# NHPP 모형에 기초한 고장 수 자료의 분석

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## 요 약

소프트웨어 신뢰도는 소프트웨어의 중요한 품질 특성 중의 하나이며, 소프트웨어 신뢰도 성장 모형은 테스트 단계 동안 신뢰도를 평가하고 신뢰도가 성장하는 양상을 파악할 수 있게 하는 도구이다. 그러므로 테스트 단계 동안 수집된 고장 자료는 적절한 소프트웨어 신뢰도 모형에 의거해 계속적으로 분석된다. 비동질 포아송 과정 모형이 적절한 소프트웨어 신뢰도 성장 모형인 경우 고장 수 자료를 분석하기 위해서 포아송 회귀 모형을 세우고 모수들은 가중 최소 자승법으로 추정하는 것이 가능하며, 이렇게 구한 가중 최소 자승 추정량은 최우 추정량과 동일한 성질을 가짐을 보일 수 있다. 이 분석 방법을 대형 시스템으로부터 수집된 실제 자료를 분석하는데 적용한다.


# Analysis of Failure Count Data Based on NHPP Models

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## ABSTRACT

An important quality characteristic of a software is the software reliability. Software reliability growth models provide the tools to evaluate and monitor the reliability growth behavior of the software during the testing phase. Therefore failure data collected during the testing phase should be continuously analyzed on the basis of some selected software reliability growth models. For the cases where nonhomogeneous Poisson process models are the candidate models, we suggest Poisson regression model, which expresses the relationship between the expected and actual failures counts in disjoint time intervals, for analyzing the failure count data. The weighted least squares method is then used to estimate the parameters in the model. The resulting estimators are equivalent to the maximum likelihood estimators. The method is illustrated by analyzing the failure count data gathered from a large-scale switching system.


## 1. Introduction

In recent years software systems such as operating systems, control programs and application programs have become more complex and larger than ever. Efforts are continuously made to produce high-quality softwares. McCall, Richards and Walters[9] proposed a useful categorization of factors that affect the software quality. An important quality attribute of a software system is the degree to which it can be relied upon to perform its intended function. Software reliability is the probability of failure free operation of a software system for a specified time in a specified environment. A number of statistical models have been developed to quantitatively evaluate the reliability

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of a software system during testing and operational phases based on its failure history. The class of nonhomogeneous Poisson process (NHPP) models is widely used as an analytical framework for describing the software failure process during the testing phase. The software reliability growth models developed by Crow [2], Musa [10], Goel and Okumoto [6], Ohba [13], Goel [4], [5], Yamada, Ohba and Osaki [15], Musa and Okumoto [12] belong to this class.

There are two types of failure data, interfailure time data and failure count data. Irrespective of which type of failure data is available, the failure data is usually analyzed by the maximum likelihood (ML) method. The ML estimators, in general the solution of ML equations, possess many desirable properties such as consistency, efficiency and asymptotic normality. Hossain and Dahiya [7] studied on the existence and uniqueness of the ML estimators. Generally the ML equations are highly complicated. It is not convenient to solve the simultaneous equations numerically. Okumoto [14] employed the least squares (LS) method to estimate the failure intensity function of the logarithmic Poisson model from the failure count data. He shows that the LS method is simple and practicable and yields the accuracy equivalent to the ML method. However, the approach of Okumoto [14] can be applied only to the models of which the failure intensity function is transformed to a simple linear function of the mean value function.

This paper considers the situation in which NHPP models are appropriate reliability growth models and the failure count data is available. Section 2 first expresses an NHPP model as a Poisson regression model, which relates the observed failure counts in disjoint time intervals to their expected values. Then the weighted LS method is used and the characteristics of the resulting weighted LS estimators are briefly discussed. An analysis procedure is suggested and illustrated in Section 3.

## 2. Transformation of NHPP Models to Poisson Regression Models

In the NHPP models the number of failures occurred during a specified testing time is treated as a random variable. An NHPP model is thus described as a stochastic process $\{N(t), t \geq 0\}$ representing the number of software failures experienced by time $t$. An NHPP with intensity function $\lambda(t)$ conforms to the 4 assumptions described in Musa, Iannino and Okumoto [11]. It is well-known that the distribution of $N(t)$ is obtained from the assumptions as a Poisson distribution with mean $m(t)=\int_0^t \lambda(s)ds$. An NHPP model is therefore characterized by its mean value function $m(t)$ or intensity function $\lambda(t)$. The followings are the mean value functions corresponding to the NHPP models proposed by Crow [2], Goel [4], [5], Goel and Okumoto [6], Ohba [13] and Yamada, Ohba and Osaki [15]. These 5 NHPP models will be considered in Section 3.

(i) Crow model

$$m(t)=\alpha t^{\beta},\ \alpha > 0,\ \beta > 0.$$

(ii) Goel and Okumoto model

$$m(t)=a(1-e^{-bt}),\ a > 0,\ b > 0.$$

(iii) Ohba model

$$m(t)=a(1-e^{-bt})/(1+ce^{-bt}),\quad a > 0,\ b > 0,\ c > 0.$$

(iv) Goel generalized model

$$m(t)=a(1-e^{-bt^{d}}),\ a > 0,\ b > 0,\ d > 0.$$

(v) Yamada, Ohba and Osaki model

$$m(t)=a[1-(1+bt)e^{-bt}],\ a > 0,\ b > 0.$$

The parameters $\alpha$ and $\beta$ in Crow model denote respectively the number of mean failures under no reliability growth and the growth rate. The common parameters $b$ in other models are the expected number of failures to be observed eventually and the failure detection rate per fault. However, $c$ represents the inflection factor and $d$ is the constant reflecting the quality of testing.

Firstly we present the procedure for transforming NHPP models into Poisson regression models. Then the corresponding estimation method will be discussed. Let $t_i$, $i=1, 2, \cdots, r$, be the times at which the values of $N(t_i)$ are recorded. Suppose that $t_0=0$, $m(t_0)=0$, and $n(t_{i-1}, t_i)=N(t_i)-N(t_{i-1})$. Specifically $n(t_{i-1}, t_i)$ denotes the number of observed failures in time interval $(t_{i-1}, t_i)$. It is well known that $n(t_{i-1}, t_i)$ also obeys the Poisson distribution with mean $m(t_i)-m(t_{i-1})$. Thus we can construct the Poisson regression model

$$\begin{aligned} n(t_{i-1}, t_i) &= E(n(t_{i-1}, t_i))+\varepsilon_i \\ &= m(t_i)-m(t_{i-1})+\varepsilon_i,\ i=1, 2, \cdots, r, \end{aligned} \tag{1}$$

where $\varepsilon_i$'s are independent errors with mean 0 and variance $m(t_i)-m(t_{i-1})$. It is not difficult to show that the Poisson regression model (1) is equivalent to the following model:

$$N(t_i)=m(t_i)+\tilde{\varepsilon}_i,\ i=1, 2, \cdots, r, \tag{2}$$

where $\tilde{\varepsilon}_i=\sum_{j=1}^{i}\varepsilon_j$ and $\tilde{\varepsilon}_i$'s have mean zero and variance-covariance matrix

$$\begin{bmatrix} m(t_1) & m(t_1) & m(t_1) & \cdots & m(t_1) \\ m(t_1) & m(t_2) & m(t_2) & \cdots & m(t_2) \\ m(t_1) & m(t_2) & m(t_3) & \cdots & m(t_3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m(t_1) & m(t_2) & m(t_3) & \cdots & m(t_r) \end{bmatrix}$$

Suppose that $t_i$ and the observed values of $n(t_{i-1}, t_i)$ are available. If a specific NHPP model is chosen, the functional form of $m(t)$ is given. The parameters in $m(t)$ are usually estimated by the ML method. In general $m(t)$ is nonlinear and corresponding ML equations are complex and nonlinear. Therefore an iterative procedure is used to find roots of the ML equations. Instead we may consider the Poisson regression model (1) and the weighted LS estimators minimizing

$$\sum_{i=1}^{r} w_i[n(t_{i-1}, t_i)-\{m(t_i)-m(t_{i-1})\}]^2, \tag{3}$$

where the weight $w_i$ is the reciprocal of the variance of $\varepsilon_i$. In order to find the values of parameters in $m(t)$ that minimize (3), we should equate the derivatives of (3) with respect to the parameters to zero. An iterative procedure is also required to compute the weighted LS estimators. However, statistical packages such as SAS provide the iterative procedure for the weighted LS method for nonlinear models. Such procedures allow us to easily compute the estimators of parameters, $m(t)$ and $E(n(t', t))$. We can also examine goodness of fit and model adequacy without any additional computation. It is also worthy of note that the weighted LS estimators are equivalent to the ML estimators. This result was shown by Frome, Kutner and Beauchamp [3]. More general results are provided in Charnes, Frome and Yu [1]. In addition, unlikely to Okumoto [14], the suggested method does not necessitate us to transform the NHPP model into a linear model.

## 3. A Procedure for Analyzing Failure Count Data

This section provides a procedure for analyzing the failure count data. In order to illustrate this procedure, we use the failure count data collected from a large-scale switching system and presented by Hwang *et al.* [8]. The switching system consists of approximately 1,500,000 lines of code. The number of failures was recorded every week and 846 failures were detected during 41 weeks. Table 1 shows the data. Hwang *et al.* [8] analyzed the data based on model (2). However, they did not take account of the variance-covariance structure of $\tilde{\varepsilon}_i$. If dependency and nonhomogeneous variance of $\tilde{\varepsilon}_i$ are disregarded, the results are still unbiased but not optimal in the sense of precision.

〈Table 1〉 Switching system data

| $t_i$ | $n(t_{i-1}, t_i)$ | $N(t_i)$ | $t_i$ | $n(t_{i-1}, t_i)$ | $N(t_i)$ |
|---|---|---|---|---|---|
| 1 | 16 | 16 | 22 | 24 | 562 |
| 2 | 13 | 29 | 23 | 27 | 589 |
| 3 | 32 | 61 | 24 | 10 | 599 |
| 4 | 60 | 121 | 25 | 21 | 620 |
| 5 | 8 | 129 | 26 | 22 | 642 |
| 6 | 30 | 159 | 27 | 19 | 661 |
| 7 | 20 | 179 | 28 | 17 | 678 |
| 8 | 56 | 235 | 29 | 13 | 691 |
| 9 | 33 | 268 | 30 | 16 | 707 |
| 10 | 34 | 302 | 31 | 9 | 716 |
| 11 | 15 | 317 | 32 | 10 | 726 |
| 12 | 47 | 364 | 33 | 20 | 746 |
| 13 | 19 | 383 | 34 | 13 | 759 |
| 14 | 15 | 398 | 35 | 24 | 783 |
| 15 | 13 | 411 | 36 | 30 | 813 |
| 16 | 26 | 437 | 37 | 3 | 816 |
| 17 | 3 | 440 | 38 | 3 | 819 |
| 18 | 18 | 458 | 39 | 13 | 832 |
| 19 | 32 | 490 | 40 | 4 | 836 |
| 20 | 18 | 508 | 41 | 10 | 846 |
| 21 | 30 | 538 | | | |

The following procedure may be adopted for analyzing the failure count data by Poisson regression approach.

(i) Select candidate NHPP models via exploratory data analysis.

(ii) Transform the candidate NHPP models into Poisson regression models.

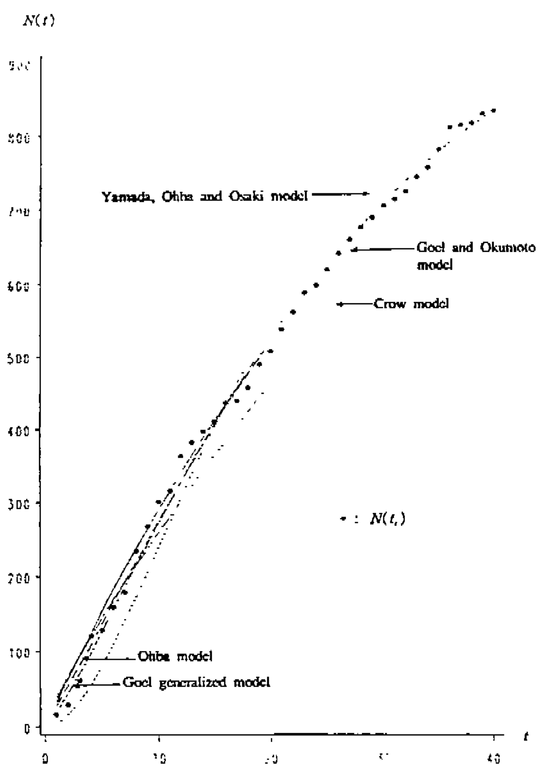
(iii) Fit the Poisson regression models to the failure count data by using an appropriate statistical procedure which supports the weighted least squares method for nonlinear regression models.

(iv) Select adequate models from the candidate models by investigating model adequacy and goodness of fit.

(v) Examining one or more model selection criteria, choose the best model from the selected adequate models.

(vi) Compute the desired quantities for evaluating software reliability.

Suppose that the candidate models are the 5 NHPP models considered in the previous section. Employing the Poisson regression model (1) and using NLIN procedure of SAS, we obtained the weighted LS estimates for the 5 NHPP models. It was found that Goel and Okumoto model, Ohba model and Goel generalized model are adequate. $N(t_i)$ and $\hat{m}(t)$'s are depicted in Fig. 1 for visual inspection. In order to select the best-fitted model from the 3 adequate




(Fig. 1) Plots of $N(t_i)$ and $\hat{m}(t_i)$'s for 5 NHPP models.

models, we consider three criteria, minimum value of (3) and maximum and sum of absolute studentized residuals. The values of these criteria are obtained and tabulated in Table 2. The values for inadequate models are also tabulated for the sake of reference. It seems to be reasonable to select Goel generalized NHPP model as the best one. The estimated mean value function and intensity function of the selected NHPP model are

$$\hat{m}(t) = 1035.2085(1 - e^{-0.0189\, t^{1.2116}})$$

and

$$\hat{\lambda}(t) = 23.6983\, t^{0.2116}\, e^{-0.0189\, t^{1.2116}}$$

If the switching system is released at time 41, the probability that the system operates during [41, 41 $+t$] without a failure is thus estimated as $e^{-\hat{\lambda}(41)t} = e^{-9.5022t}$.

〈Table 2〉 Values of 3 criteria for model selection

| model \ criteria | maximum of absolute studentized residuals | sum of absolute studentized residuals | residual sum of squares |
|---|---|---|---|
| Crow model | 2.6349 | 32.3697 | 279.7932 |
| Goel-Okumoto model | 2.3303 | 32.4912 | 228.7561 |
| Ohba model | 2.6443 | 31.9700 | 228.1015 |
| Goel generalized model | 2.6233 | 31.7055 | 217.2535 |
| Yamada, Ohba and Osaki model | 2.9711 | 30.5653 | 306.6622 |

## 4. Conclusions

The NHPP software reliability growth models and ML methods are usually adopted to analyze most software failure data. Alternatively, we suggested to express an NHPP model as a Poisson regression model and analyze the data by the weighted LS method. The resulting weighted LS estimators are equivalent to the ML estimators. The suggested method is easy to implement since most statistical packages are equipped with the procedure for the weighted LS method. It was shown by an illustrative example that the suggested method works well. Three criteria were considered in Section 3 for model selection. However, it seems to be necessary to develop other numerical and graphical methods model adequacy and predictivity. We are also interested in a similar research for the interfailure time data.

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