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THE RINGS OF INVARIANTS OF FINITE ABELIAN SUBGROUPS OF $GL(2, C)$ OF ORDER ≤ 18

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ABSTRACT. We classify up to conjugation all finite abelian subgroups of $GL(2, C)$ of order ≤ 18 and compute the generators and relations of their rings of invariants. In other words, we classify all 2-dimensional quotient singularities by an abelian group of order ≤ 18 and compute the generators and relations of their affine coordinate rings.

1. Introduction

Let G be a finite subgroup of $GL(2, \boldsymbol{C})$. Then G acts on $\boldsymbol{C}^2$ and the orbit space $\boldsymbol{C}^2/G$ becomes an affine variety whose coordinate ring is isomorphic to the ring $\boldsymbol{C}[x, y]^G$ of invariant polynomials.

The generators of the ring of invariants of G can be computed by using the Reynolds operator which is defined by the formula

$$R_G(f)(x, y) = \frac{1}{|G|} \sum_{g \in G} f(g \cdot x, y)$$

for $f(x, y) \in \boldsymbol{C}[x, y]$, and the relations among generators can be computed by applying the Groebner basis technique. See [1] for details.

In this paper, we classify up to conjugation all finite abelian subgroups of $GL(2, \boldsymbol{C})$ of small order, say, up to 18 and compute the generators and relations of their rings of invariants. In other words, we classify all 2-dimensional quotient singularities by an abelian group of order ≤ 18 and

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compute the generators and relations of their affine coordinate rings. The results are summarized in the next section.

2. List of rings of invariants

In the following list, the groups are ordered by increase of the order. For each group we give three data— the generators of the group, the generators of the ring of invariants and the relations among generators of the ring of invariants.

$\boldsymbol{Z}_2$:

1. $\left(\begin{smallmatrix}-1 & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^2,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}-1 & 0\\ 0 & -1\end{smallmatrix}\right)$; $y_1 = x^2,\ y_2 = xy,\ y_3 = y^2$; $-y_2^2 + y_1y_3$

$\boldsymbol{Z}_3$: $(\omega = exp(2\pi i/3))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^3,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^3,\ y_2 = x^2y,\ y_3 = xy^2,\ y_4 = y^3$
   ; $-y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^3,\ y_2 = xy,\ y_3 = y^3$; $-y_2^3 + y_1y_3$

$\boldsymbol{Z}_4$:

1. $\left(\begin{smallmatrix}i & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^4,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}i & 0\\ 0 & i\end{smallmatrix}\right)$; $y_1 = x^4,\ y_2 = x^3y,\ y_3 = x^2y^2,\ y_4 = xy^3,\ y_5 = y^4$
   ; $-y_4^2 + y_3y_5,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
   $-y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}i & 0\\ 0 & -1\end{smallmatrix}\right)$; $y_1 = x^4,\ y_2 = x^2y,\ y_3 = y^2$; $-y_2^2 + y_1y_3$
4. $\left(\begin{smallmatrix}i & 0\\ 0 & -i\end{smallmatrix}\right)$; $y_1 = x^4,\ y_2 = xy,\ y_3 = y^4$; $-y_2^4 + y_1y_3$

$\boldsymbol{Z}_2 \times \boldsymbol{Z}_2$:

1. $\left\langle \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right), \left(\begin{smallmatrix}-1 & 0\\ 0 & 1\end{smallmatrix}\right) \right\rangle$; $y_1 = x^2,\ y_2 = y^2$; none

$\boldsymbol{Z}_5$: $(\omega = exp(2\pi i/5))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^5,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^5,\ y_2 = x^4y,\ y_3 = x^3y^2,\ y_4 = x^2y^3,\ y_5 = xy^4,$
$y_6 = y^5$
; $-y_5^2 + y_4y_6,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^5,\ y_2 = x^3y,\ y_3 = xy^2,\ y_4 = y^5$
; $-y_3^3 + y_2y_4,\ -y_2y_3^2 + y_1y_4,\ -y_2^2 + y_1y_3$
4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1 = x^5,\ y_2 = xy,\ y_3 = y^5$; $-y_2^5 + y_1y_3$

Z_6 : $(\omega = exp(2\pi i/6))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = x^5y,\ y_3 = x^4y^2,\ y_4 = x^3y^3,\ y_5 = x^2y^4,$
$y_6 = xy^5,\ y_7 = y^6$
; $-y_6^2 + y_5y_7,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,$
$-y_4^2 + y_3y_5,\ -y_4y_5 + y_2y_7,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,$
$-y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = x^4y,\ y_3 = x^2y^2,\ y_4 = y^3$
; $-y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = x^3y,\ y_3 = y^2$; $-y_2^2 + y_1y_3$
5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = x^2y,\ y_3 = y^3$; $-y_2^3 + y_1y_3$
6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1 = x^6,\ y_2 = xy,\ y_3 = y^6$; $-y_2^6 + y_1y_3$
7. $\left(\begin{smallmatrix}\omega^2 & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1 = x^3,\ y_2 = y^2$; none

Z_7 : $(\omega = exp(2\pi i/7))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^7,\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^7,\ y_2 = x^6y,\ y_3 = x^5y^2,\ y_4 = x^4y^3,\ y_5 = x^3y^4,$
$y_6 = x^2y^5,\ y_7 = xy^6,\ y_8 = y^7$
; $-y_7^2 + y_6y_8,\ -y_6y_7 + y_5y_8,\ -y_6^2 + y_5y_7,\ -y_6^2 + y_4y_8,\ -y_5y_6 + y_4y_7,$
$-y_5^2 + y_4y_6,\ -y_5y_6 + y_3y_8,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,$
$-y_5^2 + y_2y_8,\ -y_4y_5 + y_2y_7,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,$
$-y_4y_5 + y_1y_8,\ -y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$

$-y_2^2 + y_1y_3$

3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^7,\ y_2 = x^5y,\ y_3 = x^3y^2,\ y_4 = xy^3,\ y_5 = y^7$
; $-y_4^3 + y_3y_5,\ -y_3y_4^2 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2y_4 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
4. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^3 \end{pmatrix}$; $y_1 = x^7,\ y_2 = x^4y,\ y_3 = xy^2,\ y_4 = y^7$
; $-y_3^4 + y_2y_4,\ -y_2y_3^3 + y_1y_4,\ -y_2^2 + y_1y_3$
5. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^6 \end{pmatrix}$; $y_1 = x^7,\ y_2 = xy,\ y_3 = y^7$; $-y_2^7 + y_1y_3$

Z_8 : $(\omega = exp(2\pi i/8))$

1. $\begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$; $y_1 = x^8,\ y_2 = y$; none
2. $\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^7y,\ y_3 = x^6y^2,\ y_4 = x^5y^3,\ y_5 = x^4y^4,$
$y_6 = x^3y^5,\ y_7 = x^2y^6,\ y_8 = xy^7,\ y_9 = y^8$
; $-y_8^2 + y_7y_9,\ -y_7y_8 + y_6y_9,\ -y_7^2 + y_6y_8,\ -y_7^2 + y_5y_9,\ -y_6y_7 + y_5y_8,$
$-y_6^2 + y_5y_7,\ -y_6y_7 + y_4y_9,\ -y_6^2 + y_4y_8,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,$
$-y_6^2 + y_3y_9,\ -y_5y_6 + y_3y_8,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,$
$-y_5y_6 + y_2y_9,\ -y_5^2 + y_2y_8,\ -y_4y_5 + y_2y_7,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_5^2 + y_1y_9,\ -y_4y_5 + y_1y_8,\ -y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,$
$-y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^6y,\ y_3 = x^4y^2,\ y_4 = x^2y^3,\ y_5 = y^4$
; $-y_4^2 + y_3y_5,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
4. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^3 \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^5y,\ y_3 = x^2y^2,\ y_4 = xy^5,\ y_5 = y^8$
; $-y_4^2 + y_3y_5,\ -y_3^2y_4 + y_2y_5,\ -y_3^3 + y_2y_4,\ -y_3^4 + y_1y_5,\ -y_2y_3^2 + y_1y_4,$
$-y_2^2 + y_1y_3$
5. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^4y,\ y_3 = y^2$; $-y_2^2 + y_1y_3$
6. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^5 \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^3y,\ y_3 = xy^3,\ y_4 = y^8$
; $-y_3^3 + y_2y_4,\ -y_2^2y_3^2 + y_1y_4,\ -y_2^3 + y_1y_3$
7. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^6 \end{pmatrix}$; $y_1 = x^8,\ y_2 = x^2y,\ y_3 = y^4$; $-y_2^4 + y_1y_3$
8. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^7 \end{pmatrix}$; $y_1 = x^8,\ y_2 = xy,\ y_3 = y^8$; $-y_2^8 + y_1y_3$

$Z_4 \times Z_2$:

1. $\left\langle \left(\begin{smallmatrix} i & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^4,\ y_2 = y^2$; none
2. $\left\langle \left(\begin{smallmatrix} i & 0 \\ 0 & i \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^4,\ y_2 = x^2y^2,\ y_3 = y^4$; $-y_2^2 + y_1y_3$

$Z_2 \times Z_2 \times Z_2$: none

Z_9 : $(\omega = exp(2\pi i/9))$

1. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & 1 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = y$; none
2. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = x^8y,\ y_3 = x^7y^2,\ y_4 = x^6y^3,\ y_5 = x^5y^4,$
$y_6 = x^4y^5,\ y_7 = x^3y^6,\ y_8 = x^2y^7,\ y_9 = xy^8,\ y_{10} = y^9$
; $-y_9^2+y_8y_{10},\ -y_8y_9+y_7y_{10},\ -y_8^2+y_7y_9,\ -y_8^2+y_6y_{10},\ -y_7y_8+y_6y_9,$
$-y_7^2+y_6y_8,\ -y_7y_8+y_5y_{10},\ -y_7^2+y_5y_9,\ -y_6y_7+y_5y_8,\ -y_6^2+y_5y_7,$
$-y_7^2+y_4y_{10},\ -y_6y_7+y_4y_9,\ -y_6^2+y_4y_8,\ -y_5y_6+y_4y_7,\ -y_5^2+y_4y_6,$
$-y_6y_7+y_3y_{10},\ -y_6^2+y_3y_9,\ -y_5y_6+y_3y_8,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,$
$-y_4^2+y_3y_5,\ -y_6^2+y_2y_{10},\ -y_5y_6+y_2y_9,\ -y_5^2+y_2y_8,\ -y_4y_5+y_2y_7,$
$-y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_5y_6+y_1y_{10},\ -y_5^2+y_1y_9,$
$-y_4y_5+y_1y_8,\ -y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$
3. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^2 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = x^7y,\ y_3 = x^5y^2,\ y_4 = x^3y^3,\ y_5 = xy^4,$
$y_6 = y^9$
; $-y_5^3+y_4y_6,\ -y_4y_5^2+y_3y_6,\ -y_4^2+y_3y_5,\ -y_4^2y_5+y_2y_6,\ -y_3y_4+y_2y_5,$
$-y_3^2+y_2y_4,\ -y_4^3+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$
4. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^3 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = x^6y,\ y_3 = x^3y^2,\ y_4 = y^3$
; $-y_3^2+y_2y_4,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$
5. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^4 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = x^5y,\ y_3 = xy^2,\ y_4 = y^9$
; $-y_3^5+y_2y_4,\ -y_2y_3^4+y_1y_4,\ -y_2^2+y_1y_3$
6. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^6 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = x^3y,\ y_3 = y^3,$; $-y_2^3+y_1y_3$
7. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^8 \end{smallmatrix}\right)$; $y_1 = x^9,\ y_2 = xy,\ y_3 = y^9$; $-y_2^9+y_1y_3$

$Z_3 \times Z_3$: $(\omega = exp(2\pi i/3))$

1. $\left\langle \left(\begin{smallmatrix} 1 & 0 \\ 0 & \omega \end{smallmatrix}\right), \left(\begin{smallmatrix} \omega & 0 \\ 0 & 1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^3,\ y_2 = y^3$; none

$\boldsymbol{Z}_{10}$: $(\omega = exp(2\pi i/10))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^9y,\ y_3 = x^8y^2,\ y_4 = x^7y^3,\ y_5 = x^6y^4,$
$y_6 = x^5y^5,\ y_7 = x^4y^6,\ y_8 = x^3y^7,\ y_9 = x^2y^8,\ y_{10} = xy^9,\ y_{11} = y^{10}$
; $-y_{10}^2 + y_{11}y_9,\ -y_9y_{10} + y_{11}y_8,\ -y_9^2 + y_8y_{10},\ -y_9^2 + y_7y_{11},$
$-y_8y_9 + y_7y_{10},\ -y_8^2 + y_7y_9,\ -y_8y_9 + y_6y_{11},\ -y_8^2 + y_6y_{10},\ -y_7y_8 + y_6y_9,$
$-y_7^2 + y_6y_8,\ -y_8^2 + y_5y_{11},\ -y_7y_8 + y_5y_{10},\ -y_7^2 + y_5y_9,\ -y_6y_7 + y_5y_8,$
$-y_6^2 + y_5y_7,\ -y_7y_8 + y_4y_{11},\ -y_7^2 + y_4y_{10},\ -y_6y_7 + y_4y_9,\ -y_6^2 + y_4y_8,$
$-y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_7^2 + y_3y_{11},\ -y_6y_7 + y_3y_{10},\ -y_6^2 + y_3y_9,$
$-y_5y_6 + y_3y_8,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_6y_7 + y_2y_{11},$
$-y_6^2 + y_2y_{10},\ -y_5y_6 + y_2y_9,\ -y_5^2 + y_2y_8,\ -y_4y_5 + y_2y_7,\ -y_4^2 + y_2y_6,$
$-y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_6^2 + y_{11}y_1,\ -y_5y_6 + y_{10}y_1,\ -y_5^2 + y_9y_1,$
$-y_4y_5 + y_1y_8,\ -y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^8y,\ y_3 = x^6y^2,\ y_4 = x^4y^3,\ y_5 = x^2y^4,$
$y_6 = y^5$
; $-y_5^2 + y_4y_6,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^7y,\ y_3 = x^4y^2,\ y_4 = xy^3,\ y_5 = y^{10}$
; $-y_4^4 + y_3y_5,\ -y_3y_4^3 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2y_4^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^6y,\ y_3 = x^2y^2,\ y_4 = y^5$
; $-y_3^3 + y_2y_4,\ -y_2y_3^2 + y_1y_4,\ -y_2^2 + y_1y_3$
6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^5y,\ y_3 = y^2,$; $-y_2^2 + y_1y_3$
7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^6\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^4y,\ y_3 = x^2y^3,\ y_4 = y^5$
; $-y_3^2 + y_2y_4,\ -y_2^2y_3 + y_1y_4,\ -y_2^3 + y_1y_3$
8. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^8\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = x^2y,\ y_3 = y^5$; $-y_2^5 + y_1y_3$
9. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^9\end{smallmatrix}\right)$; $y_1 = x^{10},\ y_2 = xy,\ y_3 = y^{10}$; $-y_2^{10} + y_1y_3$
10. $\left(\begin{smallmatrix}\omega^2 & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1 = x^5,\ y_2 = y^2$; none

$\boldsymbol{Z}_{11}$: $(\omega = exp(2\pi i/11))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = x^{10}y$, $y_3 = x^9y^2$, $y_4 = x^8y^3$, $y_5 = x^7y^4$, $y_6 = x^6y^5$, $y_7 = x^5y^6$, $y_8 = x^4y^7$, $y_9 = x^3y^8$, $y_{10} = x^2y^9$, $y_{11} = xy^{10}$, $y_{12} = y^{11}$
; $-y_{11}^2 + y_{10}y_{12}$, $-y_{10}y_{11} + y_{12}y_9$, $-y_{10}^2 + y_9y_{11}$, $-y_{10}^2 + y_8y_{12}$, $-y_9y_{10} + y_8y_{11}$, $-y_9^2 + y_8y_{10}$, $-y_9y_{10} + y_7y_{12}$, $-y_9^2 + y_7y_{11}$, $-y_8y_9 + y_7y_{10}$, $-y_8^2 + y_7y_9, y_9^2 + y_6y_{12}$, $-y_8y_9 + y_6y_{11}, y_8^2 + y_6y_{10}$, $-y_7y_8 + y_6y_9$, $-y_7^2 + y_6y_8$, $-y_8y_9 + y_5y_{12}$, $-y_8^2 + y_5y_{11}, y_7y_8 + y_5y_{10}$, $-y_7^2 + y_5y_9$, $-y_6y_7 + y_5y_8$, $-y_6^2 + y_5y_7$, $-y_8^2 + y_4y_{12}$, $-y_7y_8 + y_4y_{11}$, $-y_7^2 + y_4y_{10}$, $-y_6y_7 + y_4y_9, y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$, $-y_7y_8 + y_3y_{12}$, $-y_7^2 + y_3y_{11}$, $-y_6y_7 + y_3y_{10}$, $-y_6^2 + y_3y_9, y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_7^2 + y_{12}y_2$, $-y_6y_7 + y_{11}y_2$, $-y_6^2 + y_{10}y_2$, $-y_5y_6 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4, y_6y_7 + y_1y_{12}$, $-y_6^2 + y_1y_{11}, y_5y_6 + y_1y_{10}$, $-y_5^2 + y_1y_9$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7$, $-y_3y_4 + y_1y_6$, $-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$
3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = x^9y$, $y_3 = x^7y^2$, $y_4 = x^5y^3$, $y_5 = x^3y^4$, $y_6 = xy^5$, $y_7 = y^{11}$
; $-y_6^3 + y_5y_7$, $-y_5y_6^2 + y_4y_7$, $-y_5^2 + y_4y_6$, $-y_5^2y_6 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_5^3 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_4y_5^2 + y_1y_7$, $-y_3y_4 + y_1y_6$, $-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$
4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = x^8y$, $y_3 = x^5y^2$, $y_4 = x^2y^3$, $y_5 = xy^7$, $y_6 = y^{11}$
; $-y_5^2 + y_4y_6$, $-y_4^2y_5 + y_3y_6$, $-y_4^3 + y_3y_5$, $-y_4^4 + y_2y_6$, $-y_3y_4^2 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_3y_4^3 + y_1y_6$, $-y_3^2y_4 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$
5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = x^6y$, $y_3 = xy^2$, $y_4 = y^{11}$
; $-y_3^6 + y_2y_4$, $-y_2y_3^5 + y_1y_4$, $-y_2^2 + y_1y_3$
6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^7\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = x^4y$, $y_3 = xy^3$, $y_4 = y_{11}$
; $-y_3^4 + y_2y_4$, $-y_2^2y_3^3 + y_1y_4$, $-y_2^3 + y_1y_3$
7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{10}\end{smallmatrix}\right)$; $y_1 = x^{11}$, $y_2 = xy$, $y_3 = y^{11}$; $-y_2^{11} + y_1y_3$

$\boldsymbol{Z}_{12}$: $(\omega = exp(2\pi i/12))$

1. $\begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$; $y_1 = x^{12},\ y_2 = y$; none
2. $\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}$; $y_1 = x^{12},\ y_2 = x^{11}y,\ y_3 = x^{10}y^2,\ y_4 = x^9y^3,\ y_5 = x^8y^4,$
$y_6 = x^7y^5,\ y_7 = x^6y^6,\ y_8 = x^5y^7,\ y_9 = x^4y^8,\ y_{10} = x^3y^9,$
$y_{11} = x^2y^{10},\ y_{12} = xy^{11},\ y_{13} = y^{12}$
$; -y_{12}^2 + y_{11}y_{13},\ -y_{11}y_{12} + y_{13}y_{10},\ -y_{11}^2 + y_{10}y_{12},\ -y_{11}^2 + y_9y_{13},$
$-y_{10}y_{11} + y_9y_{12}, y_{10}^2 + y_9y_{11},\ -y_{10}y_{11} + y_8y_{13},\ -y_{10}^2 + y_8y_{12},$
$-y_9y_{10} + y_8y_{11},\ -y_9^2 + y_8y_{10},\ -y_{10}^2 + y_7y_{13},\ -y_9y_{10} + y_7y_{12},$
$-y_9^2 + y_7y_{11},\ -y_8y_9 + y_7y_{10},\ -y_8^2 + y_7y_9, y_9y_{10} + y_6y_{13},\ -y_9^2 + y_6y_{12},$
$-y_8y_9 + y_6y_{11},\ -y_8^2 + y_6y_{10},\ -y_7y_8 + y_6y_9,\ -y_7^2 + y_6y_8,\ -y_9^2 + y_5y_{13},$
$-y_8y_9 + y_5y_{12},\ -y_8^2 + y_5y_{11},\ -y_7y_8 + y_5y_{10},\ -y_7^2 + y_5y_9, y_6y_7 + y_5y_8,$
$-y_6^2 + y_5y_7,\ -y_8y_9 + y_4y_{13},\ -y_8^2 + y_4y_{12}, y_7y_8 + y_4y_{11},\ -y_7^2 + y_4y_{10},$
$-y_6y_7 + y_4y_9,\ -y_6^2 + y_4y_8,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_8^2 + y_{13}y_3,$
$-y_7y_8 + y_{12}y_3,\ -y_7^2 + y_{11}y_3,\ -y_6y_7 + y_3y_{10}, y_6^2 + y_3y_9,\ -y_5y_6 + y_3y_8,$
$-y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5, y_7y_8 + y_2y_{13},\ -y_7^2 + y_2y_{12},$
$-y_6y_7 + y_2y_{11},\ -y_6^2 + y_2y_{10},\ -y_5y_6 + y_2y_9,\ -y_5^2 + y_2y_8, y_4y_5 + y_2y_7,$
$-y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_7^2 + y_1y_{13}, y_6y_7 + y_1y_{12},$
$-y_6^2 + y_1y_{11},\ -y_5y_6 + y_1y_{10},\ -y_5^2 + y_1y_9, y_4y_5 + y_1y_8,\ -y_4^2 + y_1y_7,$
$-y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4, y_2^2 + y_1y_3$
3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^{12},\ y_2 = x^{10}y,\ y_3 = x^8y^2,\ y_4 = x^6y^3,\ y_5 = x^4y^4,$
$y_6 = x^2y^5,\ y_7 = y^6$
$; -y_6^2 + y_5y_7,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,$
$-y_4^2 + y_3y_5,\ -y_4y_5 + y_2y_7,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,$
$-y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
4. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^3 \end{pmatrix}$; $y_1 = x^{12},\ y_2 = x^9y,\ y_3 = x^6y^2,\ y_4 = x^3y^3,\ y_5 = y^4$
$; -y_4^2 + y_3y_5,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
5. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$; $y_1 = x^{12},\ y_2 = x^8y,\ y_3 = x^4y^2,\ y_4 = y^3$
$; -y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
6. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^5 \end{pmatrix}$; $y_1 = x^{12},\ y_2 = x^7y,\ y_3 = x^2y^2,\ y_4 = xy^7,\ y_5 = y^{12}$
$; -y_4^2 + y_3y_5,\ -y_3^3y_4 + y_2y_5,\ -y_3^4 + y_2y_4,\ -y_3^6 + y_1y_5,\ -y_2y_3^3 + y_1y_4,$

$-y_2^2 + y_1y_3$

7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^6\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = x^6y,\ y_3 = y^2$; $-y_2^2 + y_1y_3$
8. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^7\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = x^5y,\ y_3 = x^3y^3,\ y_4 = xy^5,\ y_5 = y^{12}$
; $-y_4^3 + y_3y_5,\ -y_4^2y_3 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^4 + y_1y_5,\ -y_2^3 + y_1y_3,$
$-y_2^2y_3 + y_1y_4$
9. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^8\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = x^4y,\ y_3 = y^3$; $-y_2^3 + y_1y_3$
10. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^9\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = x^3y,\ y_3 = y^4$; $-y_2^4 + y_1y_3$
11. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{10}\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = x^2y,\ y_3 = y^6$; $-y_2^6 + y_1y_3$
12. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{11}\end{smallmatrix}\right)$; $y_1 = x^{12},\ y_2 = xy,\ y_3 = y^{12}$; $-y_2^{12} + y_1y_3$

$Z_6 \times Z_2$: $(\omega = exp(2\pi i/6))$

1. $\left\langle \left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right) \right\rangle$; $y_1 = x^6,\ y_2 = y^2$; none
2. $\left\langle \left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right) \right\rangle$; $y_1 = x^6,\ y_2 = x^4y^2,\ y_3 = x^2y^4,\ y_4 = y^6$
; $-y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
3. $\left\langle \left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right) \right\rangle$; $y_1 = x^6,\ y_2 = x^2y^2,\ y_3 = y^6$; $-y_2^3 + y_1y_3$

Z_{13} : $(\omega = exp(2\pi i/13))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^{13},\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^{13},\ y_2 = x^{12}y,\ y_3 = x^{11}y^2,\ y_4 = x^{10}y^3,\ y_5 = x^9y^4,$
$y_6 = x^8y^5,\ y_7 = x^7y^6,\ y_8 = x^6y^7,\ y_9 = x^5y^8,\ y_{10} = x^4y^9,$
$y_{11} = x^3y^{10},\ y_{12} = x^2y^{11},\ y_{13} = xy^{12},\ y_{14} = y^{13}$
; $-y_{13}^2 + y_{12}y_{14},\ -y_{12}y_{13} + y_{14}y_{11},\ -y_{12}^2 + y_{11}y_{13},\ -y_{12}^2 + y_{10}y_{14},$
$-y_{11}y_{12} + y_{10}y_{13}, y_{11}^2 + y_{10}y_{12},\ -y_{11}y_{12} + y_9y_{14},\ -y_{11}^2 + y_9y_{13},$
$-y_{10}y_{11} + y_9y_{12},\ -y_{10}^2 + y_9y_{11},\ -y_{11}^2 + y_8y_{14},\ -y_{10}y_{11} + y_8y_{13},$
$-y_{10}^2 + y_8y_{12},\ -y_9y_{10} + y_8y_{11},\ -y_9^2 + y_8y_{10},\ -y_{10}y_{11} + y_7y_{14},$
$-y_{10}^2 + y_7y_{13},\ -y_9y_{10} + y_7y_{12},\ -y_9^2 + y_7y_{11},\ -y_8y_9 + y_7y_{10},\ -y_8^2 + y_7y_9,$
$-y_{10}^2 + y_6y_{14}, y_9y_{10} + y_6y_{13},\ -y_9^2 + y_6y_{12},\ -y_8y_9 + y_6y_{11},\ -y_8^2 + y_6y_{10},$
$-y_7y_8 + y_6y_9,\ -y_7^2 + y_6y_8,\ -y_9y_{10} + y_5y_{14},\ -y_9^2 + y_5y_{13}, y_8y_9 + y_5y_{12},$
$-y_8^2 + y_5y_{11},\ -y_7y_8 + y_5y_{10},\ -y_7^2 + y_5y_9,\ -y_6y_7 + y_5y_8,\ -y_6^2 + y_5y_7,$
$-y_9^2 + y_{14}y_4, y_8y_9 + y_{13}y_4,\ -y_8^2 + y_{12}y_4,\ -y_7y_8 + y_4y_{11}, y_7^2 + y_4y_{10},$

$-y_6y_7+y_4y_9,\ -y_6^2+y_4y_8,\ -y_5y_6+y_4y_7,\ -y_5^2+y_4y_6,\ -y_8y_9+y_3y_{14},$
$-y_8^2+y_3y_{13},\ -y_7y_8+y_3y_{12},\ -y_7^2+y_3y_{11},\ -y_6y_7+y_3y_{10}, y_6^2+y_3y_9,$
$-y_5y_6+y_3y_8,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,\ -y_4^2+y_3y_5,\ -y_8^2+y_2y_{14},$
$-y_7y_8+y_2y_{13},\ -y_7^2+y_2y_{12},\ -y_6y_7+y_2y_{11},\ -y_6^2+y_2y_{10}, y_5y_6+y_2y_9,$
$-y_5^2+y_2y_8,\ -y_4y_5+y_2y_7,\ -y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,$
$-y_7y_8+y_1y_{14},\ -y_7^2+y_1y_{13},\ -y_6y_7+y_1y_{12},\ -y_6^2+y_1y_{11},$
$-y_5y_6+y_1y_{10},\ -y_5^2+y_1y_9,\ -y_4y_5+y_1y_8,\ -y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,$
$-y_3^2+y_1y_5,\ -y_2y_3+y_1y_4, y_2^2+y_1y_3$

3. $\begin{pmatrix}\omega & 0\\ 0 & \omega^2\end{pmatrix}$; $y_1=x^{13},\ y_2=x^{11}y,\ y_3=x^9y^2,\ y_4=x^7y^3,\ y_5=x^5y^4,$
$y_6=x^3y^5,\ y_7=xy^6,\ y_8=y^{13}$
; $-y_7^3+y_6y_8,\ -y_6y_7^2+y_5y_8,\ -y_6^2+y_5y_7,\ -y_6^2y_7+y_4y_8,\ -y_5y_6+y_4y_7,$
$-y_5^2+y_4y_6,\ -y_6^3+y_3y_8,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,\ -y_4^2+y_3y_5,$
$-y_5y_6^2+y_2y_8,\ -y_4y_5+y_2y_7,\ -y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,$
$-y_5^2y_6+y_1y_8,\ -y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$

4. $\begin{pmatrix}\omega & 0\\ 0 & \omega^3\end{pmatrix}$; $y_1=x^{13},\ y_2=x^{10}y,\ y_3=x^7y^2,\ y_4=x^4y^3,\ y_5=xy^4,$
$y_6=y^{13}$
; $-y_5^4+y_4y_6,\ -y_4y_5^3+y_3y_6,\ -y_4^2+y_3y_5,\ -y_4^2y_5^2+y_2y_6,\ -y_3y_4+y_2y_5,$
$-y_3^2+y_2y_4,\ -y_4^3y_5+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

5. $\begin{pmatrix}\omega & 0\\ 0 & \omega^4\end{pmatrix}$; $y_1=x^{13},\ y_2=x^9y,\ y_3=x^5y^2,\ y_4=xy^3,\ y_5=y^{13}$
; $-y_4^5+y_3y_5,\ -y_3y_4^4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_3^2y_4^3+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$

6. $\begin{pmatrix}\omega & 0\\ 0 & \omega^5\end{pmatrix}$; $y_1=x^{13},\ y_2=x^8y,\ y_3=x^3y^2,\ y_4=xy^5,\ y_5=y^{13}$
; $-y_4^3+y_3y_5,\ -y_3^2y_4^2+y_2y_5,\ -y_3^3+y_2y_4,\ -y_3^4y_4+y_1y_5,\ -y_2y_3^2+y_1y_4,$
$-y_2^2+y_1y_3$

7. $\begin{pmatrix}\omega & 0\\ 0 & \omega^6\end{pmatrix}$; $y_1=x^{13},\ y_2=x^7y,\ y_3=xy^2,\ y_4=y^{13}$
; $-y_3^7+y_2y_4,\ -y_2y_3^6+y_1y_4,\ -y_2^2+y_1y_3$

8. $\begin{pmatrix}\omega & 0\\ 0 & \omega^{12}\end{pmatrix}$; $y_1=x^{13},\ y_2=xy,\ y_3=y^{13}$; $-y_2^{13}+y_1y_3$

Z_{14} : $(\omega=exp(2\pi i/14))$

1. $\begin{pmatrix}\omega & 0\\ 0 & 1\end{pmatrix}$; $y_1=x^{14},\ y_2=y$; none

2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^{14}$, $y_2 = x^{13}y$, $y_3 = x^{12}y^2$, $y_4 = x^{11}y^3$, $y_5 = x^{10}y^4$, $y_6 = x^9y^5$, $y_7 = x^8y^6$, $y_8 = x^7y^7$, $y_9 = x^6y^8$, $y_{10} = x^5y^9$, $y_{11} = x^4y^{10}$, $y_{12} = x^3y^{11}$, $y_{13} = x^2y^{12}$, $y_{14} = xy^{13}$, $y_{15} = y^{14}$
; $-y_{14}^2 + y_{13}y_{15}$, $-y_{13}y_{14} + y_{15}y_{12}$, $-y_{13}^2 + y_{12}y_{14}$, $y_{13}^2 + y_{11}y_{15}$, $-y_{12}y_{13} + y_{11}y_{14}$, $y_{12}^2 + y_{11}y_{13}$, $-y_{12}y_{13} + y_{10}y_{15}$, $-y_{12}^2 + y_{10}y_{14}$, $-y_{11}y_{12} + y_{10}y_{13}$, $-y_{11}^2 + y_{10}y_{12}$, $-y_{12}^2 + y_9y_{15}$, $-y_{11}y_{12} + y_9y_{14}$, $-y_{11}^2 + y_9y_{13}$, $-y_{10}y_{11} + y_9y_{12}$, $-y_{10}^2 + y_9y_{11}$, $-y_{11}y_{12} + y_8y_{15}$, $-y_{11}^2 + y_8y_{14}$, $-y_{10}y_{11} + y_8y_{13}$, $-y_{10}^2 + y_8y_{12}$, $-y_9y_{10} + y_8y_{11}$, $-y_9^2 + y_8y_{10}$, $-y_{11}^2 + y_7y_{15}$, $-y_{10}y_{11} + y_7y_{14}$, $-y_{10}^2 + y_7y_{13}$, $-y_9y_{10} + y_7y_{12}$, $-y_9^2 + y_7y_{11}$, $-y_8y_9 + y_7y_{10}$, $-y_8^2 + y_7y_9$, $-y_{10}y_{11} + y_6y_{15}$, $-y_{10}^2 + y_6y_{14}$, $-y_9y_{10} + y_6y_{13}$, $-y_9^2 + y_6y_{12}$, $-y_8y_9 + y_6y_{11}$, $-y_8^2 + y_6y_{10}$, $-y_7y_8 + y_6y_9$, $-y_7^2 + y_6y_8$, $-y_{10}^2 + y_{15}y_5$, $-y_9y_{10} + y_{14}y_5$, $-y_9^2 + y_{13}y_5$, $y_8y_9 + y_5y_{12}$, $-y_8^2 + y_5y_{11}$, $-y_7y_8 + y_5y_{10}$, $-y_7^2 + y_5y_9$, $-y_6y_7 + y_5y_8$, $-y_6^2 + y_5y_7$, $-y_9y_{10} + y_4y_{15}$, $-y_9^2 + y_4y_{14}$, $y_8y_9 + y_4y_{13}$, $-y_8^2 + y_4y_{12}$, $-y_7y_8 + y_4y_{11}$, $-y_7^2 + y_4y_{10}$, $-y_6y_7 + y_4y_9$, $-y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$, $-y_9^2 + y_3y_{15}$, $-y_8y_9 + y_3y_{14}$, $-y_8^2 + y_3y_{13}$, $-y_7y_8 + y_3y_{12}$, $-y_7^2 + y_3y_{11}$, $y_6y_7 + y_3y_{10}$, $-y_6^2 + y_3y_9$, $-y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_8y_9 + y_2y_{15}$, $-y_8^2 + y_2y_{14}$, $-y_7y_8 + y_2y_{13}$, $-y_7^2 + y_2y_{12}$, $-y_6y_7 + y_2y_{11}$, $-y_6^2 + y_2y_{10}$, $-y_5y_6 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_8^2 + y_1y_{15}$, $-y_7y_8 + y_1y_{14}$, $-y_7^2 + y_1y_{13}$, $-y_6y_7 + y_1y_{12}$, $-y_6^2 + y_1y_{11}$, $-y_5y_6 + y_1y_{10}$, $-y_5^2 + y_1y_9$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7$, $-y_3y_4 + y_1y_6$, $-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$

3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1 = x^{14}$, $y_2 = x^{12}y$, $y_3 = x^{10}y^2$, $y_4 = x^8y^3$, $y_5 = x^6y^4$, $y_6 = x^4y^5$, $y_7 = x^2y^6$, $y_8 = y^7$
; $-y_7^2 + y_6y_8$, $-y_6y_7 + y_5y_8$, $-y_6^2 + y_5y_7$, $-y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$, $-y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7$, $-y_3y_4 + y_1y_6$, $-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$

4. $\begin{pmatrix}\omega & 0\\ 0 & \omega^3\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^{11}y$, $y_3 = x^8y^2$, $y_4 = x^5y^3$, $y_5 = x^2y^4$, $y_6 = xy^9$, $y_7 = y^{14}$
; $-y_6^2 + y_5y_7$, $-y_5^2y_6 + y_4y_7$, $-y_5^3 + y_4y_6$, $-y_5^4 + y_3y_7$, $-y_4y_5^2 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_4y_5^3 + y_2y_7$, $-y_4^2y_5 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 - y_2y_4$, $-y_4^2y_5^2 + y_1y_7$, $-y_4^3 + y_1y_6$, $-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$
5. $\begin{pmatrix}\omega & 0\\ 0 & \omega^4\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^{10}y$, $y_3 = x^6y^2$, $y_4 = x^2y^3$, $y_5 = y^7$
; $-y_4^3 + y_3y_5$, $-y_3y_4^2 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_3^2y_4 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$
6. $\begin{pmatrix}\omega & 0\\ 0 & \omega^6\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^8y$, $y_3 = x^2y^2$, $y_4 = y^7$
; $-y_3^4 + y_2y_4$, $-y_2y_3^3 + y_1y_4$, $-y_2^2 + y_1y_3$
7. $\begin{pmatrix}\omega & 0\\ 0 & \omega^7\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^7y$, $y_3 = y^2$; $-y_2^2 + y_1y_3$
8. $\begin{pmatrix}\omega & 0\\ 0 & \omega^8\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^6y$, $y_3 = x^4y^3$, $y_4 = x^2y^5$, $y_5 = y^7$
; $-y_4^2 + y_3y_5$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_2y_3^2 + y_1y_5$, $-y_2^2y_3 + y_1y_4$, $-y_2^3 + y_1y_3$
9. $\begin{pmatrix}\omega & 0\\ 0 & \omega^9\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^5y$, $y_3 = xy^3$, $y_4 = y^{14}$
; $-y_3^5 + y_2y_4$, $-y_2^2y_3^4 + y_1y_4$, $-y_2^3 + y_1y_3$
10. $\begin{pmatrix}\omega & 0\\ 0 & \omega^{10}\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^4y$, $y_3 = x^2y^4$, $y_4 = y^7$
; $-y_3^2 + y_2y_4$, $-y_2^3y_3 + y_1y_4$, $-y_2^4 + y_1y_3$
11. $\begin{pmatrix}\omega & 0\\ 0 & \omega^{12}\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = x^2y$, $y_3 = y^7$; $-y_2^7 + y_1y_3$
12. $\begin{pmatrix}\omega & 0\\ 0 & \omega^{13}\end{pmatrix}$; $y_1 = x^{14}$, $y_2 = xy$, $y_3 = y^{14}$; $-y_2^{14} - y_1y_3$

Z_{15} : ($\omega = exp(2\pi i/15)$)

1. $\begin{pmatrix}\omega & 0\\ 0 & 1\end{pmatrix}$; $y_1 = x^{15}$, $y_2 = y$; none
2. $\begin{pmatrix}\omega & 0\\ 0 & \omega\end{pmatrix}$; $y_1 = x^{15}$, $y_2 = x^{14}y$, $y_3 = x^{13}y^2$, $y_4 = x^{12}y^3$, $y_5 = x^{11}y^4$, $y_6 = x^{10}y^5$, $y_7 = x^9y^6$, $y_8 = x^8y^7$, $y_9 = x^7y^8$, $y_{10} = x^6y^9$, $y_{11} = x^5y^{10}$, $y_{12} = x^4y^{11}$, $y_{13} = x^3y^{12}$, $y_{14} = x^2y^{13}$, $y_{15} = xy^{14}$, $y_{16} = y^{15}$
; $-y_{15}^2 + y_{14}y_{16}$, $-y_{14}y_{15} + y_{16}y_{13}$, $-y_{14}^2 + y_{13}y_{15}$, $-y_{14}^2 + y_{12}y_{16}$, $-y_{13}y_{14} + y_{12}y_{15}$, $-y_{13}^2 + y_{12}y_{14}$, $-y_{13}y_{14} + y_{11}y_{16}$, $y_{13}^2 + y_{11}y_{15}$, $-y_{12}y_{13} + y_{11}y_{14}$, $y_{12}^2 + y_{11}y_{13}$, $-y_{13}^2 + y_{10}y_{16}$, $-y_{12}y_{13} + y_{10}y_{15}$, $-y_{12}^2 + y_{10}y_{14}$, $-y_{11}y_{12} + y_{10}y_{13}$, $-y_{11}^2 + y_{10}y_{12}$, $y_{12}y_{13} + y_9y_{16}$,

$-y_{12}^2 + y_9y_{15}$, $-y_{11}y_{12} + y_9y_{14}$, $-y_{11}^2 + y_9y_{13}$, $-y_{10}y_{11} + y_9y_{12}$,
$-y_{10}^2 + y_9y_{11}, y_{12}^2 + y_8y_{16}$, $-y_{11}y_{12} + y_8y_{15}$, $-y_{11}^2 + y_8y_{14}$,
$-y_{10}y_{11} + y_8y_{13}$, $-y_{10}^2 + y_8y_{12}$, $-y_9y_{10} + y_8y_{11}$, $-y_9^2 + y_8y_{10}$,
$-y_{11}y_{12} + y_7y_{16}$, $-y_{11}^2 + y_7y_{15}$, $-y_{10}y_{11} + y_7y_{14}$, $-y_{10}^2 + y_7y_{13}$,
$-y_9y_{10} + y_7y_{12}$, $-y_9^2 + y_7y_{11}$, $-y_8y_9 + y_7y_{10}$, $-y_8^2 + y_7y_9$,
$-y_{11}^2 + y_{16}y_6$, $-y_{10}y_{11} + y_{15}y_6$, $-y_{10}^2 + y_{14}y_6$, $-y_9y_{10} + y_6y_{13}, y_9^2 + y_6y_{12}$,
$-y_8y_9 + y_6y_{11}$, $-y_8^2 + y_6y_{10}$, $-y_7y_8 + y_6y_9$, $-y_7^2 + y_6y_8$,
$-y_{10}y_{11} + y_5y_{16}$, $-y_{10}^2 + y_5y_{15}$, $-y_9y_{10} + y_5y_{14}$, $-y_9^2 + y_5y_{13}$,
$-y_8y_9 + y_5y_{12}$, $-y_8^2 + y_5y_{11}$, $-y_7y_8 + y_5y_{10}$, $-y_7^2 + y_5y_9$, $-y_6y_7 + y_5y_8$,
$-y_6^2 + y_5y_7$, $-y_{10}^2 + y_4y_{16}$, $-y_9y_{10} + y_4y_{15}$, $-y_9^2 + y_4y_{14}$, $-y_8y_9 + y_4y_{13}$,
$-y_8^2 + y_4y_{12}$, $-y_7y_8 + y_4y_{11}$, $-y_7^2 + y_4y_{10}$, $-y_6y_7 + y_4y_9$, $-y_6^2 + y_4y_8$,
$-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$, $-y_9y_{10} + y_3y_{16}$, $-y_9^2 + y_3y_{15}$, $-y_8y_9 + y_3y_{14}$,
$-y_8^2 + y_3y_{13}$, $-y_7y_8 + y_3y_{12}$, $-y_7^2 + y_3y_{11}$, $-y_6y_7 + y_3y_{10}$, $-y_6^2 + y_3y_9$,
$-y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_9^2 + y_2y_{16}$,
$-y_8y_9 + y_2y_{15}$, $-y_8^2 + y_2y_{14}$, $-y_7y_8 + y_2y_{13}$, $-y_7^2 + y_2y_{12}$,
$-y_6y_7 + y_2y_{11}$, $-y_6^2 + y_2y_{10}, y_5y_6 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$,
$-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_8y_9 + y_1y_{16}$, $-y_8^2 + y_1y_{15}$,
$-y_7y_8 + y_1y_{14}$, $-y_7^2 + y_1y_{13}$, $-y_6y_7 + y_1y_{12}$, $-y_6^2 + y_1y_{11}$,
$-y_5y_6 + y_1y_{10}$, $-y_5^2 + y_1y_9$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7$, $-y_3y_4 + y_1y_6$,
$-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$

3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^{15}$, $y_2 = x^{13}y$, $y_3 = x^{11}y^2$, $y_4 = x^9y^3$, $y_5 = x^7y^4$,
$y_6 = x^5y^5$, $y_7 = x^3y^6$, $y_8 = xy^7$, $y_9 = y^{15}$
; $-y_8^3 + y_7y_9$, $-y_7y_8^2 + y_6y_9$, $-y_7^2 + y_6y_8$, $-y_7^2y_8 + y_5y_9$, $-y_6y_7 + y_5y_8$,
$-y_6^2 + y_5y_7$, $-y_7^3 + y_4y_9$, $-y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$,
$-y_6y_7^2 + y_3y_9$, $-y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$,
$-y_6^2y_7 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$,
$-y_3^2 + y_2y_4$, $-y_6^3 + y_1y_9$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7$, $-y_3y_4 + y_1y_6$,
$-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$

4. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^3 \end{pmatrix}$; $y_1 = x^{15}$, $y_2 = x^{12}y$, $y_3 = x^9y^2$, $y_4 = x^6y^3$, $y_5 = x^3y^4$,
$y_6 = y^5$

$; -y_5^2 + y_4y_6,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$

5. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^{11}y,\ y_3 = x^7y^2,\ y_4 = x^3y^3,\ y_5 = x^2y^7,$
$y_6 = xy^{11},\ y_7 = y^{15}$
$; -y_6^2 + y_5y_7,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_4y_5^2 + y_3y_7,\ -y_4^2y_5 + y_3y_6,$
$-y_4^3 + y_3y_5,\ -y_4^3y_5 + y_2y_7,\ -y_4^4 + y_2y_6,\ -y_3y_4^2 + y_2y_5,\ -y_3^2 + y_2y_4,$
$-y_4^5 + y_1y_7,\ -y_3y_4^3 + y_1y_6,\ -y_3^2 + y_4 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$

6. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^5 \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^{10}y,\ y_3 = x^5y^2,\ y_4 = y^3$
$; -y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$

7. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^6 \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^9y,\ y_3 = x^3y^2,\ y_4 = y^5$
$; -y_3^3 + y_2y_4,\ -y_2y_3^2 + y_1y_4,\ -y_2^2 + y_1y_3$

8. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^7 \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^8y,\ y_3 = xy^2,\ y_4 = y^{15}$
$; -y_3^8 + y_2y_4,\ -y_2y_3^7 + y_1y_4,\ -y_2^2 + y_1y_3$

9. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^9 \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^6y,\ y_3 = x^3y^3,\ y_4 = y^5$
$; -y_3^2 + y_2y_4,\ -y_2^2y_3 + y_1y_4,\ -y_2^3 + y_1y_3$

10. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^{10} \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^5y,\ y_3 = y^3$; $-y_2^3 + y_1y_3$

11. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^{11} \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^4y,\ y_3 = xy^4,\ y_4 = y^{15}$
$; -y_3^4 + y_2y_4,\ -y_2^3y_3^3 + y_1y_4,\ -y_2^4 + y_1y_3$

12. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^{12} \end{pmatrix}$; $y_1 = x^{15},\ y_2 = x^3y,\ y_3 = y^5$; $-y_2^5 + y_1y_3$

13. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^{14} \end{pmatrix}$; $y_1 = x^{15},\ y_2 = xy,\ y_3 = y^{15}$; $-y_2^{15} + y_1y_3$

14. $\begin{pmatrix} \omega^3 & 0 \\ 0 & \omega^5 \end{pmatrix}$; $y_1 = x^5,\ y_2 = y^3$; none

Z_{16} : $(\omega = exp(2\pi i/16))$

1. $\begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$; $y_1 = x^{16},\ y_2 = y$; none

2. $\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}$; $y_1 = x^{16},\ y_2 = x^{15}y,\ y_3 = x^{14}y^2,\ y_4 = x^{13}y^3,\ y_5 = x^{12}y^4,$
$y_6 = x^{11}y^5,\ y_7 = x^{10}y^6,\ y_8 = x^9y^7,\ y_9 = x^8y^8,\ y_{10} = x^7y^9,$
$y_{11} = x^6y^{10},\ y_{12} = x^5y^{11},\ y_{13} = x^4y^{12},\ y_{14} = x^3y^{13},\ y_{15} = x^2y^{14},$
$y_{16} = xy^{15},\ y_{17} = y^{16}$
$; -y_{16}^2 + y_{15}y_{17},\ -y_{15}y_{16} + y_{17}y_{14},\ -y_{15}^2 + y_{14}y_{16},\ -y_{15}^2 + y_{13}y_{17},$
$-y_{14}y_{15} + y_{13}y_{16},\ -y_{14}^2 + y_{13}y_{15},\ -y_{14}y_{15} + y_{12}y_{17},\ -y_{14}^2 + y_{12}y_{16},$
$-y_{13}y_{14} + y_{12}y_{15},\ -y_{13}^2 + y_{12}y_{14},\ -y_{14}^2 + y_{11}y_{17},\ -y_{13}y_{14} + y_{11}y_{16},$

$-y_{13}^2 + y_{11}y_{15}$, $-y_{12}y_{13} + y_{11}y_{14}$, $-y_{12}^2 + y_{11}y_{13}$, $-y_{13}y_{14} + y_{10}y_{17}$,
$-y_{13}^2 + y_{10}y_{16}, y_{12}y_{13} + y_{10}y_{15}$, $-y_{12}^2 + y_{10}y_{14}$, $-y_{11}y_{12} + y_{10}y_{13}$,
$-y_{11}^2 + y_{10}y_{12}$, $-y_{13}^2 + y_9y_{17}$, $-y_{12}y_{13} + y_9y_{16}$, $-y_{12}^2 + y_9y_{15}$,
$-y_{11}y_{12} + y_9y_{14}$, $-y_{11}^2 + y_9y_{13}$, $-y_{10}y_{11} + y_9y_{12}, y_{10}^2 + y_9y_{11}$,
$-y_{12}y_{13} + y_8y_{17}$, $-y_{12}^2 + y_8y_{16}$, $-y_{11}y_{12} + y_8y_{15}$, $-y_{11}^2 + y_8y_{14}$,
$-y_{10}y_{11} + y_8y_{13}$, $-y_{10}^2 + y_8y_{12}$, $-y_9y_{10} + y_8y_{11}$, $-y_9^2 + y_8y_{10}$,
$-y_{12}^2 + y_{17}y_7$, $-y_{11}y_{12} + y_{16}y_7$, $-y_{11}^2 + y_{15}y_7$, $-y_{10}y_{11} + y_7y_{14}$,
$-y_{10}^2 + y_7y_{13}$, $-y_9y_{10} + y_7y_{12}$, $-y_9^2 + y_7y_{11}$, $-y_8y_9 + y_7y_{10}$, $-y_8^2 + y_7y_9$,
$-y_{11}y_{12} + y_6y_{17}, y_{11}^2 + y_6y_{16}$, $-y_{10}y_{11} + y_6y_{15}$, $-y_{10}^2 + y_6y_{14}$,
$-y_9y_{10} + y_6y_{13}$, $-y_9^2 + y_6y_{12}$, $-y_8y_9 + y_6y_{11}$, $-y_8^2 + y_6y_{10}$,
$-y_7y_8 + y_6y_9$, $-y_7^2 + y_6y_8, y_{11}^2 + y_5y_{17}$, $-y_{10}y_{11} + y_5y_{16}$, $-y_{10}^2 + y_5y_{15}$,
$-y_9y_{10} + y_5y_{14}$, $-y_9^2 + y_5y_{13}$, $-y_8y_9 + y_5y_{12}$, $-y_8^2 + y_5y_{11}$,
$-y_7y_8 + y_5y_{10}$, $-y_7^2 + y_5y_9$, $-y_6y_7 + y_5y_8$, $-y_6^2 + y_5y_7$, $-y_{10}y_{11} + y_4y_{17}$,
$-y_{10}^2 + y_4y_{16}$, $-y_9y_{10} + y_4y_{15}$, $-y_9^2 + y_4y_{14}$, $-y_8y_9 + y_4y_{13}$, $-y_8^2 + y_4y_{12}$,
$-y_7y_8 + y_4y_{11}$, $-y_7^2 + y_4y_{10}$, $-y_6y_7 + y_4y_9$, $-y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$,
$-y_5^2 + y_4y_6$, $-y_{10}^2 + y_3y_{17}$, $-y_9y_{10} + y_3y_{16}$, $-y_9^2 + y_3y_{15}$, $-y_8y_9 + y_3y_{14}$,
$-y_8^2 + y_3y_{13}$, $-y_7y_8 + y_3y_{12}$, $-y_7^2 + y_3y_{11}$, $-y_6y_7 + y_3y_{10}$, $-y_6^2 + y_3y_9$,
$-y_5y_6 + y_3y_8, y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$, $-y_9y_{10} + y_2y_{17}$,
$-y_9^2 + y_2y_{16}$, $-y_8y_9 + y_2y_{15}$, $-y_8^2 + y_2y_{14}$, $-y_7y_8 + y_2y_{13}$, $-y_7^2 + y_2y_{12}$,
$-y_6y_7 + y_2y_{11}$, $-y_6^2 + y_2y_{10}$, $-y_5y_6 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$,
$-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$, $-y_3^2 + y_2y_4$, $-y_9^2 + y_1y_{17}$, $-y_8y_9 + y_1y_{16}$,
$-y_8^2 + y_1y_{15}$, $-y_7y_8 + y_1y_{14}$, $-y_7^2 + y_1y_{13}$, $-y_6y_7 + y_1y_{12}$, $-y_6^2 + y_1y_{11}$,
$-y_5y_6 + y_1y_{10}$, $-y_5^2 + y_1y_9$, $-y_4y_5 + y_1y_8$, $-y_4^2 + y_1y_7, y_3y_4 + y_1y_6$,
$-y_3^2 + y_1y_5$, $-y_2y_3 + y_1y_4$, $-y_2^2 + y_1y_3$

3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^{16}$, $y_2 = x^{14}y$, $y_3 = x^{12}y^2$, $y_4 = x^{10}y^3$, $y_5 = x^8y^4$,
$y_6 = x^6y^5$, $y_7 = x^4y^6$, $y_8 = x^2y^7$, $y_9 = y^8$
; $-y_8^2 + y_7y_9$, $-y_7y_8 + y_6y_9$, $-y_7^2 + y_6y_8$, $-y_7^2 + y_5y_9$, $-y_6y_7 + y_5y_8$,
$-y_6^2 + y_5y_7$, $-y_6y_7 + y_4y_9$, $-y_6^2 + y_4y_8$, $-y_5y_6 + y_4y_7$, $-y_5^2 + y_4y_6$,
$-y_6^2 + y_3y_9$, $-y_5y_6 + y_3y_8$, $-y_5^2 + y_3y_7$, $-y_4y_5 + y_3y_6$, $-y_4^2 + y_3y_5$,
$-y_5y_6 + y_2y_9$, $-y_5^2 + y_2y_8$, $-y_4y_5 + y_2y_7$, $-y_4^2 + y_2y_6$, $-y_3y_4 + y_2y_5$,

$-y_3^2+y_2y_4,\ -y_5^2+y_1y_9,\ -y_4y_5+y_1y_8,\ -y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,$
$-y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^{13}y,\ y_3=x^{10}y^2,\ y_4=x^7y^3,$
$y_5=x^4y^4,\ y_6=xy^5,\ y_7=y^{16}$
; $-y_6^4+y_5y_7,\ -y_5y_6^3+y_4y_7,\ -y_5^2+y_4y_6,\ -y_5^2y_6^2+y_3y_7,\ -y_4y_5+y_3y_6,$
$-y_4^2+y_3y_5,\ -y_5^3y_6+y_2y_7,\ -y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,$
$-y_5^4+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^{12}y,\ y_3=x^8y^2,\ y_4=x^4y^3,\ y_5=y^4,$
; $-y_4^2+y_3y_5,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$

6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^{11}y,\ y_3=x^6y^2,\ y_4=xy^3,\ y_5=y^{16}$
; $-y_4^6+y_3y_5,\ -y_3y_4^5+y_2y_5,\ -y_3^2+y_2y_4,\ -y_3^2y_4^4+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$

7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^6\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^{10}y,\ y_3=x^4y^2,\ y_4=x^2y^5,\ y_5=y^8$
; $-y_4^2+y_3y_5,\ -y_3^2y_4+y_2y_5,\ -y_3^3+y_2y_4,\ -y_3^4+y_1y_5,\ -y_2y_3^2+y_1y_4,$
$-y_2^2+y_1y_3$

8. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^7\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^9y,\ y_3=x^2y^2,\ y_4=xy^9,\ y_5=y^{16}$
; $-y_4^2+y_3y_5,\ -y_3^4y_4+y_2y_5,\ -y_3^5+y_2y_4,\ -y_3^8+y_1y_5,\ -y_2y_3^4+y_1y_4,$
$-y_2^2+y_1y_3$

9. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^8\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^8y,\ y_3=y^2$; $-y_2^2+y_1y_3$

10. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^9\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^7y,\ y_3=x^5y^3,\ y_4=x^3y^5,$
$y_5=xy^7,\ y_6=y^{16}$
; $-y_5^3+y_4y_6,\ -y_4y_5^2+y_3y_6,\ -y_4^2+y_3y_5,\ -y_4^2y_5+y_2y_6,\ -y_3y_4+y_2y_5,$
$-y_3^2+y_2y_4,\ -y_3^2y_4^2+y_1y_6,\ -y_2y_3^2+y_1y_5,\ -y_2^2y_3+y_1y_4,\ -y_2^3+y_1y_3$

11. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{10}\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^6y,\ y_3=x^2y^3,\ y_4=y^8$
; $-y_3^3+y_2y_4,\ -y_2^2y_3^2+y_1y_4,\ -y_2^3+y_1y_3$

12. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{12}\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^4y,\ y_3=y^4$; $-y_2^4+y_1y_3$

13. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{14}\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=x^2y,\ y_3=y^8$; $-y_2^8+y_1y_3$

14. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{15}\end{smallmatrix}\right)$; $y_1=x^{16},\ y_2=xy,\ y_3=y^{16}$; $-y_2^{16}+y_1y_3$

$\boldsymbol{Z_2 \times Z_2 \times Z_4}$: none

$\boldsymbol{Z_4 \times Z_4}$:

1. $\left\langle \left(\begin{smallmatrix} 1 & 0 \\ 0 & i \end{smallmatrix}\right), \left(\begin{smallmatrix} i & 0 \\ 0 & 1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^4,\ y_2 = y^4$; none

$\boldsymbol{Z_8 \times Z_2}$: $(\omega = exp(2\pi i/8))$

1. $\left\langle \left(\begin{smallmatrix} \omega & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^8,\ y_2 = y^2$; none
2. $\left\langle \left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^8,\ y_2 = x^6y^2,\ y_3 = x^4y^4,\ y_4 = x^2y^6,$
   $y_5 = y^8$
   ; $-y_4^2 + y_3y_5,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
   $-y_2^2 + y_1y_3$
3. $\left\langle \left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega^2 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right) \right\rangle$; $y_1 = x^8,\ y_2 = x^2y^2,\ y_3 = y^8$; $-y_2^4 + y_1y_3$

$\boldsymbol{Z_{17}}$: $(\omega = exp(2\pi i/17))$

1. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & 1 \end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = y$; none
2. $\left(\begin{smallmatrix} \omega & 0 \\ 0 & \omega \end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^{16}y,\ y_3 = x^{15}y^2,\ y_4 = x^{14}y^3,\ y_5 = x^{13}y^4,$
   $y_6 = x^{12}y^5,\ y_7 = x^{11}y^6,\ y_8 = x^{10}y^7,\ y_9 = x^9y^8,\ y_{10} = x^8y^9,$
   $y_{11} = x^7y^{10},\ y_{12} = x^6y^{11},\ y_{13} = x^5y^{12},\ y_{14} = x^4y^{13},\ y_{15} = x^3y^{14},$
   $y_{16} = x^2y^{15},\ y_{17} = xy^{16},\ y_{18} = y^{17}$
   ; $-y_{17}^2 + y_{16}y_{18},\ -y_{16}y_{17} + y_{15}y_{18},\ -y_{16}^2 + y_{15}y_{17},\ -y_{16}^2 + y_{14}y_{18},$
   $-y_{15}y_{16} + y_{14}y_{17},\ -y_{15}^2 + y_{14}y_{16},\ -y_{15}y_{16} + y_{13}y_{18},\ -y_{15}^2 + y_{13}y_{17},$
   $-y_{14}y_{15} + y_{13}y_{16},\ -y_{14}^2 + y_{13}y_{15},\ -y_{15}^2 + y_{12}y_{18},\ -y_{14}y_{15} + y_{12}y_{17},$
   $-y_{14}^2 + y_{12}y_{16},\ -y_{13}y_{14} + y_{12}y_{15},\ -y_{13}^2 + y_{12}y_{14},\ -y_{14}y_{15} + y_{11}y_{18},$
   $-y_{14}^2 + y_{11}y_{17},\ -y_{13}y_{14} + y_{11}y_{16},\ -y_{13}^2 + y_{11}y_{15},\ -y_{12}y_{13} + y_{11}y_{14},$
   $-y_{12}^2 + y_{11}y_{13},\ -y_{14}^2 + y_{10}y_{18},\ -y_{13}y_{14} + y_{10}y_{17},\ -y_{13}^2 + y_{10}y_{16},$
   $-y_{12}y_{13} + y_{10}y_{15},\ -y_{12}^2 + y_{10}y_{14},\ -y_{11}y_{12} + y_{10}y_{13},\ -y_{11}^2 + y_{10}y_{12},$
   $-y_{13}y_{14} + y_9y_{18},\ -y_{13}^2 + y_9y_{17},\ -y_{12}y_{13} + y_9y_{16},\ -y_{12}^2 + y_9y_{15},$
   $-y_{11}y_{12} + y_9y_{14},\ -y_{11}^2 + y_9y_{13},\ -y_{10}y_{11} + y_9y_{12},\ -y_{10}^2 + y_9y_{11},$
   $-y_{13}^2 + y_{18}y_8,\ -y_{12}y_{13} + y_{17}y_8,\ -y_{12}^2 + y_{16}y_8,\ -y_{11}y_{12} + y_8y_{15},$
   $-y_{11}^2 + y_8y_{14},\ -y_{10}y_{11} + y_8y_{13},\ -y_{10}^2 + y_8y_{12},\ -y_9y_{10} + y_8y_{11},$
   $-y_9^2 + y_8y_{10},\ -y_{12}y_{13} + y_7y_{18},\ -y_{12}^2 + y_7y_{17},\ -y_{11}y_{12} + y_7y_{16},$

$-y_{11}^2 + y_7y_{15},\ -y_{10}y_{11} + y_7y_{14},\ -y_{10}^2 + y_7y_{13},\ -y_9y_{10} + y_7y_{12},$
$-y_9^2 + y_7y_{11},\ -y_8y_9 + y_7y_{10},\ -y_8^2 + y_7y_9,\ -y_{12}^2 + y_6y_{18},$
$-y_{11}y_{12} + y_6y_{17},\ -y_{11}^2 + y_6y_{16},\ -y_{10}y_{11} + y_6y_{15},\ -y_{10}^2 + y_6y_{14},$
$-y_9y_{10} + y_6y_{13},\ -y_9^2 + y_6y_{12},\ -y_8y_9 + y_6y_{11},\ -y_8^2 + y_6y_{10},$
$-y_7y_8 + y_6y_9,\ -y_7^2 + y_6y_8,\ -y_{11}y_{12} + y_5y_{18},\ -y_{11}^2 + y_5y_{17},$
$-y_{10}y_{11} + y_5y_{16},\ -y_{10}^2 + y_5y_{15},\ -y_9y_{10} + y_5y_{14},\ -y_9^2 + y_5y_{13},$
$-y_8y_9+y_5y_{12},\ -y_8^2+y_5y_{11},\ -y_7y_8+y_5y_{10},\ -y_7^2+y_5y_9,\ -y_6y_7+y_5y_8,$
$-y_6^2 + y_5y_7,\ -y_{11}^2 + y_4y_{18},\ -y_{10}y_{11} + y_4y_{17},\ -y_{10}^2 + y_4y_{16},$
$-y_9y_{10} + y_4y_{15},\ -y_9^2 + y_4y_{14},\ -y_8y_9 + y_4y_{13},\ -y_8^2 + y_4y_{12},$
$-y_7y_8 + y_4y_{11},\ -y_7^2 + y_4y_{10},\ -y_6y_7 + y_4y_9,\ -y_6^2 + y_4y_8,$
$-y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_{10}y_{11} + y_3y_{18},\ -y_{10}^2 + y_3y_{17},$
$-y_9y_{10} + y_3y_{16},\ -y_9^2 + y_3y_{15},\ -y_8y_9 + y_3y_{14},\ -y_8^2 + y_3y_{13},$
$-y_7y_8+y_3y_{12},\ -y_7^2+y_3y_{11},\ -y_6y_7+y_3y_{10},\ -y_6^2+y_3y_9,\ -y_5y_6+y_3y_8,$
$-y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,\ -y_4^2+y_3y_5,\ -y_{10}^2+y_2y_{18},\ -y_9y_{10}+y_2y_{17},$
$-y_9^2+y_2y_{16},\ -y_8y_9+y_2y_{15},\ -y_8^2+y_2y_{14},\ -y_7y_8+y_2y_{13},\ -y_7^2+y_2y_{12},$
$-y_6y_7+y_2y_{11},\ -y_6^2+y_2y_{10},\ -y_5y_6+y_2y_9,\ -y_5^2+y_2y_8,\ -y_4y_5+y_2y_7,$
$-y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_{10}y_9+y_1y_{18},\ -y_9^2+y_1y_{17},$
$-y_8y_9 + y_1y_{16},\ -y_8^2 + y_1y_{15},\ -y_7y_8 + y_1y_{14},\ -y_7^2 + y_1y_{13},$
$-y_6y_7+y_1y_{12},\ -y_6^2+y_1y_{11},\ -y_5y_6+y_1y_{10},\ -y_5^2+y_1y_9,\ -y_4y_5+y_1y_8,$
$-y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$

3. $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$; $y_1 = x^{17},\ y_2 = x^{15}y,\ y_3 = x^{13}y^2,\ y_4 = x^{11}y^3,\ y_5 = x^9y^4,$
$y_6 = x^7y^5,\ y_7 = x^5y^6,\ y_8 = x^3y^7,\ y_9 = xy^8,\ y_{10} = y^{17}$
$; -y_9^3 + y_8y_{10},\ -y_8y_9^2 + y_7y_{10},\ -y_8^2 + y_7y_9,\ -y_8^2y_9 + y_6y_{10},\ -y_7y_8 +$
$y_6y_9,$
$-y_7^2 + y_6y_8,\ -y_8^3 + y_5y_{10},\ -y_7^2 + y_5y_9,\ -y_6y_7 + y_5y_8,\ -y_6^2 + y_5y_7,$
$-y_7y_8^2+y_4y_{10},\ -y_6y_7+y_4y_9,\ -y_6^2+y_4y_8,\ -y_5y_6+y_4y_7,\ -y_5^2+y_4y_6,$
$-y_7^2y_8+y_3y_{10},\ -y_6^2+y_3y_9,\ -y_5y_6+y_3y_8,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,$
$-y_4^2 + y_3y_5,\ -y_7^3 + y_2y_{10},\ -y_5y_6 + y_2y_9,\ -y_5^2 + y_2y_8,\ -y_4y_5 + y_2y_7,$
$-y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,\ -y_6y_7^2 + y_1y_{10},\ -y_5^2 + y_1y_9,$
$-y_4y_5 + y_1y_8,\ -y_4^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$

4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^{14}y,\ y_3 = x^{11}y^2,\ y_4 = x^8y^3,\ y_5 = x^5y^4,$
$y_6 = x^2y^5,\ y_7 = xy^{11},\ y_8 = y^{17}$
; $-y_7^2 + y_6y_8,\ -y_6^2y_7 + y_5y_8,\ -y_6^3 + y_5y_7,\ -y_6^4 + y_4y_8,\ -y_5y_6^2 + y_4y_7,$
$-y_5^2 + y_4y_6,\ -y_5y_6^3 + y_3y_8,\ -y_5^2y_6 + y_3y_7,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,$
$-y_5^2y_6^2 + y_2y_8,\ -y_5^3 + y_2y_7,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,\ -y_3^2 + y_2y_4,$
$-y_5^3y_6 + y_1y_8,\ -y_4y_5^2 + y_1y_7,\ -y_3y_4 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,$
$-y_2^2 + y_1y_3$
5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^{13}y,\ y_3 = x^9y^2,\ y_4 = x^5y^3,\ y_5 = xy^4,$
$y_6 = y^{17}$
; $-y_5^5 + y_4y_6,\ -y_4y_5^4 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_4^2y_5^3 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_4^3y_5^2 + y_1y_6,\ -y_3^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^{12}y,\ y_3 = x^7y^2,\ y_4 = x^2y^3,\ y_5 = xy^{10},$
$y_6 = y^{17}$
; $-y_5^2 + y_4y_6,\ -y_4^3y_5 + y_3y_6,\ -y_4^4 + y_3y_5,\ -y_4^6 + y_2y_6,\ -y_3y_4^3 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_3y_4^5 + y_1y_6,\ -y_3^2y_4^2 + y_1y_5,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$
7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^7\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^{10}y,\ y_3 = x^3y^2,\ y_4 = x^2y^7,\ y_5 = xy^{12},$
$y_6 = y^{17}$
; $-y_5^2 + y_4y_6,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_3^2y_4^2 + y_2y_6,\ -y_3^3y_4 + y_2y_5,$
$-y_3^4 + y_2y_4,\ -y_3^5y_4 + y_1y_6,\ -y_3^6 + y_1y_5,\ -y_2y_3^3 + y_1y_4,\ -y_2^2 + y_1y_3$
8. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^8\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^9y,\ y_3 = xy^2,\ y_4 = y^{17}$
; $-y_3^9 + y_2y_4,\ -y_2y_3^8 + y_1y_4,\ -y_2^2 + y_1y_3$
9. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{11}\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^6y,\ y_3 = xy^3,\ y_4 = y^{17}$
; $-y_3^6 + y_2y_4,\ -y_2^2y_3^5 + y_1y_4,\ -y_2^3 + y_1y_3$
10. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{14}\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = x^3y,\ y_3 = xy^6,\ y_4 = y^{17}$
; $-y_3^3 + y_2y_4,\ -y_2^5y_3^2 + y_1y_4,\ -y_2^6 + y_1y_3$
11. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{16}\end{smallmatrix}\right)$; $y_1 = x^{17},\ y_2 = xy,\ y_3 = y^{17}$; $-y_2^{17} + y_1y_3$

Z_{18} : $(\omega = exp(2\pi i/18))$

1. $\left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = y$; none
2. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^{17}y,\ y_3 = x^{16}y^2,\ y_4 = x^{15}y^3,\ y_5 = x^{14}y^4,$
$y_6 = x^{13}y^5,\ y_7 = x^{12}y^6,\ y_8 = x^{11}y^7,\ y_9 = x^{10}y^8,\ y_{10} = x^9y^9,$

$y_{11} = x^8y^{10},\ y_{12} = x^7y^{11},\ y_{13} = x^6y^{12},\ y_{14} = x^5y^{13},\ y_{15} = x^4y^{14},$
$y_{16} = x^3y^{15},\ y_{17} = x^2y^{16},\ y_{18} = xy^{17}, y_{19} = y^{18}$
$; -y_{18}^2 + y_{17}y_{19},\ -y_{17}y_{18} + y_{16}y_{19},\ -y_{17}^2 + y_{16}y_{18},\ -y_{17}^2 + y_{15}y_{19},$
$-y_{16}y_{17} + y_{15}y_{18},\ -y_{16}^2 + y_{15}y_{17},\ -y_{16}y_{17} + y_{14}y_{19},\ -y_{16}^2 + y_{14}y_{18},$
$-y_{15}y_{16} + y_{14}y_{17},\ -y_{15}^2 + y_{14}y_{16},\ -y_{16}^2 + y_{13}y_{19},\ -y_{15}y_{16} + y_{13}y_{18},$
$-y_{15}^2 + y_{13}y_{17},\ -y_{14}y_{15} + y_{13}y_{16},\ -y_{14}^2 + y_{13}y_{15},\ -y_{15}y_{16} + y_{12}y_{19},$
$-y_{15}^2 + y_{12}y_{18},\ -y_{14}y_{15} + y_{12}y_{17},\ -y_{14}^2 + y_{12}y_{16},\ -y_{13}y_{14} + y_{12}y_{15},$
$-y_{13}^2 + y_{12}y_{14},\ -y_{15}^2 + y_{11}y_{19},\ -y_{14}y_{15} + y_{11}y_{18},\ -y_{14}^2 + y_{11}y_{17},$
$-y_{13}y_{14} + y_{11}y_{16},\ -y_{13}^2 + y_{11}y_{15},\ -y_{12}y_{13} + y_{11}y_{14},\ -y_{12}^2 + y_{11}y_{13},$
$-y_{14}y_{15} + y_{10}y_{19},\ -y_{14}^2 + y_{10}y_{18},\ -y_{13}y_{14} + y_{10}y_{17},\ -y_{13}^2 + y_{10}y_{16},$
$-y_{12}y_{13} + y_{10}y_{15},\ -y_{12}^2 + y_{10}y_{14},\ -y_{11}y_{12} + y_{10}y_{13},\ -y_{11}^2 + y_{10}y_{12},$
$-y_{14}^2 + y_{19}y_9,\ -y_{13}y_{14} + y_{18}y_9,\ -y_{13}^2 + y_{17}y_9,\ -y_{12}y_{13} + y_9y_{16},$
$-y_{12}^2 + y_9y_{15},\ -y_{11}y_{12} + y_9y_{14},\ -y_{11}^2 + y_9y_{13},\ -y_{10}y_{11} + y_9y_{12},$
$-y_{10}^2 + y_9y_{11}, y_{13}y_{14} + y_8y_{19},\ -y_{13}^2 + y_8y_{18},\ -y_{12}y_{13} + y_8y_{17},$
$-y_{12}^2 + y_8y_{16},\ -y_{11}y_{12} + y_8y_{15},\ -y_{11}^2 + y_8y_{14},\ -y_{10}y_{11} + y_8y_{13},$
$-y_{10}^2 + y_8y_{12},\ -y_9y_{10} + y_8y_{11},\ -y_9^2 + y_8y_{10},\ -y_{13}^2 + y_7y_{19},$
$-y_{12}y_{13} + y_7y_{18},\ -y_{12}^2 + y_7y_{17},\ -y_{11}y_{12} + y_7y_{16},\ -y_{11}^2 + y_7y_{15},$
$-y_{10}y_{11} + y_7y_{14},\ -y_{10}^2 + y_7y_{13},\ -y_9y_{10} + y_7y_{12},\ -y_9^2 + y_7y_{11},$
$-y_8y_9 + y_7y_{10},\ -y_8^2 + y_7y_9,\ -y_{12}y_{13} + y_6y_{19},\ -y_{12}^2 + y_6y_{18},$
$-y_{11}y_{12} + y_6y_{17},\ -y_{11}^2 + y_6y_{16},\ -y_{10}y_{11} + y_6y_{15},\ -y_{10}^2 + y_6y_{14},$
$-y_9y_{10} + y_6y_{13},\ -y_9^2 + y_6y_{12},\ -y_8y_9 + y_6y_{11},\ -y_8^2 + y_6y_{10},$
$-y_7y_8 + y_6y_9,\ -y_7^2 + y_6y_8,\ -y_{12}^2 + y_5y_{19},\ -y_{11}y_{12} + y_5y_{18},$
$-y_{11}^2 + y_5y_{17},\ -y_{10}y_{11} + y_5y_{16},\ -y_{10}^2 + y_5y_{15},\ -y_9y_{10} + y_5y_{14},$
$-y_9^2 + y_5y_{13},\ -y_8y_9 + y_5y_{12},\ -y_8^2 + y_5y_{11},\ -y_7y_8 + y_5y_{10},$
$-y_7^2 + y_5y_9,\ -y_6y_7 + y_5y_8,\ -y_6^2 + y_5y_7,\ -y_{11}y_{12} + y_4y_{19},\ -y_{11}^2 + y_4y_{18},$
$-y_{10}y_{11} + y_4y_{17},\ -y_{10}^2 + y_4y_{16},\ -y_9y_{10} + y_4y_{15},\ -y_9^2 + y_4y_{14},$
$-y_8y_9 + y_4y_{13},\ -y_8^2 + y_4y_{12},\ -y_7y_8 + y_4y_{11},\ -y_7^2 + y_4y_{10},$
$-y_6y_7 + y_4y_9,\ -y_6^2 + y_4y_8,\ -y_5y_6 + y_4y_7,\ -y_5^2 + y_4y_6,\ -y_{11}^2 + y_3y_{19},$
$-y_{10}y_{11} + y_3y_{18},\ -y_{10}^2 + y_3y_{17},\ -y_9y_{10} + y_3y_{16},\ -y_9^2 + y_3y_{15},$
$-y_8y_9 + y_3y_{14},\ -y_8^2 + y_3y_{13},\ -y_7y_8 + y_3y_{12},\ -y_7^2 + y_3y_{11},$
$-y_6y_7 + y_3y_{10},\ -y_6^2 + y_3y_9,\ -y_5y_6 + y_3y_8,\ -y_5^2 + y_3y_7,\ -y_4y_5 + y_3y_6,$

$-y_4^2+y_3y_5,\ -y_{11}y_{10}+y_2y_{19},\ -y_{10}^2+y_2y_{18},\ -y_9y_{10}+y_2y_{17},$
$-y_9^2+y_2y_{16},\ -y_8y_9+y_2y_{15},\ -y_8^2+y_2y_{14},\ -y_7y_8+y_2y_{13},\ -y_7^2+y_2y_{12},$
$-y_6y_7+y_2y_{11},\ -y_6^2+y_2y_{10},\ -y_5y_6+y_2y_9,\ -y_5^2+y_2y_8,\ -y_4y_5+y_2y_7,$
$-y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_{10}^2+y_1y_{19},\ -y_9y_{10}+y_1y_{18},$
$-y_9^2+y_1y_{17},\ -y_8y_9+y_1y_{16},\ -y_8^2+y_1y_{15},\ -y_7y_8+y_1y_{14},\ -y_7^2+y_1y_{13},$
$-y_6y_7+y_1y_{12},\ -y_6^2+y_1y_{11},\ -y_5y_6+y_1y_{10},\ -y_5^2+y_1y_9,\ -y_4y_5+y_1y_8,$
$-y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

3. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^2\end{smallmatrix}\right)$; $y_1=x^{18},\ y_2=x^{16}y,\ y_3=x^{14}y^2,\ y_4=x^{12}y^3,\ y_5=x^{10}y^4,$
$y_6=x^8y^5,\ y_7=x^6y^6,\ y_8=x^4y^7,\ y_9=x^2y^8,\ y_{10}=y^9$
$;-y_9^2+y_8y_{10},\ -y_8y_9+y_7y_{10},\ -y_8^2+y_7y_9,\ -y_8^2+y_6y_{10},\ -y_7y_8+y_6y_9,$
$-y_7^2+y_6y_8,\ -y_7y_8+y_5y_{10},\ -y_7^2+y_5y_9,\ -y_6y_7+y_5y_8,\ -y_6^2+y_5y_7,$
$-y_7^2+y_4y_{10},\ -y_6y_7+y_4y_9,\ -y_6^2+y_4y_8,\ -y_5y_6+y_4y_7,\ -y_5^2+y_4y_6,$
$-y_6y_7+y_3y_{10},\ -y_6^2+y_3y_9,\ -y_5y_6+y_3y_8,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,$
$-y_4^2+y_3y_5,\ -y_6^2+y_2y_{10},\ -y_5y_6+y_2y_9,\ -y_5^2+y_2y_8,\ -y_4y_5+y_2y_7,$
$-y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,\ -y_5y_6+y_1y_{10},\ -y_5^2+y_1y_9,$
$-y_4y_5+y_1y_8,\ -y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,$
$-y_2^2+y_1y_3$

4. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^3\end{smallmatrix}\right)$; $y_1=x^{18},\ y_2=x^{15}y,\ y_3=x^{12}y^2,\ y_4=x^9y^3,\ y_5=x^6y^4,$
$y_6=x^3y^5,\ y_7=y^6$
$;-y_6^2+y_5y_7,\ -y_5y_6+y_4y_7,\ -y_5^2+y_4y_6,\ -y_5^2+y_3y_7,\ -y_4y_5+y_3y_6,$
$-y_4^2+y_3y_5,\ -y_4y_5+y_2y_7,\ -y_4^2+y_2y_6,\ -y_3y_4+y_2y_5,\ -y_3^2+y_2y_4,$
$-y_4^2+y_1y_7,\ -y_3y_4+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

5. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^4\end{smallmatrix}\right)$; $y_1=x^{18},\ y_2=x^{14}y,\ y_3=x^{10}y^2,\ y_4=x^6y^3,\ y_5=x^2y^4,$
$y_6=y^9$
$;-y_5^3+y_4y_6,\ -y_4y_5^2+y_3y_6,\ -y_4^2+y_3y_5,\ -y_4^2y_5+y_2y_6,\ -y_3y_4+y_2y_5,$
$-y_3^2+y_2y_4,\ -y_4^3+y_1y_6,\ -y_3^2+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

6. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^5\end{smallmatrix}\right)$; $y_1=x^{18},\ y_2=x^{13}y,\ y_3=x^8y^2,\ y_4=x^3y^3,\ y_5=xy^7,$
$y_6=y^{18}$
$;-y_5^3+y_4y_6,\ -y_4^2y_5^2+y_3y_6,\ -y_4^3+y_3y_5,\ -y_4^4y_5+y_2y_6,\ -y_3y_4^2+y_2y_5,$
$-y_3^2+y_2y_4,\ -y_4^6+y_1y_6,\ -y_3^2y_4+y_1y_5,\ -y_2y_3+y_1y_4,\ -y_2^2+y_1y_3$

7. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^6\end{smallmatrix}\right)$; $y_1=x^{18},\ y_2=x^{12}y,\ y_3=x^6y^2,\ y_4=y^3$

$; -y_3^2 + y_2y_4,\ -y_2y_3 + y_1y_4,\ -y_2^2 + y_1y_3$

8. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^7\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^{11}y,\ y_3 = x^4y^2,\ y_4 = xy^5,\ y_5 = y^{18}$
$; -y_4^4 + y_3y_5,\ -y_3^2y_4^3 + y_2y_5,\ -y_3^3 + y_2y_4,\ -y_3^4y_4^2 + y_1y_5,\ -y_2y_3^2 + y_1y_4,$
$-y_2^2 + y_1y_3$
9. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^8\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^{10}y,\ y_3 = x^2y^2,\ y_4 = y^9$
$; -y_3^5 + y_2y_4,\ -y_2y_3^4 + y_1y_4,\ -y_2^2 + y_1y_3$
10. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^9\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^9y,\ y_3 = y^2$
$; -y_2^2 + y_1y_3$
11. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{10}\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^8y,\ y_3 = x^6y^3,\ y_4 = x^4y^5,\ y_5 = y^2y^7,$
$y_6 = y^9$
$; -y_5^2 + y_4y_6,\ -y_4y_5 + y_3y_6,\ -y_4^2 + y_3y_5,\ -y_4^2 + y_2y_6,\ -y_3y_4 + y_2y_5,$
$-y_3^2 + y_2y_4,\ -y_3^3 = y_1y_6,\ -y_2y_3^2 + y_1y_5,\ -y_2^2y_3 + y_1y_4,\ -y_2^3 + y_1y_3$
12. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{12}\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^6y,\ y_3 = y^3$; $-y_2^3 + y_1y_3$
13. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{14}\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^4y,\ y_3 = x^2y^5,\ y_4 = y^9$
$; -y_3^2 + y_2y_4,\ -y_2^4y_3 + y_1y_4,\ -y_2^5 + y_1y_3$
14. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{16}\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = x^2y,\ y_3 = y^9$; $-y_2^9 + y_1y_3$
15. $\left(\begin{smallmatrix}\omega & 0\\ 0 & \omega^{17}\end{smallmatrix}\right)$; $y_1 = x^{18},\ y_2 = xy,\ y_3 = y^{18}$; $-y_2^{18} + y_1y_3$

$\boldsymbol{Z_6 \times Z_3}$: $(\omega = exp(2\pi i/6),\ \rho = exp(2\pi i/3))$

1. $\left\langle \left(\begin{smallmatrix}\omega & 0\\ 0 & 1\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & 0\\ 0 & \rho\end{smallmatrix}\right) \right\rangle$; $y_1 = x^6,\ y_2 = y^3$; none
2. $\left\langle \left(\begin{smallmatrix}\omega & 0\\ 0 & \omega\end{smallmatrix}\right), \left(\begin{smallmatrix}1 & 0\\ 0 & \rho\end{smallmatrix}\right) \right\rangle$; $y_1 = x^6,\ y_2 = x^3y^3,\ y_3 = y^6$; $-y_2^2 + y_1y_3$

References

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