

An Adaptive Controller with Fuzzy Compensator for Nonlinear Time-Varying Systems

비선형 시변 시스템을 위한 퍼지 보상기를 가진 적응 제어기

Ko-Dong Park, Wan-Soo Jeon, Jong-Hwa Kim, Man-Hyung Lee

(박 거 동, 전 완 수, 김 중 화, 이 만 형)

요 약 : 본 논문에서는 비선형 시변 시스템을 제어할 경우 제어시스템의 안정성을 보장하고 성능을 향상시키기 위한 새로운 적응제어 구조를 전개하였다. 주어진 플랜트가 선형 시불변이라는 가정하에 표준 기준모델 적응제어기가 적용될 경우 발생하는 출력오차는 플랜트의 비선형 시변특성으로 인하여 점근적으로 0에 수렴되지 않는다. 이 때 미지의 출력오차를 점근적으로 0에 수렴시키는 방법으로 퍼지보상기를 사용하였으며 결과적으로 플랜트의 비선형 시변 특성을 보상하는 효과를 얻을 수 있었다. 퍼지보상기로는 출력오차등의 조건에 따라 이득이 변하는 퍼지 PID 보상기를 도입하여 안정하게 설계되도록 노력하였다. 또한 출력오차를 점근적으로 0에 수렴시키는 것은 표준 기준 모델 적응제어기 내부의 모든 파라미터와 신호가 유한하게 됨을 의미하기 때문에, 제어시스템 전체의 안정도를 보장할 뿐만 아니라 결과적으로 과도응답 성능을 향상시킬 수 있게 되었다. 몇 가지 예제를 대상으로 시뮬레이션을 수행하고 그 결과를 분석함으로써 비선형 시변 시스템을 제어할 경우 본 논문에서 전개된 새로운 적응제어 구조의 타당성을 확인하였다.

Keywords: model reference adaptive control(MRAC), fuzzy compensator, global stability, transient response, bounded parameters and signals

I. Introduction

When uncertain parameters are contained in a controlled system or parameters are unknown, it is not easy to design a suitable controller using conventional control methods which necessitate complete mathematical model, because of the deficient model information. In order to overcome this problem a new method named as adaptive control appeared. Adaptive control is the method which updates control parameters used to generate control input in a way that the output of the unknown plant follows that of a reference model so that the output error goes to zero asymptotically.

Recently, new adaptive control methods have been developed and their interest are concentrated in performance improvement such as transient response and steady state response[1] in addition to stability. Since the standard model reference adaptive control(MRAC) scheme has bad transient behavior when bounded disturbances are present, modified MRAC schemes[2-4] had been proposed to improve the transient response. Zhihua Qu et al. proposed model reference robust control(MRRC) scheme[5] which introduced the concept the robust control into the MRAC in order to cope with uncertainties and slow-varying nonlinearities. However, since these modified schemes have been developed through mathematical modifications and structural modifications in the standard structure, there still remain some problems such as mathematical complexity and difficulty in the proof of stability for applications. Especially in case a plant is modeled as highly nonlinear and/or time-varying system,

those schemes cannot be applied in order to assure global stability and to improve performance of it, because they cannot satisfy several assumptions of adaptive control schemes based on the linear time-invariant assumption.

Therefore, in this paper, a new adaptive control structure, titled as model reference adaptive control with fuzzy compensator(MRACF), is developed and suggested in order to improve transient response and intuitively assure robustness against nonlinearity and time-varying characteristics of plant under disturbances. The structure is only the simple form which is combined the standard MRAC with a fuzzy compensator. The role of the fuzzy compensator is to compensate the unknown nonlinear time-varying characteristics of the given plant so that it forces the output error of the MRAC to converge to zero asymptotically. As long as the output error is converged to zero asymptotically by the fuzzy compensator, parameters and all internal signals of the MRAC are bounded by the adaptation ability of standard MRAC. Resultantly the stability of the overall control system is assured and the transient performance is improved. Through simulation studies the effectiveness of suggested MRACF is proved.

II. Model Reference Adaptive Control with Fuzzy Compensator(MRACF)

When a controlled system is modeled as a nonlinear time-varying system, the standard MRAC scheme itself is not adequate to control it. Therefore, a new control scheme which is developed now based on the standard MRAC must have the ability to handle the effect of modeling error and uncertainty due to model approximation as a linear time-invariant system. And also it must handle the effect of disturbances. In order to solve

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박거동 : 한국해양대학교 제어계측공학과 박사과정

전완수 : 국방과학연구소

김중화 : 한국해양대학교 자동차·정보공학부

이만형 : 부산대학교 기계기술연구소

these problems, a new adaptive control scheme, named as MRACF, is suggested. Its structure is shown in Fig.1 and has only the simple form which is combined the standard MRAC with a fuzzy compensator.

The concept of the MRACF is as follows.

1) The MRAC is applied to control the unknown nonlinear time varying plant which is assumed to be linear time-invariant by using an appropriate reference model. Hence, the global stability of the control system would not be assured and the response would naturally be bad.

2) Then a fuzzy compensator regulates the output error of the MRAC in the manner that it makes the output error converge to zero asymptotically. The steady state output error of the MRAC can be considered to be caused by the modeling error due to the nonlinear time-varying characteristics of the given plant, because the output error must converge to zero asymptotically in case the given plant is linear time-invariant. Therefore, a fuzzy control theory must be chosen carefully in the aspect that the correct design of a fuzzy compensator means the effective compensation of the nonlinear time-varying characteristics and the asymptotic behavior of the fuzzy compensator becomes the basics of the global stability of the overall control system. The nonlinear fuzzy PID controller[7] which evolves an asymptotic behavior was adopted to guarantee the asymptotic stability of the fuzzy compensator in this paper.

3) As long as the output error of the MRAC converges to zero asymptotically by the fuzzy compensator, because all the signals and parameters of the MRAC remain bounded, the overall system can be stable and the transient response can be improved.

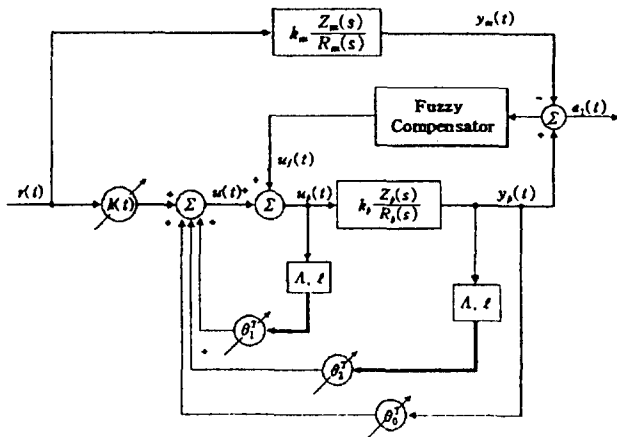


Fig. 1. The structure of the MRACF proposed in this paper.

2.1 The standard MRAC system[6]

If a plant to be controlled is linear time-invariant, it is represented by the following differential equations.

$$\begin{aligned} \dot{x}_p &= A_p x_p + b_p u(t) \\ y_p &= h_p^T x_p \end{aligned} \tag{1}$$

where A_p is an $(n \times n)$ matrix, h_p and b_p are n -vectors, $u: \mathbb{R}^+ \rightarrow \mathbb{R}$ is the input, $y_p: \mathbb{R}^+ \rightarrow \mathbb{R}$ is the

output, and $x_p: \mathbb{R}^+ \rightarrow \mathbb{R}^n$ is the n -dimensional state vector of the plant. The transfer function $W_p(s)$ of the plant may be represented as

$$W_p(s) = h_p^T (sI - A_p)^{-1} b_p \triangleq k_p \frac{Z_p(s)}{R_p(s)} \tag{2}$$

$W_p(s)$ is strictly proper with $Z_p(s)$ a monic Hurwitz polynomial of degree $m (\leq n-1)$, $R_p(s)$ a monic polynomial of n , and k_p a constant parameter. We further assume that only m , n , and the sign of k_p are known.

A reference Model M represents the behavior expected from the plant when it is augmented with a suitable controller. The model has a reference input $r(t)$ which is piecewise-continuous and uniformly bounded and an output $y_m(t)$. The transfer function of the model, denoted by $W_m(s)$, may be represented as

$$W_m(s) = k_m \frac{Z_m(s)}{R_m(s)} \tag{3}$$

where $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree $n-1$ and n respectively, and k_m is a positive constant.

2.1.1 The controller structure

The controller is described completely by the differential equation

$$\begin{aligned} \dot{\omega}_1(t) &= \Lambda \omega_1(t) + \ell u(t) \\ \dot{\omega}_2(t) &= \Lambda \omega_2(t) + \ell y_p(t) \\ \omega(t) &\triangleq [\kappa(t), \omega_1^T(t), y_p(t), \omega_2^T(t)]^T \\ \theta(t) &\triangleq [k(t), \theta_1^T(t), \theta_0(t), \theta_2^T(t)]^T \\ u(t) &= \theta^T(t) \omega(t) \end{aligned} \tag{4}$$

where $k: \mathbb{R}^+ \rightarrow \mathbb{R}$, $\theta_1, \omega_1: \mathbb{R}^+ \rightarrow \mathbb{R}^{n-1}$, $\theta_0: \mathbb{R}^+ \rightarrow \mathbb{R}$, $\theta_2, \omega_2: \mathbb{R}^+ \rightarrow \mathbb{R}^{n-1}$, and Λ is an $(n-1) \times (n-1)$ stable matrix.

The overall system can also be represented as

$$\begin{aligned} \begin{bmatrix} \dot{x}_p \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} &= \begin{bmatrix} A_p & 0 & 0 \\ 0 & \Lambda & 0 \\ \ell h_p^T & 0 & \Lambda \end{bmatrix} \begin{bmatrix} x_p \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} b_p \\ \ell \\ 0 \end{bmatrix} [\theta^T(t) \omega(t)] \\ y_p &= h_p^T x_p \end{aligned} \tag{5}$$

The following parameter errors are defined as

$$\begin{aligned} \psi(t) &\triangleq k(t) - k^*, \quad \phi_0(t) \triangleq \theta_0(t) - \theta_0^*, \quad \phi_1(t) \triangleq \theta_1(t) - \theta_1^* \\ \phi_2(t) &\triangleq \theta_2(t) - \theta_2^*, \quad \phi(t) \triangleq [\psi(t), \phi_1^T(t), \phi_0(t), \phi_2^T(t)]^T. \end{aligned}$$

Then the state can also be written as

$$\dot{x} = A_c x + b_c [k^* r + \phi^T \omega]; \quad y_p = h_c^T x \tag{6}$$

where $x = [x_p^T, \omega_1^T, \omega_2^T]^T$, $h_c = [h_p^T, 0, 0]^T$,

$$A_c = \begin{bmatrix} A_p + \theta_0^* b_p h_p^T & b_p \theta_1^{*T} & b_p \theta_2^{*T} \\ \ell \theta_0^* h_p^T & \Lambda + \ell \theta_1^{*T} & \ell \theta_2^{*T} \\ \ell h_p^T & 0 & \Lambda \end{bmatrix}, \quad b_c = \begin{bmatrix} b_p \\ \ell \\ 0 \end{bmatrix}. \tag{7}$$

When $\phi(t) \equiv 0$ that is $\theta(t) = \theta^*$, (6) also represents the nonminimal reference model which can be described by the $(3n-2)$ th order differential equation

$$\dot{x}_{mc} = A_c x_{mc} + b_c k^* r; \quad y_m = h_c^T x_{mc} \quad (8)$$

where $x_{mc} = [x_p^{*T}, \omega_1^{*T}, \omega_2^{*T}]^T$,

$$h_c^T (sI - A_c)^{-1} b_c = \frac{k_p}{k_m} W_m(s)$$

2.1.2 The error equation

The error equation between model and plant may be expressed as

$$\begin{aligned} \dot{e}(t) &= A_c e(t) + b_c [\phi^T(t) \omega(t)] \\ e_1(t) &= h_c^T e(t) \end{aligned} \quad (9)$$

where $e(t) \triangleq x(t) - x_{mc}(t)$ and $e_1 = y_p - y_m$. The output error e_1 is given by

$$e_1(t) = \frac{k_p}{k_m} W_m(s) \phi^T(t) \omega(t) \quad (10)$$

2.1.3 The adaptive laws

1) Relative degree $n^* (= n - m) = 1$

Because a model can be chosen which has a strictly positive real transfer function, the parameter error vector $\phi(t)$ is updated according to the control law

$$\dot{\phi} = \dot{\theta} = -\text{sgn}(k_p) e_1(t) \omega(t) \quad (11)$$

then the state error $e(t)$ and parameter error $\phi(t)$ are bounded. Since e_1 as well as the output of the reference model are bounded, y_p is bounded and $\omega(t)$ is bounded so that $e(t) \rightarrow 0$ as $t \rightarrow \infty$ or $|e_1(t)| \rightarrow 0$ as $t \rightarrow \infty$.

2) Relative degree $n^* \geq 2$

case (i) k_p known

The augmented error method suggested by Monopoli can suitably modify the error equation in order to implement a stable adaptive law. The augmented error can now be expressed as

$$\begin{aligned} \varepsilon_1(t) &= \bar{\phi}^T \bar{\zeta}(t) + \delta_1(t), \\ \delta_1(t) &= \bar{\theta}^* T \bar{\zeta}(t) - W_m(s) \bar{\theta}^{*T} \omega(t) \end{aligned} \quad (12)$$

where $\bar{\zeta}(t) \triangleq W_m(s) I \bar{\omega}(t)$, $\bar{\phi} = \bar{\theta} - \bar{\theta}^*$,

$\bar{\theta}^{*T} = [\theta_1^{*T}, \theta_0^{*T}, \theta_2^{*T}]$, $\bar{\theta}^T \triangleq [\theta_1^T, \theta_0, \theta_2^T]$, $\bar{\omega}^T \triangleq [\omega_1^T, y_p, \omega_2^T]$, and $\delta_1(t)$ is an exponentially decaying signal due to initial conditions. The adaptive law having the form

$$\dot{\bar{\phi}}(t) = -\varepsilon_1(t) \bar{\zeta}(t) \quad (13)$$

would suffice to assure stability.

case (ii) k_p unknown

Since the feedforward gain $k(t)$ has to be adjusted, the augmented error $\varepsilon_1(t)$ must contain an additional gain. The augmented error can now be expressed as

$$\varepsilon_1 = \frac{k_p}{k_m} \phi^T \zeta + \phi_1 e_2 + \delta_2(t),$$

$$\delta_2(t) = \frac{k_p}{k_m} (\theta^{*T} \zeta - W_m(s) \theta^{*T} \omega) \quad (14)$$

where $\zeta = W_m(s) I \omega$, $k_1(t) = k_p / k_m + \phi_1(t)$, and $\delta_2(t)$ is an exponentially decaying term due to initial conditions. Adaptive laws for adjusting $\phi(t)$ and $\phi_1(t)$ are given by

$$\begin{aligned} \dot{\phi} &= -\text{sgn}(k_p) \frac{\varepsilon_1 \zeta}{1 + \zeta^T \zeta} \\ \dot{\phi}_1 &= -\frac{\varepsilon_1 e_2}{1 + \zeta^T \zeta} \end{aligned} \quad (15)$$

These adaptive laws assure the global boundedness of all the signals in the overall adaptive system and that $\lim_{t \rightarrow \infty} e_1(t) = 0$.

2.2 Design of a Fuzzy Compensator

In case the given plant is represented by a nonlinear time-varying system, a compensator is required in order to compensate the function of the standard MRAC against modeling error resulted from linearization and time-varying characteristics. To do this role, a fuzzy compensator is adopted and the configuration of it is shown in Fig. 2 suggested in [7]. With three inputs $e_1(nT)$, $r_1(nT)$, and $a_1(nT)$, the structure of the fuzzy compensator is composed of two independent parallel fuzzy control blocks which contain fuzzy control rules and defuzzifier respectively. The incremental output of the fuzzy compensator is formed by algebraically adding the two outputs of fuzzy control blocks.

The notations employed in Fig. 2 are as follows :

$$\begin{aligned} e_1(nT) &= \text{sampling}[e_1(t)] \big|_{t=nT}, \quad e_1^* = GE * e_1(nT) \\ r_1(nT) &= [e_1(nT) - e_1(nT - T)] / T \\ r_1^* &= GR * r_1(nT) \\ a_1(nT) &= [r_1(nT) - r_1(nT - T)] / T \\ &= [e_1(nT) - 2e_1(nT - T) + e_1(nT - 2T)] / T^2 \\ a_1^* &= GA * a_1(nT), \quad u_f(nT) = du_f(nT) + u_f(nT - T) \\ du_f(nT) &= GU_f * du_f(nT) \\ du_f(nT) &= dU_{1f}(nT) + dU_{2f}(nT) \end{aligned}$$

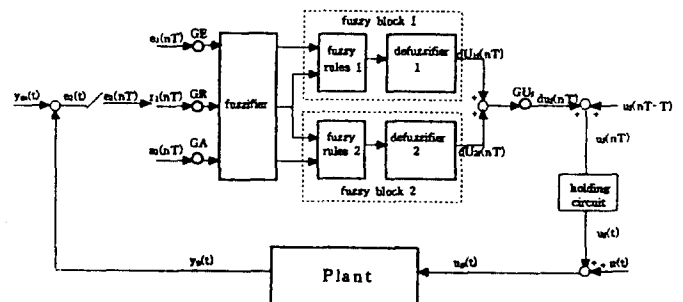
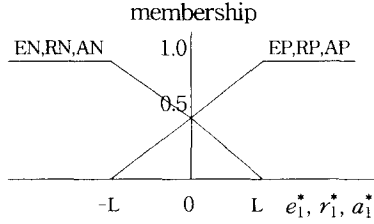


Fig. 2. Configuration of the fuzzy compensator.

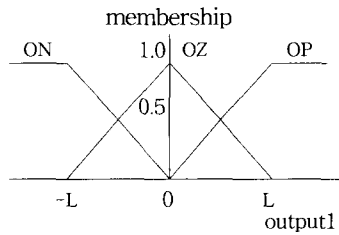
2.2.1 Fuzzification algorithms for scaled inputs and outputs of the fuzzy compensator

The fuzzification algorithms for scaled inputs and outputs are shown in Fig. 3. The fuzzy set "error" has two members EP(error_positive) and EN (error_negative) ; the fuzzy set "rate" has two members RP(rate_positive) and RN(rate_negative) ; the fuzzy set "acc" also has two members AP(acc_positive) and AN(acc_negative). The

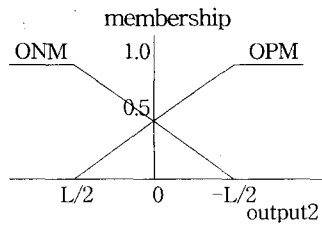
fuzzy set "output1" has three members OP(output_ positive), OZ(output_zero) and ON(output_negative) for the fuzzification of incremental output of fuzzy block 1. The fuzzy set "output2" has two members OPM(output_ positive_middle) and ONM(output_negative_middle) for the fuzzification of incremental output of fuzzy block 2.



(a) The inputs of fuzzy compensator.



(b) The fuzzified output1 of fuzzy block 1.



(c) The fuzzified output2 of fuzzy block 2.

Fig. 3. Fuzzification algorithms for fuzzy compensator.

2.2.2 Fuzzy rules and fuzzy logics for evaluation of the fuzzy rules

For fuzzy block 1, four linear fuzzy rules are given as:

- (R1)₁ : if error = EP and rate = RP then output = ON
- (R2)₁ : if error = EP and rate = RN then output = OZ
- (R3)₁ : if error = EN and rate = RP then output = OZ
- (R4)₁ : if error = EN and rate = RN then output = OP

For fuzzy block 2, four linear fuzzy rules are given as:

- (R1)₂ : if rate = RP and acc = AP then output = ONM
- (R2)₂ : if rate = RP and acc = AN then output = OPM
- (R3)₂ : if rate = RN and acc = AP then output = ONM
- (R4)₂ : if rate = RN and acc = AN then output = OPM

The eight different combinations of scaled error and scaled rate constituting inputs to the rules are shown graphically in Fig. 4 for the block 1. For the block 2, the eight different combinations of scaled rate and scaled acc are shown in Fig. 5[7].

2.2.3 Defuzzification algorithm

The defuzzified output of a fuzzy set is defined as the center of area method.

$$dU = \frac{\sum(\text{membership of member}) * (\text{value of member})}{\sum(\text{memberships})} \quad (16)$$

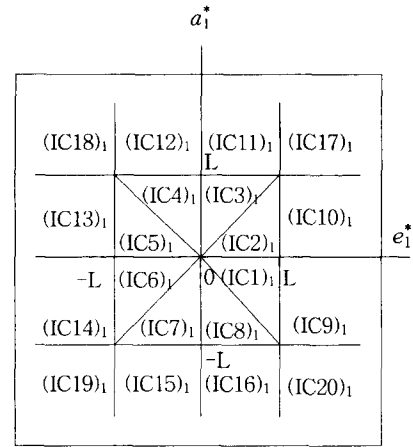


Fig. 4. Possible input combinations of e_1^* and r_1^* for fuzzy block 1.

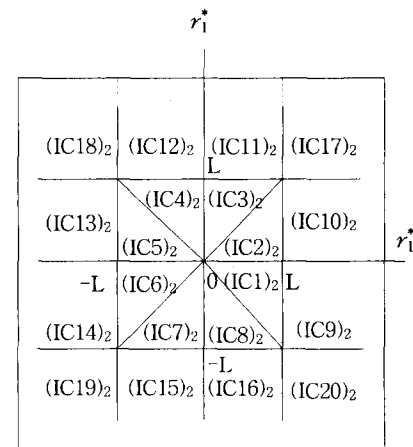


Fig. 5. Possible input combinations of r_1^* and a_1^* for fuzzy block 2.

When the above method is applied to each fuzzy control block, the defuzzified output of the fuzzy block 1 at sampling time nT , $dU_{1f}(nT)$, can be described by the following two equations.

$$\begin{aligned} & \text{If } GR * |r_1(nT)| \leq GE * |e_1(nT)| \leq L, \\ & dU_{1f}(nT) = -\frac{0.5 * L}{2L - GE * |e_1(nT)|} \\ & \quad * [GE * e_1(nT) + GR * r_1(nT)] \end{aligned} \quad (17)$$

$$\begin{aligned} & \text{If } GE * |e_1(nT)| \leq GR * |r_1(nT)| \leq L, \\ & dU_{1f}(nT) = -\frac{0.5 * L}{2L - GR * |r_1(nT)|} \\ & \quad * [GE * e_1(nT) + GR * r_1(nT)] \end{aligned} \quad (18)$$

The defuzzified output of the fuzzy block 2 at sampling time nT , $dU_{2f}(nT)$, can be given by the following two equations.

$$\begin{aligned} & \text{If } GA * |a_1(nT)| \leq GR * |r_1(nT)| \leq L, \\ & dU_{2f}(nT) = -\frac{0.25 * L}{2L - GR * |r_1(nT)|} [A * a_1(nT)] \end{aligned} \quad (19)$$

$$\begin{aligned} & \text{If } GR * |r_1(nT)| \leq GA * |a_1(nT)| \leq L, \\ & dU_{2f}(nT) = -\frac{0.25 * L}{2L - GA * |a_1(nT)|} [GA * a_1(nT)] \end{aligned} \quad (20)$$

Conclusively, the incremental output of fuzzy compensator can be divided into four different forms according to the following conditions :

1) If $GR * |r_1(nT)| \leq GE * |e_1(nT)| \leq L$ and

$$GA * |a_1(nT)| \leq GR * |r_1(nT)| \leq L,$$

$$\begin{aligned} du_r(nT) = & -\frac{0.5 * L * GU}{2L - GE * |e_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)] \\ & - \frac{0.25 * L * GU}{2L - GR * |r_1(nT)|} [GA * a_1(nT)] \end{aligned} \quad (21)$$

2) If $GR * |r_1(nT)| \leq GE * |e_1(nT)| \leq L$ and

$$GR * |r_1(nT)| \leq GA * |a_1(nT)| \leq L,$$

$$\begin{aligned} du_r(nT) = & -\frac{0.5 * L * GU}{2L - GE * |e_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)] \\ & - \frac{0.25 * L * GU}{2L - GA * |a_1(nT)|} [GA * a_1(nT)] \end{aligned} \quad (22)$$

3) If $GE * |e_1(nT)| \leq GR * |r_1(nT)| \leq L$ and

$$GA * |a_1(nT)| \leq GR * |r_1(nT)| \leq L,$$

$$\begin{aligned} du_r(nT) = & -\frac{0.5 * L * GU}{2L - GR * |r_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)] \\ & - \frac{0.25 * L * GU}{2L - GR * |r_1(nT)|} [GA * a_1(nT)] \end{aligned} \quad (23)$$

4) If $GE * |e_1(nT)| \leq GR * |r_1(nT)| \leq L$ and

$$GR * |r_1(nT)| \leq GA * |a_1(nT)| \leq L,$$

$$\begin{aligned} du_r(nT) = & -\frac{0.5 * L * GU}{2L - GR * |r_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)] \\ & - \frac{0.25 * L * GU}{2L - GA * |a_1(nT)|} [GA * a_1(nT)] \end{aligned} \quad (24)$$

If scaled error, rate and/or acc are not within the interval $[-L, L]$, the incremental output of the fuzzy compensator is obtained from the combinations of incremental outputs for the fuzzy blocks given in Table 3 and 4 in [7].

In this section, the model reference adaptive controller with fuzzy compensator (MRACF) was developed. The role of a fuzzy compensator is to compensate the effect of the nonlinear time-varying characteristics of the given plant. And the fuzzy compensator discussed thus far has the structure which can control the nonlinear or uncertain systems in an asymptotically stable fashion. But this structure by itself cannot be applied to control an unstable plant because it needs bounded output data of the given plant in order to decide fuzzification algorithms for the fuzzy inputs. Here is the idea to combine the MRAC with the fuzzy compensator in order to control the plant which is unstable nonlinear time-varying. The unstable characteristics can be controlled to be stable by the MRAC which is assumed to be applied to a linear time-invariant subsystem of the plant. Afterwards the nonlinear time-varying characteristics can be compensated by the fuzzy compensator through the regulation of the output error in a way that the fuzzy compensator makes the output error converge to zero asymptotically. Therefore, as long as the fuzzy compensator is designed correctly, the output error converges to zero asymptotically. Then all the internal signals and parameters in the MRAC remain bounded.

Conclusively the global boundedness of the overall control system can be achieved through the following theorem.

Theorem 1: If the fuzzy compensator discussed in this paper is designed correctly for the a nonlinear time-varying plant in a way that it makes the output error of the MRAC converge to zero asymptotically, all the signals and parameters in the MRAC remain bounded and the MRACF is uniformly asymptotically stable.

Proof: The fuzzy control theory adopted in this paper is the theory which assures the asymptotic stability of a fuzzy control system if design factors are selected appropriately. Therefore, as long as the fuzzy compensator makes the output error of the MRAC converge to zero asymptotically, $\lim_{t \rightarrow \infty} e_1(t) = 0$ always holds true. It means the following inequality is satisfied.

$$\lim_{t \rightarrow \infty} \int_0^t |e_1(\tau)| d\tau < \infty \quad (25)$$

because the contradiction of (25) is the contradiction of the asymptotic convergence of $e_1(t)$. And (25) means also $|e_1(t)| < \infty$ so that $e_1 \in L^1 \cap L^\infty$. From (10) the state error $e(t)$ satisfies $e(t) \in L^1 \cap L^\infty$ because $W_m(s)$ is a strictly positive real function so that $e(t)$ has the same growth rate as $e_1(t)$. As $e_1(t)$ converges to zero, the plant output y_p remains bounded because y_m is bounded. This means the boundedness of u and $\dot{e}(t)$, that is $\dot{e}(t) \in L^\infty$. Therefore, all the signals and parameters remain bounded.

Conclusively, the MRACF is uniformly asymptotically stable and it can improve the performance of the nonlinear time-varying plant. ■

III. Simulation and results

When the plant is nonlinear and/or time-varying in the presence of bounded disturbance, simulations were executed for the standard MRAC, MRACF, and fuzzy control scheme adopted in this paper in order to compare the results.

In simulations, the fixed controller parameters Λ and ℓ in (11) were chosen as $\Lambda = -1$ and $\ell = 1$. The high frequency gain k_p was assumed to be known and the control parameters $\theta_1(t)$, $\theta_0(t)$ and $\theta_2(t)$ were adjusted.

Simulation 1: In case the plant is nonlinear in the presence of bounded disturbance

$$\text{plant : } \dot{y} = y + 0.5y^2 + \dot{u} + u + v$$

$$\text{model : } \dot{y} = -y + r$$

$$\text{input : } r(t) = 5 \cos t + 15 \cos 5t$$

$$\text{disturbance : } v(t) = 0.2 \sin t + 0.5 \sin y_p + y_p^2 \cos t$$

Simulation 2: In case the plant is time-varying in the presence of bounded disturbance

$$\text{plant : } \dot{y} = -\dot{y} + (2.0 + \sin t)y + \dot{u} + u + v$$

$$\text{model : } \dot{y} = -y + r$$

$$\text{input : } r(t) = 5.0 \cos t + 15 \cos 5t$$

$$\text{disturbance : } v(t) = 0.5 \sin t + e_1 \cos 2t + 0.5 e_1^2 \cos t$$

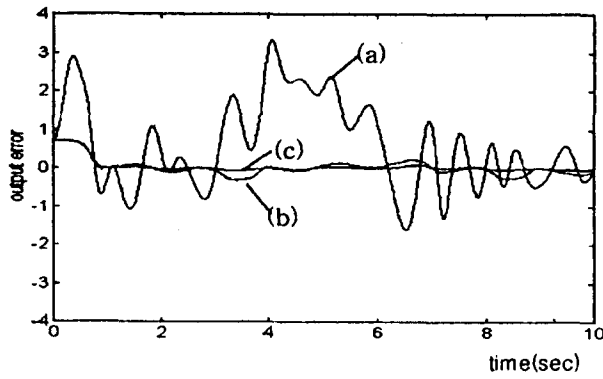


Fig. 6. Comparison of output errors between the reference model and (a) the standard MRAC (b) fuzzy control only (c) the MRACF.

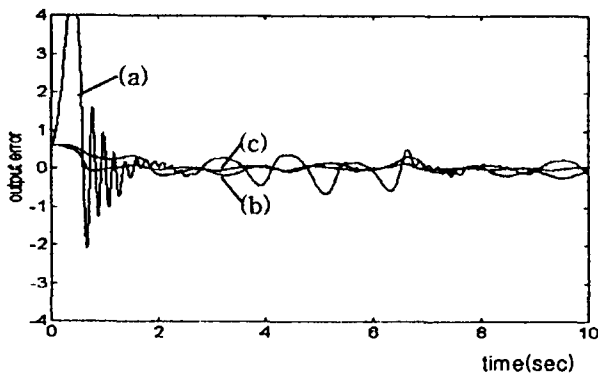


Fig. 7. Comparison of output errors between the reference model and (a) the standard MRAC (b) fuzzy control only (c) the MRACF.

Simulation 3: In case the plant is nonlinear and time-varying in the presence of bounded disturbance

$$\text{plant : } \dot{y} = \dot{y} + (2.0 + \cos t)y^2 + \dot{u} + u + v$$

$$\text{model : } \dot{y} = -y + r$$

$$\text{input : } r(t) = 5 \cos t + 20 \cos 5t$$

$$\text{disturbance : } v(t) = 0.5 \sin t + e_1 \cos 2t + 0.5 e_1^2 \cos t$$

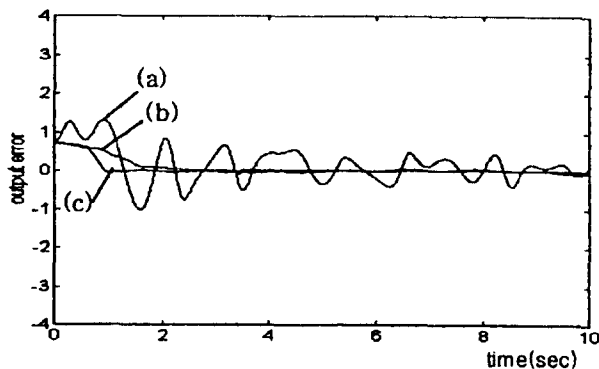


Fig. 8. Comparison of output errors between the reference model and (a) the standard MRAC (b) the fuzzy control only (c) the MRACF.

The above simulations were executed using the inputs which were persistently excited. The persistent excitation input means that the reference input for the plant is time-varying and unknown so that the tracking control should be accomplished.

As shown in Fig. 6 ~ Fig. 8, the output error e_1 does not converge to zero as $t \rightarrow \infty$ in the standard MRAC. While the output errors in the MRACF and the fuzzy control scheme converge to zero as $t \rightarrow \infty$, but it is seen that the response of the MRACF is better than that of the fuzzy control scheme. For this reason it is naturally considered that the compensation ability of the fuzzy compensator was added to the adaption ability of the standard MRAC. As shown in Fig. 9, the parameter error of the MRAC is not bounded as $t \rightarrow \infty$ and the increasing rate is high, but the parameter error of the MRACF is nearly constant in spite of the nonlinear and time-varying characteristics, so that the MRACF control system is marginally stable.

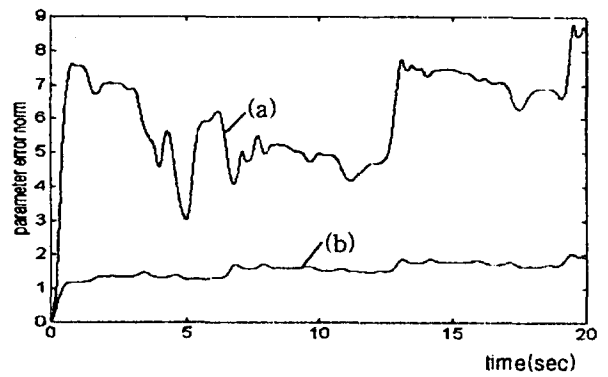


Fig. 9. The norm of the parameter error vector (a) the standard MRAC (b) the MRACF.

Conclusively, it was proved that the MRACF scheme is adequate to control a nonlinear time-varying and unknown plant. And also it was proved that the fuzzy compensator played a important role of supervising and compensating the standard MRAC scheme.

IV. Conclusion

In this paper, a MRACF scheme was suggested in order to improve performance and assure stability of the overall control system for nonlinear and time-varying systems with unknown disturbances. The scheme improved performance by using the fuzzy compensator supervising the MRAC and assured global stability in the sense that the output error and all internal signals were bounded by the fuzzy compensator, while the fuzzy compensator should be designed stably. The MRACF scheme exhibits better performance than those of the standard MRAC and only the fuzzy control scheme adopted in this paper.

The requirement of the MRACF for the prior information about the plant is equal to that of the standard MRAC. And the fuzzy compensator adopted in this paper has analytical control law, so that the MRACF scheme can be easily applied to nonlinear time-varying systems and can naturally improve performance.

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박 거 등

1972년 1월 17일생. 1994년 2월 한국해양대학교 제어계측공학과 학사. 1996년 2월 동대학교 제어계측공학과 석사. 1996년 ~ 현재 동대학원 제어계측공학과 박사과정. 연구분야는 적응제어, 퍼지제어이다.

김 종 화

1958년 11월 14일생. 1981년 부산대학교 기계공학과 학사. 1985년 동대학교 기계공학과 석사. 1989년 동대학교 기계공학과 박사. 현재 한국해양대학교 자동화·정보공학부 부교수. 관심분야는 굴삭기의 자동화와 제어, 계층적 제어구조를 가지는 로봇 제어기 설계, 퍼지제어를 응용한 불확실 모델의 강인한 제어.

전 완 수

현재 국방과학연구소 연구실장.

이 만 형

제어·자동화 시스템공학 논문지 제 3권, 제 1호 참조.