization method in [6] is a little modified by employing a linear adaptive filter (instead of a linear inverse filter as in [6]) in the design of a nonlinear compensator, to minimize the total distortion in the output.

In the next chapter, Volterra series modeling of weakly nonlinear systems is considered, and in Chapter III, a new adaptive pre-processing method is presented. Also, filter coefficient updating algorithms for the adaptive compensation of the pure nonlinear distortion are derived in Chapter IV, and, finally, some simulation

접수일자 : 1996. 11. 1., 수정완료 : 1997. 9. 10.
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※ 본 논문은 한국 과학 재단(961-0917-080-1)에서 지원하여 연구하였습니다.

results are provided in Chapter V to demonstrate the validity of the proposed pre-processing technique.

II. Modeling of weakly nonlinear Volterra systems

The system output, s[n], of a weakly nonlinear system, which can be modeled by a third-order Volterra series with input y[n], can be expressed by where $H_L[\cdot]$ is

$$\begin{aligned} s[n] &= \sum_{L=1}^{3} H_L[y[n]] \\ &= \sum_{i=0}^{N-1} h_1[i]y[n-i] \\ &+ \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h_2[i,j]y[n-i]y[n-j] \\ &+ \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} h_3[i,j,k]y[n-i]y[n-j]y[n-k] \end{aligned} \quad (35)$$

a Lth-order Volterra operator and $h_1[i]$, $h_2[i,j]$, $h_3[i,j,k]$ are linear, quadratic, and cubic Volterra kernels, respectively. Also, N is the system memory size. For the identification of such a weakly nonlinear system, a third-order adaptive Volterra filter (AVF) and a recursive least-squares (RLS) algorithm can be utilized to estimate the respective first-order, second-order, and third-order system kernels in an adaptive way.

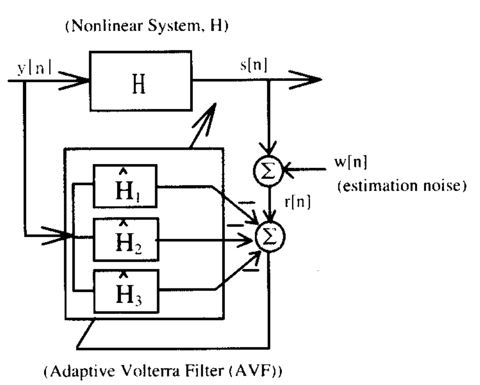

Fig. 1. Identification of a nonlinear Volterra system.

Let's consider $e_0[n]$ which is defined as the error between the system(H) output and the Volterra model output (see Fig. 1):

$$e_0[n] = s[n] - \sum_{L=1}^{3} H_L[y[n]] \quad (2)$$

By minimizing $E[e_0^2[n]]$, we can get the estimated Volterra kernels of the system H. Then, we get

$$s[n] \approx \sum_{L=1}^{3} H_L[y[n]] \quad (3)$$

From these estimated Volterra kernels, the filter coefficients of a linear inverse filter(LIF) and of a linear adaptive filter(LAF) in the proposed pre-processor (see Fig. 2) can be updated for the adaptive compensation of both linear and nonlinear distortions, which will be discussed in more detail in the next two chapters (Chapter III and IV).

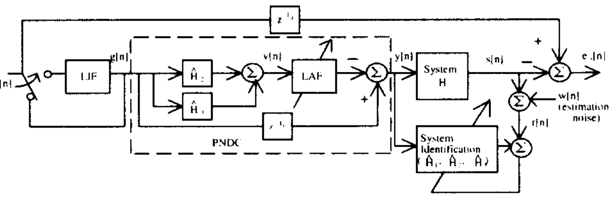

Fig. 2. The structure of the proposed adaptive pre-processing system.

III. Structure of the proposed pre-processor

The block diagram of the pre-processor proposed in this paper is presented in Fig. 2, where (i) from I/O data of the weakly nonlinear Volterra system, the linear, quadratic, and cubic Volterra kernels can be obtained by applying the adaptive Volterra filter(AVF) and the RLS algorithm, (ii) the estimated nonlinear Volterra kernels (i.e., $\hat{H}_2$, and $\hat{H}_3$) are copied to the pure nonlinear distortion compensation(PNDC) at every iteration, where the PNDC is introduced to compensate for the nonlinear distortion, (iii) before the LIF operates, x[n] is equal to g[n], and after the LIF operates, g[n] is equal to $H_1^{-1}[x[n]]$ (then, r[n] becomes $x[n-T_d]$: here, T_d is a delay), and (iv) the linear inverse filter (LIF), $H_1^{-1}[\cdot]$, which is introduced to compensation for the linear distortion (see Fig. 2), starts to operate only after the nonlinear distortion in the system output decreases to some degree(see Fig. 4).

IV. The LAF coefficient update algorithm

The LAF filter coefficients of the pure nonlinear distortion compensator(PNDC) as described in Fig. 2 can be updated by minimizing the mean square errors, $E(e_1^2[n])$, and by applying the steepest descent algorithm, where $e_1[n]$ is defined as the difference between $\hat{H}_1[g[n-T_F]]$ and the system model output, r[n], of the weakly nonlinear system ($e_1[n]$ is the purely nonlinear distortion part in the system output and T_F is the delay of LAF filter: see (4) and Fig. 2):

$$e_1[n] = \hat{H}_1[g[n-T_F]] - r[n] \quad (4)$$

Also, the input, y[n], to the weakly nonlinear system, can be expressed as follows:

$$y[n] = \left[-\sum_{m=0}^{M-1} f[m]v[n-m] + g[n-T_F]\right] \quad (5)$$

where

$$\begin{aligned} v[n] &= \sum_{L=2}^{3} \hat{H}_L[g[n]] \\ &= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \hat{h}_2[i,j]g[n-i]g[n-j] \\ &+ \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} \hat{h}_3[i,j,k]g[n-i]g[n-j]g[n-k] \end{aligned} \quad (6)$$

and f[m]'s (m=0, 1, ..., M-1), are the LAF filter coefficients (see Fig. 2) for the nonlinear distortion compensation. Also, (M-1) is the order of the LAF and $\hat{H}_L[\cdot]$ represents the estimated Lth-order Volterra operator of the nonlinear system to be linearized.

To find the LAF filter coefficients optimal in a mean square error sense, the method of steepest-descent[7] can be utilized, which allows the LAF coefficients (i) to be updated in a direction opposite to that of the derivative of the mean-squared-error or cost function (gradient vector) evaluated with respect to the filter coefficient vector and (ii) to converge to an optimum at which each gradient value is zero (note that the LAF is a linear filter). By applying the steepest descent algorithm to (4)-(6), we can get the following:

$$\begin{aligned} f_{n+1}[m] &= f_n[m] - \frac{\mu}{2}\frac{\partial e_1^2[n]}{\partial f_n[m]} \\ &= f_n[m] + \mu e_1[n]\frac{\partial r[n]}{\partial f_n[m]} \\ &(m=0,\cdots,M-1) \end{aligned} \tag{7}$$

where

$$r[n] \approx \sum_{L=1}^{3} \hat{H}_L[y[n]] + w[n] \tag{8}$$

$$\frac{\partial r[n]}{\partial f_n[m]} = \sum_{r=0}^{N-1} \frac{\partial r[n]}{\partial y[n-r]}\frac{\partial y[n-r]}{\partial f_n[m]} \tag{9}$$

$$\begin{aligned} \frac{\partial r[n]}{\partial y[n-r]} &= \hat{h}_1[r] \\ &+2\sum_{i=0}^{N-1}\hat{h}_2[r,i]y[n-i] \\ &+3\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\hat{h}_3[r,i,j]y[n-i]y[n-j] \end{aligned} \tag{10}$$

$$\frac{\partial y[n-r]}{\partial f_n[m]} = -v[n-r-m] \tag{11}$$

From (8)-(11), the LAF filter coefficients, f[m]'s (m=0, 1, ..., M-1), can be obtained adaptively.

V. Computer simulation

To demonstrate the validity of the proposed approach, three simulations are considered:

1. Linearization of a memoryless nonlinear system:

The system considered in the simulation is a third-order memoryless Volterra system which is expressed by

$$\begin{aligned} r[n] &= H[y[n]] \\ &= y[n] + 0.2y^2[n] + 0.5y^3[n] \end{aligned} \tag{12}$$

In this simulation, the LAF becomes also memoryless and its coefficient, f[0], which is optimal in the mean square error sense, is given by 0.4749. Futhermore, Fig. 3 shows the input-output relationship (linearized outputs) when the magnitude of the input is on the interval [-1,1], where the Curve-(a) is the system output without any compensation, the Curve-(b) is the linearized output obtained by applying the Gao's linearization method[6], and the Curve-(c) is the linearized output obtained by the proposed method (also, the Curve-(d) in Fig. 3 indicates the perfectly linearized output). Note that the second-order nonlinear terms can be removed by the Gao's method[6], but, higher-order nonlinear terms still remains in the

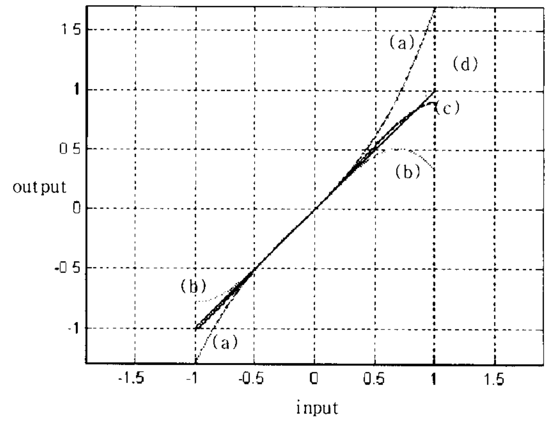

Fig. 3. The linearized output curves:
Curve-(a): Output without compensation.
Curve-(b): Output by the Pth-order inverse (P=2).
Curve-(c): Output by the proposed method.
Curve-(d): 45° line.

output (the Gao's method is equivalent to 2nd-order inverse). In particular, the linearization approach proposed in this paper, where the LAF filter coefficients are updated by minimizing total (linear and nonlinear) distortions, leads to better linearization performance than the Gao's method[6] (see Fig. 3).

2. Comparison with the commonly-used pth-order inverse method

In this simulation, we compare the performance and convergence properties of our approach with those of the Pth-order inverse method[8]. The following Volterra system, which was considered in [8], is also adopted for comparison:

$$r[n] = H_1[y[n]] + H_3[y[n]] + w[n] \tag{13}$$

where

$$\begin{aligned} H_1[y[n]] &= 0.8300y[n] \\ &+0.7097y[n-1] \\ &+0.1659y[n-2] - 0.2463y[n-3] \end{aligned} \tag{14}$$

$$\begin{aligned} H_3[y[n]] &= 1.8943y^3[n] \\ &+0.3157y[n-1]\,y[n-2]\,y[n-3] \\ &+0.3157y[n-1]\,y[n-3]\,y[n-2] \\ &+0.3157y[n-2]\,y[n-1]\,y[n-3] \\ &+0.3157y[n-2]\,y[n-3]\,y[n-1] \\ &+0.3157y[n-3]\,y[n-1]\,y[n-2] \\ &+0.3157y[n-3]\,y[n-2]\,y[n-1] \end{aligned} \tag{15}$$

Note that the linear part of the Volterra system has minimum phase. Also, the input is chosen as a Gaussian random signal with 30 dB SNR as in [8].

The performance criterion (i.e., normalized minimum mean square error: NMSE) is defined by

$$10\log_{10}\left(\frac{E[(d[n-T_d]-z[n])^2]}{E[d^2[n-T_d]]}\right) \tag{16}$$

where d[n] is equal to $H_1[x[n-T_F]]$ before the LIF operates, as described in Chapter III and in Fig. 2, and d[n] becomes $x[n-T_d]$ (here, T_d is the possible delay from LIF and LAF) after the LIF is connected.

The simulation results in Fig. 3 show that (i) the nonlinear distortions were reduced by about 16 dB in the first part (when the LIF is off) and (ii) in the second part (at iteration number 3000, when the LIF starts to operate), the normalized minimum mean square(NMSE) decreases to about -23[dB] at iteration number 6000. Note that, in the Pth-order inverse method[8], where the system parameters assumed to be known (identification was not performed), the -23[dB] level of the NMSE curve was reached at iteration number 30000. In our approach, the system identification procedure is also performed in an adaptive way, but in spite of this additional computational burden, much (about 5 times) faster and more stable converging characteristcs can be obtained in our approach under the same conditions as in [8].

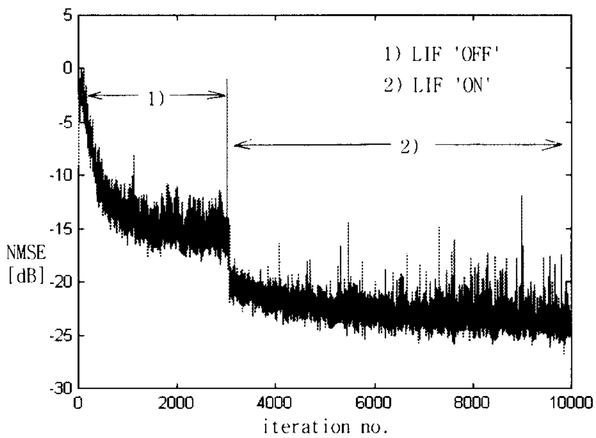

Fig. 4. NMSE obtained by the proposed approach.
1) Reduction of nonlinear distortion,
2) Reduction of total distortions.

3. Application to the compensation for the satellite communication channel

In this simulation, the performance of the proposed nonlinear pre-processor is tested by utilizing a baseband model of a satellite communication channel[9], as shown in Fig. 5: the transmit and receiver filters are given by TX=[0.8, 0.1] and RX=[0.9, 0.2, 0.1], respectively, and the nonlinearities of TWT(traveling wave tube) may be characterized by the following AM/AM and AM/PM conversions:

$$A(r)=\frac{\alpha_a r}{1+\beta_a r^2} \quad (17)$$

$$\Phi(r)=\frac{\alpha_\phi r^2}{1+\beta_\phi r^2} \quad (18)$$

In (17)-(18), $\alpha_a=2$, $\beta_a=1$, $\alpha_\phi=\pi/3$, $\beta_\phi=1$, and r is the input amplitude. If θ is the phase of the TWT input, then the amplitude and phase of the TWT output can be expressed by $A(r)$ and $\Phi(r)+\theta$. Then, the satellite channel can be modeled by Volterra series with odd-order nonlinearities[3][4][9]. For this simulation, its third-order Volterra model, and 16-PSK signals with 0.68 magnitude (as the input to the pre-processor) are applied.

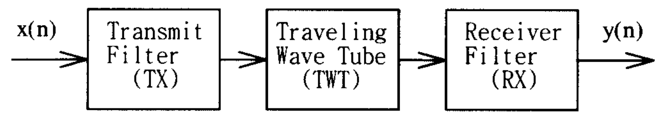

Fig. 5. The basedband model of satellite communication channel.

To verify the performance of the proposed method, the Gao's linearization approach is simulated at the same condition and compared with the proposed method. From the simulation results, the NMSE(normalized minimum mean square error) of the Gao's method is 0.0053(-22.7dB) and that of the proposed pre-processing technique is 0.0012(-29.2dB) in case of the 2000-point test signals. The scattering diagram at the output of the nonlinear channel when the proposed approach is applied is shown in Fig. 6.

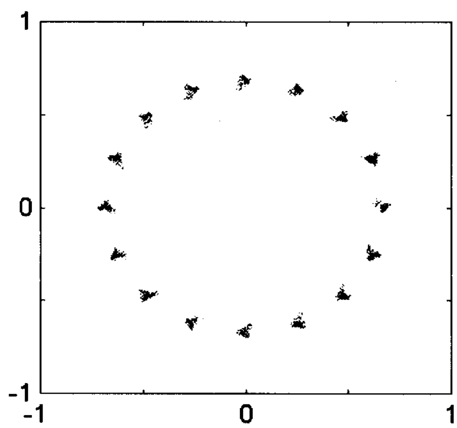

Fig. 6. The compensated output of a satellite communication channel.

VI. Conclusion

In this paper, a new pre-processing technique to linearize a weakly nonlinear system is presented. In particular, the proposed pre-processor has (i) a simpler structure, consisting of a linear inverse filter (LIF) and a pure nonlinear distortion compensator (PNDC), and (ii) faster convergence property than the Pth-order inverse method. The reason is that, while all Volterra kernel coefficients in the Pth-order inverse method are required to be updated at every iteration, only the LAF coefficients in the proposed linearization approach need to be updated, which leads to (i) considerable reduction of structural complexity and computational burden and (ii) easy implementation of the linearization algorithm. Also, the computer simulations demonstrate the good performance of the proposed adaptive pre-processing method. From these encouraging simulation results, we are going as a next step to analyze real experimental data and utilize the derived results for the linearization of real weakly nonlinear systems.

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