

Design of a Discrete-Time H_∞ Controller with Preview Action

예견 기능을 가진 이산시간 H_∞ 제어기의 설계

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요 약 : 이산시간 H_∞ 제어에 의한 최적 예견제어기를 제안한다. 기존의 H_∞ 제어기는 미지의 외란만 고려한 것이고, LQ에 의한 예견제어기는 예견 가능한 외란을 보상하도록 설계되었다. 본 연구에서는 예견 가능한 외란과 미지의 외란이 동시에 가해지는 동적 시스템의 전달함수 행렬의 infinity norm을 최소화하는 피드백제어기가 동시에 설계된다. 제어기의 설계는 full-information H_∞ 제어 이론을 따르나, 그 유도 과정은 LQ에 기초한 예견제어기와 유사하게 이루어진다. 설계된 H_∞ 예견 계인 행렬은 LQ예견 계인 행렬과 유사한 구조를 갖는다. 전달함수 행렬의 infinity norm이 ∞ 로 갈수록 H_∞ 예견 계인 행렬은 LQ에 의한 것에 접근한다. LQ 예견 계인 행렬은 H_∞ 예견 계인 행렬의 부분 집합임이 입증한다.

Keywords : H_∞ , preview, LQ, control, disturbance, full-information

I. Introduction

Most of control systems are designed in the form of feedback structure depending on the error signals in the current instant without considering future information, such as tracking commands and measurable disturbances into the systems. But if we know the future input informations and these are fully reflected on a control law, it is anticipated that the resulting control system may have better performances. Thus, several researches have been performed on the preview control since 1960's, but most of them only illustrate its importance with simplified examples. The preview controller, in general, has a two-degree-of-freedom structure and its design procedure involves separate optimization of the preview filter and the feedback controller.

Tomizuka[1] presented the general optimal preview controller taking account of measurement noises as well as stochastic informations. He established the seminal researches on the region of the preview control through the successive contributions. Tomizuka and Rosenthal[2] presented a digital controller with PID type structure and Peng and Tomizuka[3] also derived a preview controller by a frequency shaped LQ control in the continuous time domain. Huisman *et al.*[4] have derived a control strategy with preview for an active suspension based on LQ control. But the dynamical systems, in general, are corrupted with unknown bounded disturbances. LQ controller syntheses don't consider the informations about the unmeasurable disturbances acting into the plant when designing a controller.

It is more common in modern times to design a controller by considering the unknown disturbances into the plant. The effects of the disturbances on the systems should be fully considered in order to design control

systems to get better performance. H_∞ control is known as a controller design method to be able to handle minimization of the maximum output energy for all command and disturbance inputs with bounded energy[5].

Most of results on the preview control have been derived from LQ control theory, but preview controllers based on the H_∞ control have not appeared yet. This paper presents a design method of the preview controller based on discrete-time full-information(FI)/state-feedback(SF) H_∞ control theory for the case where some plant disturbances are measured and/or even previewable and others are unmeasurable.

The preview and feedback controller are designed to minimize the worst case RMS value of the regulated variables when the bounded unknown disturbances and the previewable disturbances hit the dynamical plants. Namely, both the preview and feedback controller are simultaneously optimized so that the infinity norm of the transfer function matrix from both the previewable and unmeasurable disturbances to the regulated variables is minimized. The FI H_∞ controller for the continuous-time case uses only the state signal $x(t)$, if $D_{11}=0$ in the generalized plant. $D_{11}=0$ means that the feed through term of the unmeasurable disturbances $w(k)$ to the regulated output does not exist in the generalized plant. But the discrete-time FI H_∞ controller uses both $x(k)$ and $w(k)$ even in the case $D_{11}=0$ [6]. Namely, the controller needs the unknown disturbances. So, a state feedback controller and the related preview controller are derived in this design, even though problem formulation and solving an algebraic Riccati equation are based on the FI controller design scheme. Since the preview controller is synthesized by considering the unknown disturbances as well as the known disturbances, the designed controller may be well suited to the real applications.

The controller derivation procedure is analogous to the LQ-based scheme, but is more complicated because the discrete algebraic Riccati equation for the full-information H_∞ controller takes a sophisticated form. The generalized plant model for designing a controller requires some assumptions to get a stabilizing controller as surveyed in Doyle *et al.*[7] and Green and Limebeer[8].

It is assumed that N time step preview measurement of previewable disturbance v is possible, which means that $v(k), v(k+1), \dots, v(k+N)$ are known in advance. A sequence of these known signals are considered as measurable states of a dynamic system. We obtain the state-feedback controller for the augmented dynamic system which consists of the original system and the measurable states. Then, the feedback controller for the dynamic system state and a feedforward controller for the previewable disturbance are designed.

The designed preview gain matrix has a similar structure as the LQ-based one. The gain matrix is sensitively determined by the infinity norm of the transfer function from the disturbance to the regulated output. As the infinity norm γ of the transfer function matrix tends to ∞ , the preview gains approach to the gains by the LQR. That is, the LQ-based preview gains are subsets of the state-feedback H_∞ -based preview gains. To illustrate the performances of the proposed controller, a SISO first-order plant is chosen as a design example. It is investigated that the external disturbance can be sufficiently attenuated by the H_∞ controller with preview compensation. Simulation results show that the H_∞ controller with the preview compensation has better regulating performances than one without compensation.

II. Controller synthesis

1. A discrete-time full-information(state-feedback) H_∞ control

Consider a FI(SF) MIMO control system described by the block diagram of Fig. 1. The linear time invariant system has a state-space description of the plant. The controller K , generates the control signal u , according to

$$u = K \begin{bmatrix} x \\ w \end{bmatrix} \tag{1}$$

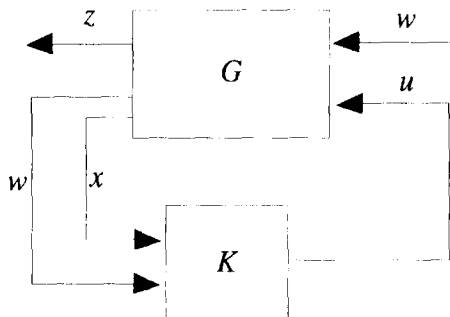


Fig. 1. The full-information configuration.

where w , is an l -dimensional exogenous input, u , is an m -dimensional control, x is an n -dimensional state, and z

is a p -dimensional objective signal.

The generalized plant G for a full-information system in the discrete-time domain is defined by linear difference equations.

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ \begin{bmatrix} I \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ I \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

Assumptions :

- 1) (A, B_2) is stabilizable,
- 2) $D_{12}'D_{12} > 0$;
- 3) $\text{rank} \begin{bmatrix} A - e^{j\theta}I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + m$, for all $\theta \in (-\pi, \pi]$.

Additional assumptions such that $D_{11} = 0$ and $D_{12}'D_{12} = I$ are not necessary in this scheme, since these only produce slight simplifications in the formulas. Define the combined signal vectors and a diagonal matrix represented by:

$$\psi(k) = \begin{bmatrix} w \\ u \end{bmatrix}, \eta(k) = \begin{bmatrix} z \\ x \end{bmatrix}, \text{ and } J = \begin{bmatrix} I_p & 0 \\ 0 & -\gamma^2 I_l \end{bmatrix}$$

The relevant state-space systems are written as

$$\begin{bmatrix} x(k+1) \\ \eta(k) \end{bmatrix} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} x(k) \\ \psi(k) \end{bmatrix}$$

where

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ 0 & I_l & 0 \end{bmatrix}$$

Two matrices R and L are introduced as follows to simplify H_∞ controller synthesis:

$$R = D_s' J D_s + B_s' X_\infty B_s \tag{2}$$

and

$$L = D_s' J C_s + B_s' X_\infty A_s \tag{3}$$

The matrices R and L can be partitioned conformably with $\psi = [w' \ u']'$

$$R = \begin{bmatrix} R_1 & R_2' \\ R_2 & R_3 \end{bmatrix}, \quad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

with $R_3 \in m \times m$, $L_2 \in m \times n$.

Theorem 1[8]: There exists a causal, linear, time-invariant and stabilizing full-information controller such that $\|F_\infty(G, K)\|_\infty < \gamma$ is satisfied if and only if the Riccati equation

$$X_\infty = C_s' J C_s + A_s' X_\infty A_s - L' R^{-1} L$$

has a solution such that $A_s - B_s R^{-1} L$ is asymptotically stable and $X_\infty \geq 0$ and $R_1 < 0$. Since a FI controller needs informations about exogenous signal $w(k)$ as well as the states $x(k)$, a state-feedback control law is more suitable to design a real controller. In this case, a stabilizing central full-state feedback controller that achieves the objective is given by

$$K = -(R_3 - R_2 R_1^{-1} R_2')^{-1} (L_2 - R_2 R_1^{-1} L_1) \quad (4)$$

2. An H_∞ controller with preview compensation

An H_∞ controller with preview compensation will be synthesized in the analogous method to Tomizuka[1] in this section. Consider a discrete-time linear time invariant dynamical system with a previewable disturbance and an unknown bounded disturbance. The state-space equations are given by

$$x(k+1) = \Phi x(k) + \Gamma_1 u(k) + \Gamma_2 u(k) + \Gamma_d v(k) \quad (5)$$

$$y(k) = Cx(k) \quad (6)$$

where $v(k)$ and $w(k)$ are the q -dimensional previewable disturbance and the l -dimensional unmeasurable disturbance, respectively. It is assumed that N time step preview measurement of v is possible, which means that $v(k), v(k+1), \dots, v(k+N)$ are known in advance.

It is supposed that future values of v beyond the time $k+N$ are the same as that of the $v(k+N)$, since the effects of the previewable disturbances have time correlations. Actually, the disturbances in the distant future hardly have effects on the system at the current time. A sequence of these known signals can make up measurable states of a dynamic system without the loss of generality. Expressing a sequence of previewable disturbances as states, these can be expressed as

$$x_d(k+1) = \Phi_d x_d(k) + E_d v(k+N)$$

where

$$x_d(k) = \begin{bmatrix} v(k) \\ v(k+1) \\ \vdots \\ v(k+N-1) \end{bmatrix}, \Phi_d = \begin{bmatrix} 0 & I & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} \quad (7)$$

, and $E_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix}$.

The augmented state-space equation including the previewable disturbances expressed as (7) is given by

$$\begin{bmatrix} x(k+1) \\ x_d(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_{pd} \\ 0 & \Phi_d \end{bmatrix} \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} \Gamma_2 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ E_d \end{bmatrix} v(k+N) \quad (8)$$

where $\Gamma_{pd} = [\Gamma_d \ 0 \ \dots \ 0]$.

Define the matrices for convenience as follows:

$$\overline{\Phi} = \begin{bmatrix} \Phi & \Gamma_{pd} \\ 0 & \Phi_d \end{bmatrix}, \overline{\Gamma}_1 = \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix}, \overline{\Gamma}_2 = \begin{bmatrix} \Gamma_2 \\ 0 \end{bmatrix}, \overline{E}_d = \begin{bmatrix} 0 \\ E_d \end{bmatrix}$$

$$\overline{C} = [C \ 0], \overline{x}(k) = \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix}$$

Then, the state-space equations are rewritten as

$$\overline{x}(k+1) = \overline{\Phi} \overline{x}(k) + \overline{\Gamma}_1 w(k) + \overline{\Gamma}_2 u(k) + \overline{E}_d v(k+N) \quad (9)$$

$$y(k) = \overline{C} \overline{x}(k) \quad (10)$$

The transfer function matrix for the augmented system is given as:

$$G_a = \left[\begin{array}{c|cc} \overline{\Phi} & \overline{\Gamma}_1 & \overline{\Gamma}_2 \\ \hline \overline{C} & 0 & 0 \end{array} \right]$$

where $v(k+N)$ is not included because it is the distant future signal which has little effects on the system at the current time. The FI control problem for the above descriptions will now be cast into a two-port framework. The objective is to keep the plant output small despite unknown bounded external disturbances acting on the system. The control signal is also included in the objective signal z to prevent actuator saturation. This will insure that the rank condition on D_{12} in the assumptions of the H_∞ control problems should be satisfied.

The block diagram of the generalized plant is shown in Fig. 2. The W_1 and W_2 are weighting matrices for the regulated output and the control input, respectively. The W_1 is positive semi-definite and the W_2 is positive definite. These also bound the output energy.

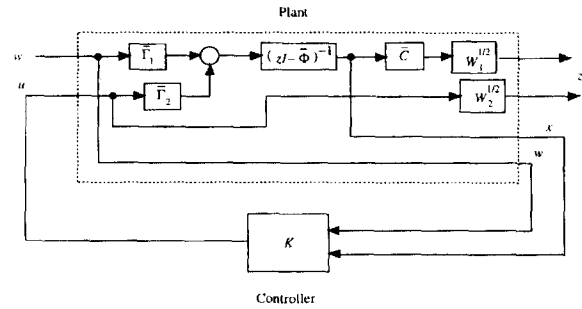


Fig. 2. Two-port formulation.

Obtaining the regulated outputs whose average RMS power is to be minimized in the formulation,

$$z = \begin{bmatrix} W_1^{1/2} G_a \begin{bmatrix} w \\ u \end{bmatrix} \\ W_2^{1/2} u \end{bmatrix} \quad (11)$$

It is assumed that $W_1^{1/2} = (W_1^{1/2})'$ and $W_2^{1/2} = (W_2^{1/2})'$. Deriving the generalized state space model from the relationships between inputs and outputs,

$$G_g = \left[\begin{array}{c|cc} \overline{\Phi} & \overline{\Gamma}_1 & \overline{\Gamma}_2 \\ \hline W_1^{1/2} \overline{C} & 0 & 0 \\ 0 & 0 & W_2^{1/2} \\ I & 0 & 0 \\ 0 & I & 0 \end{array} \right] \quad (12)$$

The relevant state space systems for designing an H_∞ controller are written as[8]

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \left[\begin{array}{c|cc} \overline{\Phi} & \overline{\Gamma}_1 & \overline{\Gamma}_2 \\ \hline W_1^{1/2} \overline{C} & 0 & 0 \\ 0 & 0 & W_2^{1/2} \\ 0 & I & 0 \end{array} \right] \quad (13)$$

The algebraic Riccati equation(ARE) for a discrete-time FI H_∞ controller is

$$X_\infty = C_s' J C_s + A_s' X_\infty A_s - L' R^{-1} L \quad (14)$$

Calculating the terms composing of the ARE,

$$R = \begin{bmatrix} -\gamma^2 I_l + \overline{\Gamma}_1' X_\infty \overline{\Gamma}_1 & \overline{\Gamma}_1' X_\infty \overline{\Gamma}_2 \\ \overline{\Gamma}_2' X_\infty \overline{\Gamma}_1 & W_2 + \overline{\Gamma}_2' X_\infty \overline{\Gamma}_2 \end{bmatrix},$$

$$L = \begin{bmatrix} \overline{\Gamma}_1' X_\infty \overline{\Phi} \\ \overline{\Gamma}_2' X_\infty \overline{\Phi} \end{bmatrix},$$

$$C_s' J C_s = \overline{C}' W_1 \overline{C}.$$

Applying the above results to the ARE,

$$X_\infty = A_s' [X_\infty - X_\infty' B_s R^{-1} B_s' X_\infty] A_s + \overline{C}' W_1 \overline{C}. \quad (15)$$

The ARE is often rewritten as

$$X_\infty = A_s' M_\infty A_s + \overline{C}' W_1 \overline{C} \quad (16)$$

where

$$M_\infty = X_\infty - X_\infty' B_s R^{-1} B_s' X_\infty. \quad (17)$$

Partitioning the matrices X_∞ and M_∞ conformably with $x(k)$ and $x_d(k)$,

$$X_\infty = \begin{bmatrix} X_{xx} & X_{xd} \\ X_{xd}' & X_{dd} \end{bmatrix}, \text{ and } M_\infty = \begin{bmatrix} M_{xx} & M_{xd} \\ M_{xd}' & M_{dd} \end{bmatrix}.$$

With the partitioned forms of X_∞ and M_∞ ,

$$\begin{bmatrix} X_{xx} & X_{xd} \\ X_{xd}' & X_{dd} \end{bmatrix} = \begin{bmatrix} \Phi' & 0 \\ \Gamma_{pd}' & \Phi_d' \end{bmatrix} \begin{bmatrix} M_{xx} & M_{xd} \\ M_{xd}' & M_{dd} \end{bmatrix} \begin{bmatrix} \Phi & \Gamma_{pd} \\ 0 & \Phi_d \end{bmatrix} + \begin{bmatrix} C' W_1 C & 0 \\ 0 & 0 \end{bmatrix} \quad (18)$$

where $M_p = \Gamma_{pd}' M_{xx} + \Phi_d' M_{xd} + (\Gamma_{pd}' M_{xd} + \Phi_d' M_{dd}) \Phi_d$.

From (18), it results that

$$X_{xx} = \Phi' M_{xx} \Phi + C' W_1 C, \quad (19)$$

$$X_{xd} = \Phi' M_{xx} \Gamma_{pd} + \Phi' M_{xd} \Phi_d. \quad (20)$$

Obtaining M_{xx} and M_{xd} with the similar method,

$$\begin{bmatrix} M_{xx} & M_{xd} \\ M_{xd}' & M_{dd} \end{bmatrix} = \begin{bmatrix} X_{xx} & X_{xd} \\ X_{xd}' & X_{dd} \end{bmatrix} - \begin{bmatrix} X_{xx} \Gamma_1 & X_{xx} \Gamma_2 \\ X_{xd}' \Gamma_1 & X_{xd}' \Gamma_2 \end{bmatrix} R^{-1} \begin{bmatrix} \Gamma_1' X_{xx} & \Gamma_1' X_{xd} \\ \Gamma_2' X_{xx} & \Gamma_2' X_{xd} \end{bmatrix}. \quad (21)$$

Define a new matrix described by:

$$\begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} = \begin{bmatrix} X_{xx} & \Gamma_1 & X_{xx} & \Gamma_2 \\ X_{xd}' & \Gamma_1 & X_{xd}' & \Gamma_2 \end{bmatrix}. \quad (22)$$

Then, the matrix M_∞ is given as a more concise form.

$$\begin{bmatrix} M_{xx} & M_{xd} \\ M_{xd}' & M_{dd} \end{bmatrix} = \begin{bmatrix} X_{xx} & X_{xd} \\ X_{xd}' & X_{dd} \end{bmatrix} - \begin{bmatrix} \overline{X}_1 R^{-1} \overline{X}_1' & \overline{X}_1 R^{-1} \overline{X}_2' \\ \overline{X}_2 R^{-1} \overline{X}_1' & \overline{X}_2 R^{-1} \overline{X}_2' \end{bmatrix}. \quad (23)$$

There exist the matrices M_{xx} and M_{xd} of the matrix M_∞ such that

$$M_{xx} = X_{xx} - \overline{X}_1 R^{-1} \overline{X}_1', \quad (24)$$

and

$$M_{xd} = X_{xd} - \overline{X}_1 R^{-1} \overline{X}_2'. \quad (25)$$

Lemma 1 : A closed-loop system matrix $\Phi_{c_{xx}}$ of the FI H_∞ control system for the states $x(k)$ excluding $x_d(k)$ is $\Phi_{c_{xx}} = (I - [\Gamma_1 \ \Gamma_2] R^{-1} \begin{bmatrix} \Gamma_1' \\ \Gamma_2' \end{bmatrix} X_{xx}) \Phi$.

Proof : The closed-loop system matrix Φ_c of the FI H_∞ control system is given as $A_s - B_s R^{-1} L$. Rearranging Φ_c ,

$$\Phi_c = \begin{bmatrix} \Phi - \overline{X}_3 R^{-1} \overline{X}_4 & \Gamma_{pd} - \overline{X}_3 R^{-1} \overline{X}_5 \\ 0 & \Phi_d \end{bmatrix} \quad (26)$$

where

$$\overline{X}_3 = [\Gamma_1 \ \Gamma_2], \quad \overline{X}_4 = \begin{bmatrix} \Gamma_1' X_{xx} \Phi \\ \Gamma_2' X_{xx} \Phi \end{bmatrix},$$

$$\overline{X}_5 = \begin{bmatrix} \Gamma_1' (X_{xx} \Gamma_{pd} + X_{xd} \Phi_d) \\ \Gamma_2' (X_{xx} \Gamma_{pd} + X_{xd} \Phi_d) \end{bmatrix}.$$

Since Φ_c is a block matrix, the characteristic polynomial of Φ_c is just the product of the characteristic polynomial of $\Phi - \overline{X}_3 R^{-1} \overline{X}_4$ and the characteristic polynomial of the Φ_d . Therefore, $\Phi - \overline{X}_3 R^{-1} \overline{X}_4$ is a closed-loop system matrix $\Phi_{c_{xx}}$ for the state $X(k)$.

$$\Phi_{c_{xx}} = (I - [\Gamma_1 \ \Gamma_2] R^{-1} \begin{bmatrix} \Gamma_1' \\ \Gamma_2' \end{bmatrix} X_{xx}) \Phi \quad (27)$$

Note that the closed-loop system matrix $\Phi_{c_{xx}}$ is determined by the state-space description of only the $x(k)$.

The matrix R can be written in a simple form

$$R = \begin{bmatrix} -\gamma^2 I_l + \Gamma_1' X_{xx} \Gamma_1 & \Gamma_1' X_{xx} \Gamma_2 \\ \Gamma_2' X_{xx} \Gamma_1 & W_2 + \Gamma_2' X_{xx} \Gamma_2 \end{bmatrix}. \quad (28)$$

Lemma 2 : The solutions of partitioned matrices X_{xx} and X_{xd} of the ARE are given as

$$X_{xx} = \Phi_{c_{xx}}' X_{xx} \Phi + C' W_1 C \quad (29)$$

and

$$\begin{aligned} X_{xd}(0) &= \Phi_{c_{xx}}' X_{xx} \Gamma_d \\ X_{xd}(1) &= \Phi_{c_{xx}}' X_{xd}(0) = (\Phi_{c_{xx}}')^2 X_{xx} \Gamma_d \\ &\vdots \\ X_{xd}(N-1) &= (\Phi_{c_{xx}}')^N X_{xx} \Gamma_d \end{aligned} \quad (30)$$

where $X_{xd} = [X_{xd}(0) \ X_{xd}(1) \ \cdots \ X_{xd}(N-1)]$.

Proof : By using (19), (20) and (27), X_{xx} and X_{xd} are obtained as follows:

$$\begin{aligned} X_{xx} &= \Phi' (X_{xx} - \overline{X}_1 R^{-1} \overline{X}_1') \Phi + C' W_1 C \\ &= \Phi_{c_{xx}}' X_{xx} \Phi + C' W_1 C, \end{aligned} \quad (31)$$

$$\begin{aligned} X_{xd} &= \Phi' (X_{xx} - \overline{X}_1 R^{-1} \overline{X}_1') \Gamma_{pd} \\ &\quad + \Phi' (X_{xd} - \overline{X}_1 R^{-1} \overline{X}_2') \Phi_d \\ &= \Phi' (I - X_{xx} [\Gamma_1 \ \Gamma_2] R^{-1} \begin{bmatrix} \Gamma_1' \\ \Gamma_2' \end{bmatrix}) X_{xx} \Gamma_{pd} \\ &\quad + \Phi' (I - X_{xx} [\Gamma_1 \ \Gamma_2] R^{-1} \begin{bmatrix} \Gamma_1' \\ \Gamma_2' \end{bmatrix}) X_{xd} \Phi_d. \end{aligned} \quad (32)$$

Recalling the closed loop system matrix $\Phi_{c_{xx}}$, the matrix

X_{xd} becomes

$$X_{xd} = \Phi_{c_{xx}}' X_{xx} \Gamma_{pd} + \Phi_{c_{xx}}' X_{xd} \Phi_d \quad (33)$$

Since the Φ_d has a form of a companion matrix and Γ_{pd} has a simple structure, X_{xd} can have a partitioned form.

$$X_{xd} = [X_{xd}(0) \ X_{xd}(1) \ \dots \ X_{xd}(N-1)] \quad (34)$$

Obtaining the elements of X_{xd} ,

$$\begin{aligned} X_{xd}(0) &= \Phi_{c_{xx}}' X_{xx} \Gamma_d \\ X_{xd}(1) &= \Phi_{c_{xx}}' X_{xd}(0) = (\Phi_{c_{xx}}')^2 X_{xx} \Gamma_d \\ &\vdots \\ X_{xd}(N-1) &= (\Phi_{c_{xx}}')^N X_{xx} \Gamma_d \end{aligned} \quad (35)$$

The control gain matrix K of the augmented system can also be partitioned as K_x and K_d conformably with $x(k)$ and $x_d(k)$ and the control input for the system is a sum of the control input by K_x and the control input by K_d . Therefore,

$$u(k) = K \bar{x}(k) = [K_x \ K_d] \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix}. \quad (36)$$

From the (35), it is concluded that the elements of the matrix X_{xd} are completely determined from the closed-loop system matrix $\Phi_{c_{xx}}$ and the solution of the algebraic Riccati equation X_{xx} . We define the generalized plant of the original dynamical system hit by previewable and unmeasurable disturbances as follows:

$$G(z) = \begin{bmatrix} \Phi & \Gamma_1 & \Gamma_2 \\ W_1^{1/2} C & 0 & 0 \\ 0 & 0 & W_2^{1/2} \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}. \quad (37)$$

Theorem 2: Suppose that following conditions hold.

a) $[\Phi, \Gamma_2]$ is stabilizable.

b) $\text{rank} \begin{bmatrix} \Phi - e^{j\theta} I & \Gamma_2 \\ \begin{bmatrix} W_1^{1/2} C \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ W_2^{1/2} \end{bmatrix} \end{bmatrix} = n + m$, for all $\theta \in (-\pi, \pi]$.

Then, the discrete-time stabilizing state feedback and preview control gains satisfying $\|F_r(G, K)\|_\infty < \gamma$ for both previewable and unmeasurable disturbances are given as

$$K_x = R_p \Omega X_{xx} \Phi \quad (38)$$

$$\begin{aligned} K_d(0) &= R_p \Omega X_{xx} \Gamma_d \\ K_d(1) &= R_p \Omega (\Phi_{c_{xx}}') X_{xx} \Gamma_d \\ &\vdots \\ K_d(N) &= R_p \Omega (\Phi_{c_{xx}}')^N X_{xx} \Gamma_d. \end{aligned} \quad (39)$$

where $K_d = [K_d(0) \ K_d(1) \ \dots \ K_d(N)]$ consistently with the number of the preview time step.

Proof: A stabilizing state feedback controller is given

by

$$K = -(R_3 - R_2 R_1^{-1} R_2')^{-1} (L_2 - R_2 R_1^{-1} L_1).$$

Define R_l for convenience:

$$R_l = (R_3 - R_2 R_1^{-1} R_2')^{-1}.$$

From (28),

$$\begin{aligned} R_l &= W_2 + \Gamma_2' X_{xx} \Gamma_2 \\ &\quad - \Gamma_2' X_{xx} \Gamma_1 (-\gamma^2 I_l + \Gamma_1' X_{xx} \Gamma_1)^{-1} \\ &\quad \Gamma_1' X_{xx} \Gamma_2. \end{aligned} \quad (40)$$

Expressing L_1 and L_2 as simple forms,

$$\begin{aligned} L_1 &= \bar{\Gamma}_1' X_\infty \bar{\Phi} \\ &= [\Gamma_1' \ 0] \begin{bmatrix} X_{xx} & X_{xd} \\ X_{xd} & X_{dd} \end{bmatrix} \begin{bmatrix} \Phi & \Gamma_{pd} \\ 0 & \Phi_d \end{bmatrix} \\ &= \Gamma_1' [X_{xx} \Phi \ X_{xx} \Gamma_{pd} + X_{xd} \Phi_d] \end{aligned} \quad (41)$$

and

$$\begin{aligned} L_2 &= \bar{\Gamma}_2' X_\infty \bar{\Phi} = [\Gamma_2' \ 0] X_\infty \bar{\Phi} \\ &= \Gamma_2' [X_{xx} \Phi \ X_{xx} \Gamma_{pd} + X_{xd} \Phi_d]. \end{aligned} \quad (42)$$

From (22), (41) and (42),

$$\begin{aligned} L_2 - R_2 R_1^{-1} L_1 &= \\ \Gamma_2' (I - X_{xx} \Gamma_1 (-\gamma^2 I_l + \Gamma_1' X_{xx} \Gamma_1)^{-1} \Gamma_1') \\ &\quad [X_{xx} \Phi \ X_{xx} \Gamma_{pd} + X_{xd} \Phi_d]. \end{aligned} \quad (43)$$

For simplicity, a new matrix is defined as:

$$\Omega = \Gamma_2' (I - X_{xx} \Gamma_1 (-\gamma^2 I_l + \Gamma_1' X_{xx} \Gamma_1)^{-1} \Gamma_1'). \quad (44)$$

Then,

$$L_2 - R_2 R_1^{-1} L_1 = \Omega [X_{xx} \Phi \ X_{xx} \Gamma_{pd} + X_{xd} \Phi_d]. \quad (45)$$

With Ω and R_l , the control gain matrix is

$$\begin{aligned} K &= [K_x \ K_d] = R_l (L_2 - R_2 R_1^{-1} L_1) \\ &= R_p \Omega [X_{xx} \Phi \ X_{xx} \Gamma_{pd} + X_{xd} \Phi_d]. \end{aligned} \quad (46)$$

The resulting feedback gain matrix and the preview gain matrix are

$$K_x = R_p \Omega X_{xx} \Phi, \quad (47)$$

$$K_d = R_p \Omega (X_{xx} \Gamma_{pd} + X_{xd} \Phi_d). \quad (48)$$

Since $X_{xd} = [X_{xd}(0) \ X_{xd}(1) \ \dots \ X_{xd}(N-1)]$, K_d can be rewritten as

$$\begin{aligned} K_d(0) &= R_p \Omega X_{xx} \Gamma_d \\ K_d(1) &= R_p \Omega X_{xd}(0) \\ &\vdots \\ K_d(N) &= R_p \Omega X_{xd}(N-1). \end{aligned} \quad (49)$$

Therefore, the preview gains are obtained as

$$\begin{aligned} K_d(0) &= R_p \Omega X_{xx} \Gamma_d \\ K_d(1) &= R_p \Omega (\Phi_{c_{xx}}') X_{xx} \Gamma_d \\ &\vdots \\ K_d(N) &= R_p \Omega (\Phi_{c_{xx}}')^N X_{xx} \Gamma_d. \end{aligned} \quad (50)$$

The preview gain is completely derived from the $\Phi_{c_{xx}}$ and X_{xx} . If we obtain the state-feedback gain matrix and the related solution of the ARE, we can determine the preview gain matrix from the state-space model of the original dynamical plant instead of the augmented plant. Therefore, the obtained $K=[K_x \ K_d]$ satisfies $\|F_\gamma(G, K)\|_\infty < \gamma$ for both previewable and unmeasurable disturbances. The preview controller is completely determined with X_{xx} and $\Phi_{c_{xx}}$ of the original dynamical system. The feedback gains depend on the infinity norm and the selected weighting matrices. Note that the preview length doesn't have effects on them. The feedback gains are exactly the same as those obtained in the general SF H_∞ controller problem. The preview gains are inherited from the feedback gains. The previewable disturbances are compensated by preview action before they hit the system as shown in Fig. 3.

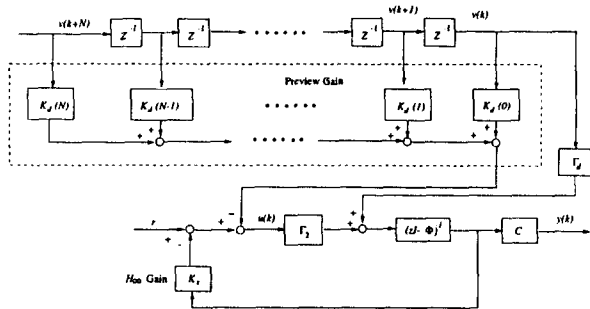


Fig. 3. The closed-loop control system with preview.

It is interesting to note that as γ tends to ∞ in (38) and (39), the H_∞ preview controller actually approaches the LQ preview controller. As $\gamma \Rightarrow \infty$, $\Omega = \Gamma_2'$ and $R_I = W_2 + \Gamma_2' X_{xx} \Gamma_2$.

Rewriting the gains,

$$K_x = (W_2 + \Gamma_2' X_{xx} \Gamma_2) \Gamma_2' X_{xx} \Phi \quad (51)$$

$$K_d(N) = (W_2 + \Gamma_2' X_{xx} \Gamma_2) \Gamma_2' (\Phi_{c_{xx}}')^N X_{xx} \Gamma_d \quad (52)$$

where W_2 is equivalent to the control weighting matrix in the LQ cost functional. Eq. (51) and (52) are exactly the same as LQ gain and the related preview gain. This is similar to the relationships between H_2 and H_∞ controller.

III. A design example

Consider an example that will give us a feel for the properties of the H_∞ preview control. This is done for the following simple first-order SISO system:

$$G_p = \begin{bmatrix} 0.966 & 0.098 \\ 1 & 0 \end{bmatrix}. \quad (53)$$

The plant model for an H_∞ controller design is shown in Fig. 4. It is assumed that the unknown and measurable disturbances hit the plant in the input side and the dimensions of both the disturbances are 1's for

simplicity. The disturbance input matrices, Γ_1 and Γ_d , are also assumed to be the same as the control input matrix Γ_2 . The regulated output vector z to be minimized is chosen as

$$z = \begin{bmatrix} W_1^{1/2} G_p(u+w) \\ W_2^{1/2} u \end{bmatrix}. \quad (54)$$

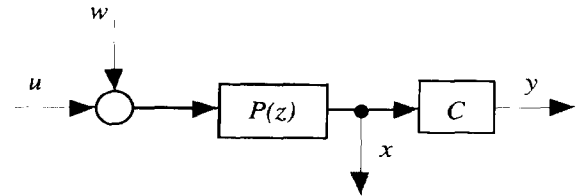


Fig. 4. The input and output signals for the plant.

The output weighting matrix W_1 and the control weighting matrix W_2 are properly selected to fit design specifications. The future 5 time step disturbances are assumed to be measurable in the present time, which implies that the preview length N is chosen as 5 in the design. An generalized plant for designing an SF H_∞ controller is written as

$$G = \begin{bmatrix} 0.966 & 0.098 & 0.098 \\ W_1^{1/2} & 0 & 0 \\ 0 & 0 & W_2^{1/2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \quad (55)$$

Obtaining a discrete ARE,

$$X_{xx} = 0.966^2 M_{xx} + W_1, \quad (56)$$

$$M_{xx} = X_{xx} - [1 \ 1] R^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} 0.966^2 X_{xx}^2. \quad (57)$$

The matrix R is written as:

$$R = \begin{bmatrix} -\gamma^2 + 0.098^2 X_{xx} & 0.098^2 X_{xx} \\ 0.098^2 X_{xx} & W_2 + 0.098^2 X_{xx} \end{bmatrix}. \quad (58)$$

In the design, the control weighting matrix W_2 and the output weighting matrix W_1 are equally given as 1 and γ is done as 0.98 to illustrate the design scheme. Solving a discrete ARE, its solution and feedback gain matrix are obtained. The control gain and the closed-loop system matrix are as follows:

$$K_x = 5.3585 \quad \text{and} \quad \Phi_c = 0.4426.$$

Since the eigenvalue of Φ_c is inside the unit circle, the control gain is a stabilizing one. From the equation $K_d(N) = R_I \Omega (\Phi_c')^N X_{xx} \Gamma_d$, the preview gain matrix is

$$K_{d(N)} = [0.18 \ 0.08 \ 0.04 \ 0.02 \ 0.01 \ 0.01]$$

The transfer functions of the closed-loop system with/without the preview compensation from the disturbance to the plant output are

$$G_{dp} = \begin{bmatrix} \Phi_c & \bar{E}_d \\ [C \ 0] & 0 \end{bmatrix} \quad \text{and} \quad G_{dc} = \begin{bmatrix} \Phi & \Gamma_d \\ [C \ 0] & 0 \end{bmatrix}.$$

The transfer functions $G_{dp}(z)$ and $G_{dc}(z)$ are plotted in Fig. 5. It can be seen that the preview compensation improves the ability of the previewable disturbance rejection. Note that the preview controller shows good performance in the low frequency region which most of the disturbances exist.

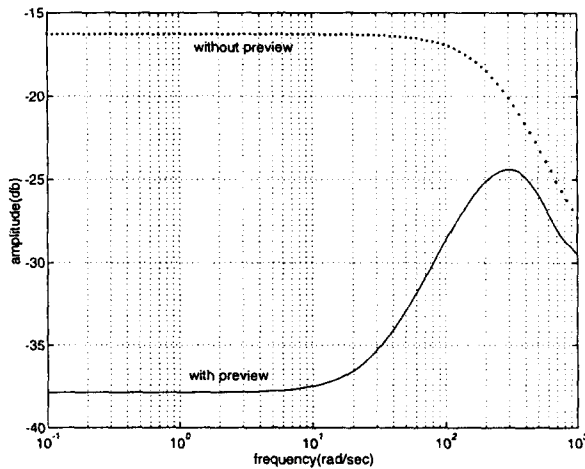


Fig. 5. Frequency responses of $G_{dp}(z)$ and $G_{dc}(z)$.

To examine the performance of the designed controller, time responses are checked through simulation. A previewable disturbance to the plant for test is given as

$$v(t) = 10 \sin(2\pi t) + 10 \sin(4\pi t).$$

Fig. 6 illustrates the time responses of plant outputs with/without preview controller.

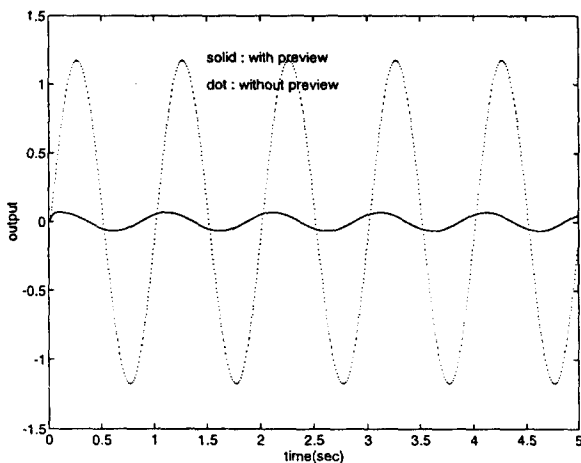


Fig. 6. Time responses of the plant output.

It is evident that the preview control has significantly reduced the effects of the sine wave disturbance compared with only the state feedback controller. This results from the fact that the preview gain compensated the external disturbance before the effects of the disturbance appear in the plant output.

Since the SF H_∞ controller with the preview compensation has a two-degree-of-freedom structure, the ma-

gnitudes of the resulting control inputs by the feedback gain and the preview gain are investigated. The control input by the preview gain in the feedforward loop is much larger than one by the feedback gain as shown in Fig. 7. Therefore, it is conjectured that the attenuating ability for the previewable disturbance is mostly done by the preview action rather than the state feedback.

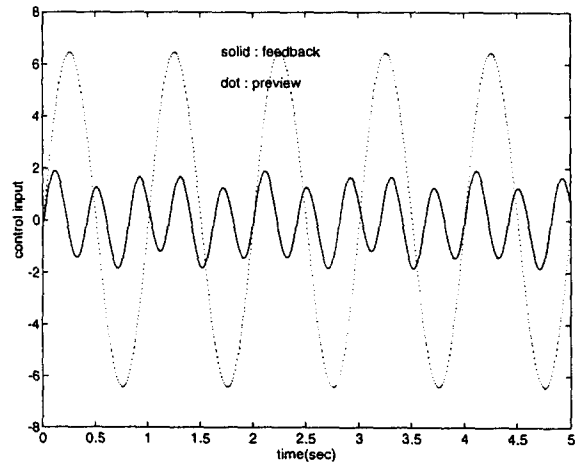


Fig. 7. Control inputs by the state feedback and the preview gains.

Note that the H_∞ controller with the preview compensation gives better results than simple H_∞ control with slightly larger control input. Regulating performances of two kinds of the preview controller are compared: one based on H_∞ controller design and the other based on the LQ control theory.

The control and the output weighting matrices in the design of the LQ-based preview controller are given 0.01 and 1 to have the similar magnitude of the control input. Fig. 8 illustrates the outputs attenuated by both the controllers. The H_∞ -based preview controller shows better regulation ability, despite the slightly larger control inputs as shown in Fig. 9. This results from the fact that as γ approaches small the output energy is minimized.

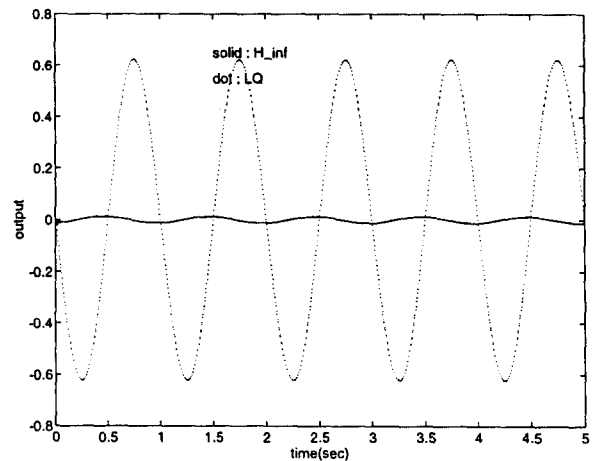


Fig. 8. Time responses of the plant outputs.

Fig. 10 depicts the preview gains for selected values of

γ over the preview time step. As γ approaches an optimal value, the preview gain gets large. The effects of future informations adjacent to the present time becomes more prominent when the γ has a smaller value.

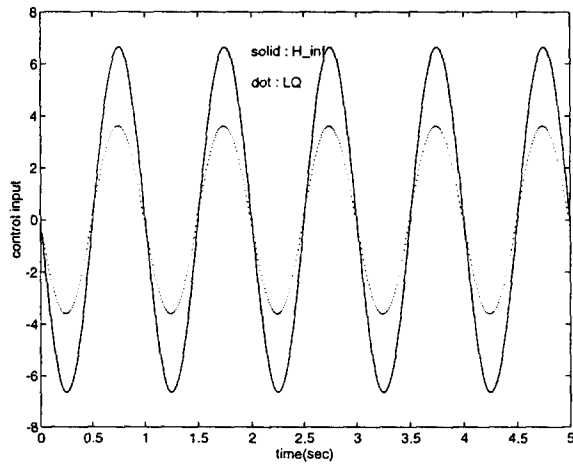


Fig. 9. Time responses of the control inputs.

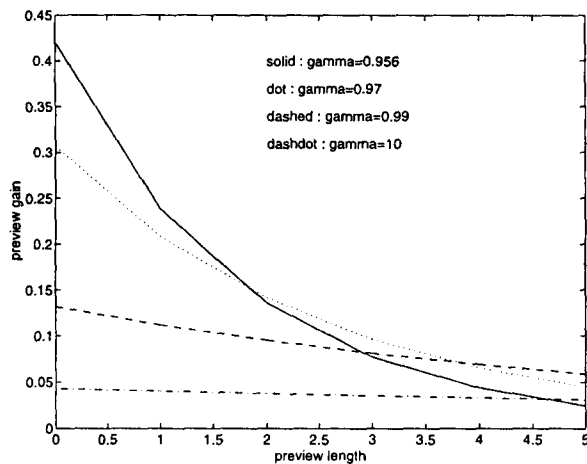


Fig. 10. Preview gains with respect to the preview step.

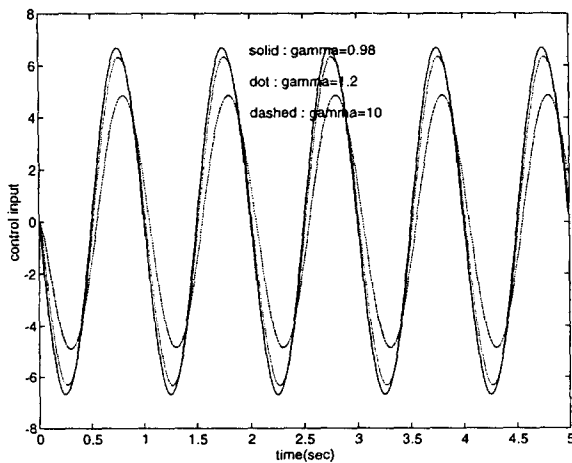


Fig. 11. Control inputs by the preview gains.

This behavior will become more clarified if the control inputs with respect to γ are investigated. Fig. 11 shows the control inputs by the preview gains for the variation of γ . As γ approaches the optimum, the control input by

the preview compensation gets large, while the ones by the state feedback does converse behaviors as depicted in Fig. 12. As the γ approaches an optimal value, the preview controller dominates the whole controller.

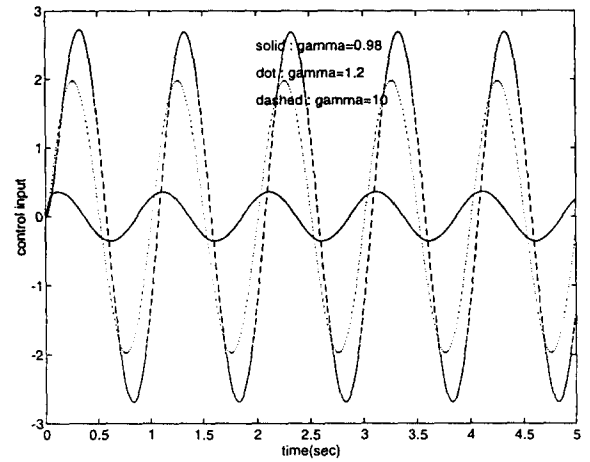


Fig. 12. Control inputs by the state feedback gains.

The variations of the preview gains according to the W_1 and W_2 are also shown in Fig. 13 and 14. As W_1 gets larger and W_2 does smaller, the resulting preview gains become prominent over the preview step.

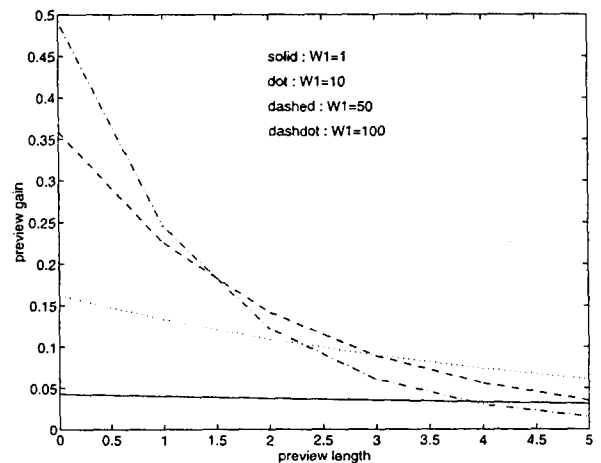


Fig. 13. Preview gains with respect to the W_1 .

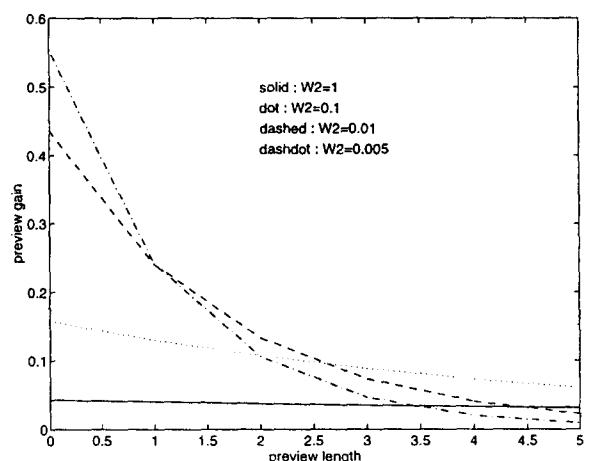


Fig. 14. Preview gains with respect to the W_2 .

IV. Conclusions

A new preview control law is presented based on the discrete-time state-feedback H_∞ control scheme. The derivation procedure is analogous to the scheme by LQR. The synthesized control law also considers the unknown bounded disturbances as well as the measurable ones. The preview and feedback controller are designed by simultaneous optimization so that the infinity norm of the transfer function matrix from both the previewable and unmeasurable disturbances to the regulated variables is minimized. As the infinity norm of the transfer function tends to ∞ , the feedback gain and preview gain approach to LQ-based ones, which is the similar relationship of H_2 and H_∞ . A first-order SISO system as a design example was given to demonstrate the control law and simulated to ascertain the control performance. The time responses of the preview controller showed better performances compared to the simple H_∞ controller.

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