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광범위한 안정 영역을 갖는 출력만을 이용한 제어기 설계

(Output Feedback Semiglobal Stabilization for A Nonlinear System)

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요 약

모델의 불확실성이 있는 다변수 비선형 시스템을 안정화 시킬 수 있는 출력만을 이용한 가변 구조 제어 방법에 의한 제어기를 설계하였다. 외란이 존재하는 경우에도 상태변수를 잘 예측 할 수 있는 고 이득 관찰기를 사용하였으며 고 이득 관측기의 사용에 따른 상태변수의 피킹 현상을 줄이기 위해 제한치를 갖는 제어기를 사용하였다. 또한 광범위한 영역(any compact set)에서도 안정화가 가능한 제어기를 설계하였다.

Abstract

We consider the stabilization of a class of multivariable nonlinear system using variable structure output feedback control. A high-gain observer is used to estimate state variable while rejecting the effect of the disturbances. We design a globally bounded output feedback variable structure controller that semi-globally stabilize the closed-loop system, while state variables do not exhibit a peaking.

I. Introduction

Variable structure control(VSC) has been successfully used to achieve control tasks such as stabilization or tracking due to the robustness to modeling uncertainty and external disturbance for linear systems or nonlinear systems. Most of work on VSC assumes that measurement of state variable is available to feedback controller [1]. There have been some efforts to develope output feedback VSC. Two different schemes have been studied on output feedback VSC. One of them is the static output feedback VSC [5] which does not use an observer to estimate the state variable of the system. However, this scheme restricts the system to be relative degree one. The other one is observer-based controller that can be used for relative degree higher than one systems. Papers [2] used high-gain observers to reject disturbance due to modeling uncertainty and imperfect feedback cancellation of nonlinearity. However, the use of high-gain observer for relative degree higher than one systems results in peaking in the state variables and shrinking of region of attraction [3]. The paper [8], motivated by [3], showed that a globally bounded VSC with high-gain observer can stabilize a class of nonlinear system with no peaking. The paper obtained a regional result, i.e., an estimate of region of attraction is a some limited compact set which depend on controller. In fact, the region of attraction in the paper can be estimated after the design of controller. In this paper we generalize the paper [8] in two direction. First, we develop a controller design scheme that can achieve

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semiglobal output feedback stabilization, namely, a region of attraction of the closed-loop system could be any given compact set. The difference between semiglobal stabilization and global stabilization is well described in [9, page 483]. A good example in which a controller can achieve the semiglobal stabilization but not the global stabilization can be found in the same book. Second, we use a less conservative assumption on the uncertainty of input coefficient matrix. We consider a feedback linearizable nonlinear system whose initial condition belongs to any given compact set. A robust-high gain observer is constructed to estimate the state variable. The use of the high-gain observer and globally bounded control enable to show that the state estimation error decays to arbitrarily small values during a short transient period. A globally bounded VSC is designed such that region of attraction of the closed-loop system contains the given compact set as well as a reaching condition is satisfied when the state estimation error becomes arbitrarily small value. After the trajectories reaching the sliding surface, stability of closed-loop system is established using a standard Lyapunov argument.

Ⅱ. Problem Statement

We consider the multivariable nonlinear system

$$\dot{w} = f(w) + \sum_{i=1}^{m} g_i(w)u_i \tag{1}$$

$$y = h(w) \tag{2}$$

where $w \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^m$ is the measured output. We assume that f, g_i, and h are sufficiently smooth and globally definded function, and f(0) = 0, h(0) = 0. We make a following assumption on the system (1)-(2).

Assumption 1 *For all* $w \in R^n$,

- *The system (1)-(2) has a uniform vector relative degree* $\{r_1, \cdots, r_m\}$, *i.e.,* $L_{g_j} h_i(w) = \cdots = L_{g_j} L_f^{r_i-2} = 0$ *for all* $1 \le i, j \le m$, *and the matrix* $A(w) = \{a_{ij}(w)\} = \{L_{g_j} L_f^{r_i-1} h_i(w)\}$ *is nonsingular.*
- $n = r_1 + \cdots + r_m$.
- *The mapping* $x = T(w)$, *defined by* $x_j^i = L_f^{j-1} h_i(w)$ *for* $1 \le j \le r_i$, $1 \le i \le m$ *and* $x = [x_1^1, \cdots, x_{r_1}^1, \cdots, x_1^m, \cdots, x_{r_m}^m]^T$, *is a proper map, i.e.,* $\lim_{\|x\| \to \infty} \|T(x)\| = \infty$ [10].

The first two(2) of the assumptions used are a necessary and sufficient condition for the mapping $x = T(w)$ to be a local diffeomorphism in the neighborhood of every $w \in R^n$ [6]. The change of variables $x = T(w)$ transforms the system (1)-(2) into the normal form

$$\dot{x} = Ax + B[F(x) + G(x)u] \tag{3}$$

$$y = Cx \tag{4}$$

where $A = block\ diag[A_1, \cdots, A_m]$, $B = block\ diag[B_1, \cdots, B_m]$, $C = block\ diag[C_1, \cdots, C_m]$, the matrices (A_i, B_i) are Brunowsky controllable canonical form, and $C_i = [1, 0, \cdots, 0]_{1 \times r_i}$. The properness of the mapping $x = T(w)$ ensures that it is a global diffeomorphism, c.f., [10]. Therefore, the normal form is defined for all $x \in R^n$.

Remark 1 *To ensure the global diffeomorphism, some vector field should be complete in [8, proposition 9.1.1]. However, it is not easy to check a vector field is complete in general, meanwhile, a properness of the mapping can be easily checked, since we have a explicit form of local transformation.*

Let $F_o(x)$ and $G_o(x)$ be known nominal models of $F(x)$ and $G(x)$, respectively. Suppose that $F_o(x)$ and $G_o(x)$ are sufficiently smooth; $F_o(0) = 0$, and $G_o(x)$ is nonsingular for all $x \in R^n$. We also assume that $F_o(x)$ and $G_o(x)$ are globally bounded. This can be always achieved by saturating the given nominal functions outside a bounded domain of interest, as it will be illustrated later on. The uncertainty in equation (3) satisfies the matching condition, which is a typical assumption in the design of robust variable structure control [1, 7]. We make the following assumption on the uncertainty.

Assumption 2

- *For every compact set $U \in R^n$, and $\forall x \in U$, there is a scalarnonnegative locally Lipschitz function $\rho_i(x)$ such that*

$$| F^i(x) - F_o^i(x) | \leq \rho_i(x) \tag{5}$$

where $F^i(x)$ and $F_o^i(x)$ are the ith components of vectors $F(x)$ and $F_o(x)$, respectively.

- *The matrix $[I-N]$ is an M-matrix. where N is an $m \times m$ matrix such that each component of N is an upper bound on the absolute value of the corresponding component of the matrix $[G(x)G_o^{-1}(x) - I]$, $\forall x \in U$.*

The definition of an M-matrix can be found in [7]. Notice that since $G(x)$ and $G_o^{-1}(x)$ are bounded, the matrix N is well defined. Note that $\rho_i(x)$ and matrix N could depend on the set U.

Remark 2

A typical assumption of uncertainty on the input coefficient matrix is $\|I - G(x)G_o^{-1}(x)\|_\infty < 1$, e.g., [8]. *It can be shown that the second assumption of Assumption 2 is less conservative than $\|I - G(x)G_o^{-1}(x)\|_\infty < 1$.*

Ⅲ. Controller Design

We use the following observer to estimate the state x

$$\dot{\hat{x}} = A\hat{x} + B[F_o(\hat{x}) + G_o(\hat{x})u] + D(\varepsilon)LC(x - \hat{x}) \tag{6}$$

where $L = block\ diag[L_1, \cdots, L_m]$, $L_i = [\alpha_1^i, \cdots, \alpha_{r_i}^i]^T$, $D(\varepsilon) = block\ diag[D_1(\varepsilon), \cdots, D_m(\varepsilon)]$, and $D_i(\varepsilon) = diag[1/\varepsilon, \cdots, 1/(\varepsilon^{r_i})]$. Note that a design parameter ε is a positive constant to be specified later on. We choose the observer gain α_j^i such that (A-LC) is a Hurwitz matrix. Let $e_j^i = x_j^i - \hat{x}_j^i$ be the estimation error and define the scaled variables $\zeta_j^i = (1/\varepsilon^{r_i - j})e_j^i$. It can be shown that the closed-loop system is given by

$$\dot{x} = Ax + B[F(x) + G(x)u(\hat{x})] \tag{7}$$

$$\begin{aligned} \varepsilon\dot{\zeta} &= (A - LC)\zeta + \varepsilon B[F(x) - F_o(\hat{x}) \\ &+ (G(x) - G_o(\hat{x}))u(\hat{x})] \end{aligned} \tag{8}$$

where $\zeta = [\zeta_1^1, \cdots, \zeta_{r_1}^1, \cdots, \zeta_1^m, \cdots, \zeta_{r_m}^m]^T$. For small ε, the closed-loop system is a singularly perturbed system with x as the slow variable and ζ as the fast one. We choose the sliding surface $\sigma(\hat{x}) = [\sigma_1(\hat{x}), \cdots, \sigma_m(\hat{x})]^T$ such that $\sigma_i(\hat{x}) = \hat{x}_{r_i}^i + m_{r_i-1}^i \hat{x}_{r_i-1}^i + \cdots + m_1^i \hat{x}_1^i$, $1 \leq i \leq m$ where m_j^i are chosen such that $\tilde{A}_i$ is Hurwitz where

$$\tilde{A}_i = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ -m_1^i & -m_2^i & \cdots & -m_{r_i-2}^i & -m_{r_i-1}^i \end{bmatrix}$$

Rewrite $\sigma(\hat{x})$ as $\sigma(\hat{x}) = M\hat{x}$ where $M = block\ diag[M_1, \cdots, M_m]$, $M_i = [m_1^i \cdots m_{r_i-1}^i\ 1]$. We consider a control input of the form $u_j = \varphi_j(\hat{x}) + \nu_j(\hat{x})sgn(\sigma_j(\hat{x}))$ where $\varphi_j(\hat{x})$ and $\nu_j(\hat{x})$ are continuous and globally bounded functions. We will specify u_j later on. To show semiglobal stabilization, we need to show that a region of attraction of the closed-loop system (7)-(8) can be made arbitrarily large. To this end, we assume that initial condition of state variable $x(0)$ belongs to any given compact set $\bar{\Omega}_c$. Define the matrices $\tilde{A} = block\ diag[\tilde{A}_1, \cdots, \tilde{A}_m]$ and $\tilde{B} = block\ diag[\tilde{B}_1, \cdots, \tilde{B}_m]$ where $\tilde{B}_i = [0, \cdots, 1]^T_{1 \times (r_i - 1)}$ for $1 \leq i \leq m$. Since $\tilde{A}$ is a Hurwitz matrix, for any positive definite matrix Q, there is a symmetric positive definite matrix P such that $P\tilde{A} + \tilde{A}^T P = -Q$ [7]. Define $z_i = [x_1^i, \cdots, x_{r_i-1}^i]^T$, $z = [z_1, \cdots, z_m]^T$, and $V(z) = z^T Pz$. Since $\|Mx\|$ and $V(z)$ is radially unbounded, i.e., $\lim_{x \to \infty} \|Mx\| = \infty$ and $\lim_{x \to \infty} V(x) = \infty$, we can choose c_{so} and c_{zo} large enough such that $\Omega_o = \{x \in R^n \mid \|Mx\| \leq c_{so}, \sqrt{V(z)} \leq c_{zo}\}$ and $\bar{\Omega}_o \subset \Omega_c$. Choose c_{sr} and c_{zr} such that $\Omega_r = \{x \in R^n \mid \|Mx\| \leq c_{sr}, \sqrt{V(z)} \leq c_{zr}\}$ where $c_{sr} > c_{so}$, $c_{zr} > c_{zo}$, $c_{zr} = r(a_2/a_1)c_{sr}$, , $r > 1$,

$a_1=\lambda_{\min}(Q)/\lambda_{\max}(P)$, and $a_2=2\|P\hat{B}\|/\sqrt{\lambda_{\min}(P)}$. The set Ω_r is taken as the region of interest in our analysis. Note that $\Omega_o\subset\Omega_r$. The motivation for choosing Ω_r in this form will become clear as we proceed with the analysis. Let $\Omega_1=\{\zeta\in R^n \mid \|\zeta\|< c/\varepsilon^{(\gamma-1)}\}$ where $\gamma=\max_i r_i$ for all $i=1,\cdots,m$ and c is an arbitrary positive constant. Define a set

$$\Omega=\Omega_o\times\Omega_1 \tag{9}$$

Note that the set Ω_1 implies that any point within the oder of 1 distance from Ω_o is allowed as an initial condition for $\hat{x}$.

Lemma 1

Consider the singularly perturbed system (7)-(8) and suppose that Assumption 2 is satisfied. Then, for all $(x(0),\zeta(0))\in\Omega$, there exists ε_1 and $T_1=T_1(\varepsilon)\le T_3$ such that for all $0<\varepsilon<\varepsilon_1$, $\|\zeta\|<k\varepsilon$ for all $t\in[T_1,T_4)$ where T_3 is a finite time and $T_4>T_3$ is the first time $x(t)$ exits from the set Ω_r.

Proof : See [8, Lemma 1]

It is shown in the proof of Lemma that the fast variable ζ decays very rapidly during a short time period $[0,\ T_1]$ due to the use of the globally bounded control. We design the control input $u(\hat{x})$ such that a sliding mode condition is satisfied when $\|\zeta\|<k\varepsilon$ and $(x,\hat{x})\in\Omega_r\times\Omega_r$. This will be done by showing that $\sigma(\hat{x})^T\dot{\sigma}(\hat{x})<0$ as long as $\sigma(\hat{x})\neq 0$. We have

$$\begin{aligned}\sigma^T(\hat{x})\dot{\sigma}(\hat{x}) &= \sigma^T(\partial\sigma/\partial\hat{x})\dot{\hat{x}}\\ &=\sigma^T M[A\hat{x}+B(F_o(\hat{x})+G_o(\hat{x})u(\hat{x}))+D(\varepsilon)LC(x-\hat{x})]\\ &=\sigma^T M[A\hat{x}+B(F_o(\hat{x})+G_o(\hat{x})u(\hat{x}))+(1/\varepsilon)\tilde{D}(\varepsilon)LC\zeta]\end{aligned} \tag{10}$$

where $\tilde{D}(\varepsilon)=\text{block diag}[\tilde{d}_1(\varepsilon),\cdots,\tilde{d}_m(\varepsilon)]$ and $\tilde{d}_i(\varepsilon)=\text{diag}[\varepsilon^{r_i-1},\varepsilon^{r_i-2},\cdots,\varepsilon,1]$. It can be shown that the last term on the right-hand side of equation (10) is given by

$$(1/\varepsilon)\tilde{D}(\varepsilon)LC\zeta=(1/\varepsilon)\tilde{D}(\varepsilon)LC\eta\pm B\phi_1(\hat{x})+O(\varepsilon) \tag{11}$$

for $\hat{x}\in\Omega$, where $\phi_1(\hat{x})=F(\hat{x})-F_o(\hat{x})$. Therefore we need an estimate of $(1/\varepsilon)\tilde{D}(\varepsilon)LC\eta$ to design the control input u such that a sliding mode condition is satisfied. It can be shown that

$$(1/\varepsilon)M\tilde{D}(\varepsilon)LC\eta=(1/\varepsilon)M\tilde{D}(0)L\int_{T_1}^{t}Ce^{(A-LC)(t-l)/\varepsilon}\times B\{G(\hat{x})G_o^{-1}(\hat{x})-I\}G_0(\hat{x})v(l)dl+O(\varepsilon)$$

for $t\in[T_1+\varepsilon\ln(1/\varepsilon),T_4)$ where $v(t)\in K\{u(t)\}$ for almost all t and the convex hull $K\{u(t)\}$ is defined in [4]. Let k_{ui} be an essential upper bound of the absolute value of ith component for vector $G_o(\cdot)u$ to be specified. Choose the observer gain α_j^i such that all eigen values of $(A-LC)$ are real and negative. It can be verified that

$$\left|\left[(1/\varepsilon)M\tilde{D}(0)L\int_{T_1}^{t}Ce^{(A-LC)(t-l)/\varepsilon}B\{G(\hat{x})G_o^{-1}(\hat{x})-I\}\times G_o(\hat{x})v(l)dl\right]_i\right|\le[Nk_u]_i \tag{12}$$

where $[\cdot]_i$ denotes the ith component of a vector, and $k_u=[k_{u1},\ldots,k_{um}]^T$. Define the constant k_{si} by the inequality

$$|[-M(A\hat{x}+BF_o(\hat{x}))-\rho(\hat{x})\,\text{sgn}(\sigma)]_i|\le k_{si} \tag{13}$$

for almost every $\hat{x}\in\Omega$, where $\rho(\cdot)=\text{diag}[\rho_1(\cdot),\ldots,\rho_m(\cdot)]$ and $\text{sgn}(\sigma)=[\text{sgn}(\sigma_1),\ldots,\text{sgn}(\sigma_m)]^T$. Notice that k_{si} can be calculated since $F_o(\cdot)$ and $\rho(\cdot)$ are known. Define the vector

$$\bar{x}=(I-N)^{-1}Nk_s+\delta \tag{14}$$

where $\delta>0$ is a vector such that $(I-N)\delta>0$ and $k_s=[k_{s1},\ldots,k_{sm}]^T$. Since the matrix $(I-N)$ is an M-matrix, such a vector δ always exists[9]. Consider the function

$\psi(\hat{x})=G_o^{-1}(\hat{x})[-MA\hat{x}-F_o(\hat{x})-(\rho(\hat{x})+x)\text{sgn}(\sigma(\hat{x}))]$

where $x=\text{diag}[\bar{x}_1,\ldots,\bar{x}_n]$ and $\bar{x}_i$ is the ith component of the vector $\bar{x}$. We take the control input u as $\psi(\hat{x})$, saturated outside the set Ω_r. In particular, let $\psi^1(\hat{x})=-G_o^{-1}(\hat{x})M(A\hat{x}+BF_o(\hat{x}))$, $\psi^2(\hat{x})=-G_o^{-1}(\hat{x})(\rho(\hat{x})+x)$, $S_i^j=\max_{\hat{x}\in\Omega_r}|\psi_i^j(\hat{x})|$, and take

$$u_i(\hat{x})=S_i^1\,\text{sat}(\psi_i^1(\hat{x})/S_i^1)+S_i^2\,\text{sat}(\psi_i^2(\hat{x})/S_i^2)\text{sgn}(\sigma_i(\hat{x})) \tag{15}$$

where $sat(\cdot)$ is the saturation function and $\psi^i_i(\hat{x})$ denotes the ith component of the vector $\psi^i(\hat{x})$. Inside the set Ω_r, we have

$$u(\hat{x}) = \psi(\hat{x}) \tag{16}$$

where $u(\cdot) = [u_1, \ldots, u_m]^T$. Hence, k_u in inequality (12) can be taken by

$$k_u = k_s + \bar{x} \tag{17}$$

So far we show that how do we make a globally bounded control input. Moreover, the control input (15) satisfies a sliding mode condition when $\hat{x} \in \Omega_r$. Using (10) and (11),

$$\sigma^T \dot{\sigma} = \sigma^T[-(\rho(\cdot) + x)sgn(\sigma) + (1/\varepsilon)\hat{D}(\varepsilon)LC\eta \pm \phi_1(\cdot) + O(\varepsilon)]$$

Using (5), (12), and (17), we obtains the following inequality,

$$\begin{aligned}\sigma^T \dot{\sigma} &\le -\bar{\sigma}^T[\bar{x} - N(k_s + \bar{x})] + \varepsilon k \sum_{i=1}^{n} |\sigma_i| \\ &= -\bar{\sigma}^T[(I-N)\bar{x} - Nk_s] + \varepsilon k \sum_{i=1}^{n} |\sigma_i|\end{aligned}$$

where $\bar{\sigma} = [|\sigma_1|, \ldots, |\sigma_m|]^T$. After substituting (14), $\sigma^T \dot{\sigma} \le -\bar{\sigma}^T (I-N)\delta + \varepsilon k \sum_{i=1}^{n} |\sigma_i|$. Utilizing the fact that $(I-N)\delta > 0$, we get $\sigma^T \dot{\sigma} < -\bar{\delta}\bar{\sigma}^T$ for sufficiently small ε where $\bar{\delta} > 0$ and k is some positive constant. We summarize our findings in the following lemma.

Lemma 2

Consider the singularly perturbed system (7)-(8) with the control input (16). Suppose that Assumption 2 is satisfied, $\|\xi\| < k\varepsilon$, *and* $(x, \hat{x}) \in \Omega_r \times \Omega$, *for* $t \in [T_1, T_4)$, *and* ε *is small enough. Then, for almost all* $t \in [T_1 + \varepsilon \ln(1/\varepsilon), T_4)$, *the sliding mode condition* $\sigma^T \dot{\sigma} < 0$ *is satisfied for sufficiently small* ε.

Lemma 1 implies that the errors between the state variable x and the estimate $\hat{x}$ becomes small enough after the short period of time due to the use of high gain observer. After the error $x - \hat{x}$ becoming small enough, we show that the control input u (16) satisfies a sliding mode condition in Lemma 2. It is shown in [8, Lemma 3] that for sufficiently small ε, $\|\xi\| < k\varepsilon$ and $(x, \hat{x}) \in \Omega_r \times \Omega_r$, for all $t \ge T$, where $T = T_1(\varepsilon) + \varepsilon \ln(1/\varepsilon)$. Since a sliding mode condition is satisfied, the trajectories of (6) reach the sliding manifold $\sigma(\hat{x}) = 0$ within a finite time and stay in the manifold thereafter. Using a standard Lyapunov argument on the sliding manifold, we can arrive the following theorem.

Theorem 1

Consider globally defined the system (7)-(8). Suppose that Assumption 2 are satisfied. Let the observer gain be chosen as in (6) and the control input be chosen as in (16). Then there is $\varepsilon_2 > 0$ such that for all $0 < \varepsilon < \varepsilon_2$, the closed-loop system (7)-(8) and (16) is uniformly ultimately bounded with respect to the set $\Omega_\varepsilon = \{(x, \xi) \in R^n \times R^n \mid \|x\| \le k_{ps}\sqrt{\varepsilon}, \|\xi\| < \hat{k}_1 \varepsilon\}$, for some constants k_{ps} and $\hat{k}_1$ and Ω, defined by (9), is an estimate of the region of attraction. Moreover, suppose that $G(x) = G_o(x)$ $\forall x \in R^n$. Then there is $\varepsilon_3 > 0$ such that for all $0 < \varepsilon < \varepsilon_3$, the origin of the closed-loop system (7)-(8) and (16) is asymptotically stable and Ω, defined by (9), is an estimate of region of attraction.

Proof : See [8, Theorem 1]

Remark 3

- *The control input u used in [8] satisfied a sliding mode condition with a different control input u of this paper. Using a standard Lyapunov analysis technique, a stability analysis of the closed-loop system performed on a sliding manifold in [8]. We show that a sliding mode condition is satisfied in Lemma 2. The same stability analysis method used in [8] can be applied. Hence we omit the proof of the Theorem 1.*
- *We state Theorem 1 in* x*-coordinate for convenience. Since the mapping* $T(\cdot)$ *is a diffeomorphism, the property of Theorem 1 hold in* w*-coordinate.*

V. Example

Consider the following globally defined MIMO system

$$\begin{aligned} \dot{x} &= Ax + B[F(x) + G(x)u] \\ y &= Cx \end{aligned} \quad (18)$$

where

$A = block\ diag[A_1, A_2]$, $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$,

$B = block\ diag[B_1, B_2]$, $B_i = [0\ 1]^T$,

$C = block\ diag[C_1, C_2]$, $C_i = [1\ 0]$,

$F = [\theta_1 \sin(x_1)\ \ \theta_2 x_3^2]^T$, $u = [u_1\ u_2]^T$,

$G(x) = \begin{bmatrix} 1 & 0.2 \\ 0.3 & 1 \end{bmatrix}$, and unknown constant $\theta_i (i=1,2) \in [-0.8,\ 0.8]$. Suppose that $\|x(0)\| \le 1$. Notice that we assume that the initial condition of x is in the inside of unit ball for convenience. Our goal is a design of output feedback controller that stabilize the system represented by (18). Take the nominal functions $F_o(\cdot) = 0$ and $G_o(\cdot) = \begin{bmatrix} 1 & 0.2 \\ 0.3 & 1 \end{bmatrix}$. We construct the observer a

$\dot{\hat{x}} = A\hat{x} + B\{G_o(\cdot)u\} + D(\varepsilon)LC(x - \hat{x})$

where

$L = block\ diag[L_1, L_2]$, $L_i = [3\ 2]^T$,

$D(\varepsilon) = block\ diag[D_1(\varepsilon), D_2(\varepsilon)]$, $D_i(\varepsilon) = diag[1/\varepsilon, 1/\varepsilon^2]$.

One can verify that $(A-LC)$ is a Hurwitz matrix. The sliding surface is chosen by $\sigma(\hat{x}) = M\hat{x}$ where $M = block\ diag[M_1, M_2]$ and $M_i = [2\ 1]$. We take the set $\Omega_r = \{x \in R^4 |\ \|Mx\| \le 4,\ \sqrt{V(x)} \le 3\}$ where $V(x) = x_1^2 + x_3^2$. It can be verified that the set $\|x(0)\| < 1 \in \Omega_r$. It can be also verified that for $x \in \Omega_r$, $|F^1(x) - F_o^1(x)| \le |x_1|$ and $|F^2(x) - F_0^2(x)| \le x_3^2$. Define $\phi(\hat{x}) = G_o^{-1}[-MA\hat{x} - (\rho(\hat{x}) + \kappa)sgn(\sigma)]$ where $\rho(\hat{x}) = diag[\rho_1(\cdot), \rho_2(\cdot)]$, $\rho_1(\cdot) = |x_1|$, $\rho_2(\cdot) = x_3^2$, and $\kappa = diag[0.3,\ 0.3]$. Let $\psi^1(\hat{x}) = -G_o^{-1}MA\hat{x}$, $\psi^2(\hat{x}) = -G_o^{-1}(\rho(\cdot) + \kappa)$ and take a control input as $u_i(\hat{x}) = S_i^1 sat(\psi_i^1(\hat{x})/S_i^1) + S_i^2 sat(\psi_i^2(\hat{x})/S_i^2) sgn(\sigma_i(\hat{x}))$ for $i = 1,2$ where $S_i^j = \max_{x \in \Omega_r} |\psi_i^j(\hat{x})|$. We determine $S_i^1 = 10 (i=1,2)$, $S_1^2 = 3.3$, and $S_2^2 = 9.3$. It should be emphasized that the control input depends on initial condition. We simulate the response for $\theta_1 = \theta_2 = 0.8$, $x(0) = [1\ 0\ 1\ 0]^T$, and $\hat{x}(0) = [0\ 0\ 0\ 0]^T$ with $\varepsilon = 0.01$.

Figure 1 shows that the origin of closed-loop system is asymptotically stable, which is predicted by Theorem 1 since $G(x) = G_o(x)$. One can observe that state variables $x_1(t)$, $x_2(t)$ do not exhibit peaking due to the use of globally bounded control input. Peaking is evident in $\hat{x}_2(t)$, but it is not our concern since the estimate is just a computed value, not physical variable. We give the plot of $x_1(t)$, $x_2(t)$ and their estimates. One can verify that similar result can be obtained for

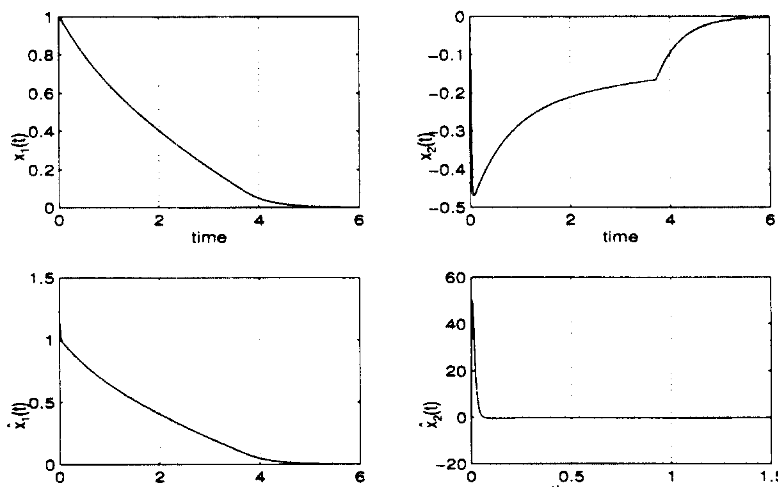

그림 1. 상태변수 및 관측치

Fig. 1. The plot of state variables and their estimates.

$x_3(t)$, $x_4(t)$ and their estimates. Figure 2 shows that the control input u is saturated during a short transient time, which is a consequence of saturating outside the set Ω_r.

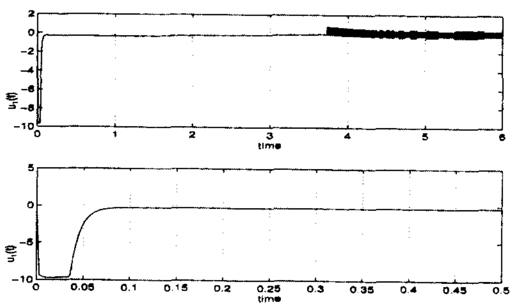

그림 2. 제어 입력 $u_1(t)$
Fig. 2. The plot of control input $u_1(t)$.

Ⅵ. Concluding Remark

We have designed an output feedback VSC that region of attraction of the closed-loop system contains any given compact set. The controller could depend on initial condition. But this will not cause a problem, since we know a range of initial condition before we design the controller. The closed-loop system is uniformly ultimately bounded with respect to Ω_ϵ, which can be made arbitrarily small by decreasing ϵ. Moreover, we can achieve asymptotic stability when there is no uncertainty on the input coefficient matrix. We require that the diagonal components of $\{G(\cdot)G_o^{-1}-I\}$ dominate the off diagonal components by requiring $(I-N)$ to be M-matrix. This requirement is less conservative than $\|G(\cdot)G_o^{-1}-I\|<1$ as we required in [8].

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