RANDOM GENERALIZED SET-VALUED COMPLEMENTARITY PROBLEMS*

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1. Introduction

Complementaity problem theory developed by Lemke [10], Cottle and Dantzig [8] and others in the early 1960s and thereafter, has numerous applications in diverse fields of mathematical and engineering sciences. And it is closely related to variational inequality theory and fixed point theory. Recently, fixed point methods for the solving of nonlinear complementarity problems were considered by Noor et al. [11 , 12]. Also complementarity problems related to variational inequality problems were investigated by Chang [1] , Cottle [7] and others.

The study of random complementarity problem theory is lying at the intersection of complementarity problem theory and probabilistic theory. Random complementarity problem theory and random variational inequality theory are probabilistic generalizations of classical complementarity problem theory and classical variational inequality theory respectively. A far-reaching interest in the domain and a vast amount of mathematical activity have led to many remarkable new results and viewpoints yielding insights even into traditional questions. Some problems for random variational inequalities have been considered by Chang et al. [1, 4, 5] and Tan [13]. Especially, Lee et al. [9] investigated random vector variational inequalities and random noncooperative vector equilibrium as random generalizations of vector variational inequalities studied by Chen and Yang [6]. Recently, a class of

random set-valued quasi-complementarity problems was discussed by
Chang and Huang [2, 3].

Inspired and motivated by the recent researches in this field, we in-
troduce new classes of random generalized set-valued complementarity
problems and construct new random algorithms.

This paper is concerned with the convergence of random iterative
sequences obtained by the new random iterative algorithms for the fol-
lowing random set-valued complementarity problems, called a random
generalized strongly set-vlaued complementarity problems (RGSSCP),
and others.

Find measurable mappings f, g and $h : \Omega \to \mathbb{R}^n$ such that for $t \in \Omega$

$$(1.1) \qquad \begin{cases} f(t) \geq 0, \\ g(t) \in F(t, f(t)), \\ h(t) \in G(t, f(t)), \\ g(t) + h(t) \geq 0 \text{ and} \\ (f(t), g(t) + h(t)) = 0, \end{cases}$$

where F and $G : \Omega \times \mathbb{R}^n \to 2^{\mathbb{R}^n}$ are random set-valued mappings.

2. Preliminaries and formulations

Throughout this paper let $(\Omega, \mathcal{A})$ be a measurable space. By $\mathcal{B}(\mathbb{R}^n)$
we denote a Borel σ-field in $\mathbb{R}^n$, $(\cdot, \cdot)$ and $\| \cdot \|$ denote an inner product
and a norm on $\mathbb{R}^n$ respectively. In the sequel we will use the following
notations :

$$2^{\mathbb{R}^n} = \{A : A \subset \mathbb{R}^n\}.$$
$$C(\mathbb{R}^n) = \{A \subset \mathbb{R}^n : \text{ nonempty, compact }\}.$$
$$H(\cdot, \cdot) \text{ will denote the Hausdorff metric on } C(\mathbb{R}^n).$$

For any $x = (x_1, x_2, \cdots, x_n), \quad y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n$, we denote

$$|x| = (|x_1|, |x_2|, \cdots, |x_n|)$$

and define an order "$\geq$" on $\mathbb{R}^n$ as $x \geq y$ if and only if $x_i \geq y_i, \quad i = 1, 2, \cdots, n.$

DEFINITION 2.1. A mapping $f : \Omega \to \mathbb{R}^n$ is called *measurable*, if for any $B \in \mathcal{B}(\mathbb{R}^n)$, the set $\{t \in \Omega : f(t) \in B\} \in \mathcal{A}$.

DEFINITION 2.2. A mapping $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ is called a *random operator*, if for any given $x \in \mathbb{R}^n$, $T(\cdot, x) : \Omega \to \mathbb{R}^n$ is measurable. A random operator T is called *continuous*, if for any given $t \in \Omega$, the mapping $T(t, \cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is continuous.

DEFINITION 2.3. A set-valued mapping $V : \Omega \to 2^{\mathbb{R}^n}$ is called *measurable*, if for any $B \in \mathcal{B}(\mathbb{R}^n)$, the set $V^{-1}(B) = \{t \in \Omega : V(t) \cap B \neq \phi\} \in \mathcal{A}$.

DEFINITION 2.4. A mapping $f : \Omega \to \mathbb{R}^n$ is called a *measurable selection* of a set-valued measurable mapping $V : \Omega \to 2^{\mathbb{R}^n}$, if f is measurable and for any $t \in \Omega$, $f(t) \in V(t)$.

DEFINITION 2.5. A mapping $F : \Omega \times \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is called a *random set-valued mapping*, if for any $x \in \mathbb{R}^n$, $F(\cdot, x)$ is a set-valued measurable mapping, A random set-valued mapping $F : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ is called *H-continuous*, if for any $t \in \Omega$, $F(t, \cdot)$ is continuous in the Hausdorff metric H.

REMARK 2.1. If $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ is a random operator, then problem (1.1) is equivalent to find measurable mappings $f, g : \Omega \to \mathbb{R}^n$ such that for any $t \in \Omega$, $\quad g(t) \in G(t, f(t))$ and

$$(2.1) \quad f(t) \geq 0, \quad T(t, f(t)) + g(t) \geq 0, \quad (f(t), T(t, f(t)) + g(t)) = 0,$$

which is called a random generalized set-valued complementarity problem.

REMARK 2.2. If $T, S : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ are two random operators, then problem (1.1) is equivalent to find a measurable mapping $f : \Omega \to \mathbb{R}^n$ such that for any $t \in \Omega$,

$$(2.2) \qquad f(t) \geq 0, \quad T(t, f(t)) + S(t, f(t)) \geq 0,$$
$$(f(t), T(t, f(t)) + S(t, f(t))) = 0,$$

which is called a random generalized complementarity problem.

REMARK 2.3. In the determinate case, the problem (1.1) is equivalent to find x, y and $z \in \mathbb{R}^n$ such that $y \in F(x)$, $\quad z \in G(x)$ and

$$(2.3) \qquad x \geq 0, \quad y + z \geq 0, \quad (x, y + z) = 0,$$

where F, $\quad G : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ are set-valued mappings, which is called a generalized strongly set-valued complementarity problem.

Obviously, problems (1.1) and (2.3) include many complementarity problems as special cases.

Problem (1.1) can be written as

$$(2.4) \qquad f(t) \geq 0, \quad k(t) = g(t) + h(t) \geq 0, \quad (f(t), k(t)) = 0,$$

where $k : \Omega \to \mathbb{R}^n$ is a measurable mapping. We now consider the following change of variables :

$$(2.5) \quad f(t) = \frac{1}{2}(|h(t)| + h(t)), \quad k(t) = (\lambda(t)\rho(t))^{-1}(|h(t)| - h(t)).$$

where $\lambda : \Omega \to (0, 1)$ and $\rho : \Omega \to (0, \infty)$ are both measurable functions and $h : \Omega \to \mathbb{R}^n$ is a measurable mapping. Clearly, $f(t) \geq 0$ an $k(t) \geq 0$. From (2.5) we know that problem (2.4) is equivalent to fin measurable mappings f, g and $h : \Omega \to \mathbb{R}^n$ such that for any $t \in \Omega$, $f(t) \in F(t, \frac{1}{2}(|h(t)| + h(t)))$, $\quad g(t) \in G(t, \frac{1}{2}(|h(t)| + h(t)))$ and

$$(2.6) \qquad h(t) = \frac{1}{2}(|h(t)| + h(t)) - \frac{1}{2}\lambda(t)\rho(t)(f(t) + g(t)).$$

3. Random algorithm

We first introduce the following lemmas in [2, 3].

LEMMA 3.1. Let $F : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ be a H-continuous randc set-valued mapping. Then for any measurable mapping $f : \Omega \to \mathbb{R}^n$. mapping $F(\cdot, f(\cdot)) : \Omega \to C(\mathbb{R}^n)$ is set-valued measurable.

LEMMA 3.2. Let $V, W : \Omega \to C(\mathbb{R}^n)$ be two set-valued measural mappings and $f : \Omega \to \mathbb{R}^n$ a measurable selection of V. Then the exists a measurable selection $g : \Omega \to \mathbb{R}^n$ of W such that

$$\|f(t) - g(t)\| \leq H(V(t), W(t)) \text{ for all } t \in \Omega.$$

Based on the formulations in section 2, we now construct a n random algorithm for the random generalized strongly set-valued co plementarity problem (RGSSCP) (1.1).

ALGORITHM 3.1. Let $F, G : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ be two H-continuous random set-valued mappings. For any given measurable mapping $h_0 : \Omega \to \mathbb{R}^n$, two set-valued mappings $F(\cdot, \frac{1}{2}(|h_0(\cdot)| + h_0(\cdot)))$ and $G(\cdot, \frac{1}{2}(|h_0(\cdot)| + h_0(\cdot))) : \Omega \to C(\mathbb{R}^n)$ are measurable by Lemma 3.1. It follows that there exist measurable selections $f_0(\cdot) : \Omega \to \mathbb{R}^n$ of $F(t, \frac{1}{2}(|h_0(t)| + h_0(t)))$ and $g_0(\cdot) : \Omega \to \mathbb{R}^n$ of $G(t, \frac{1}{2}(|h_0(t)| + h_0(t)))$ respectively. Letting

$$
\begin{aligned}
h_1(t) = {} & \frac{1}{2}(1 - \lambda(t))(|h_0(t)| + h_0(t)) \\
& + \frac{1}{2}\lambda(t)(|h_0(t)| + h_0(t) - \rho(t)(f_0(t) + g_0(t))),
\end{aligned}
$$

where $\lambda(t)$ and $\rho(t)$ are the same as in (2.5), it is easy to see that $h_1 : \Omega \to \mathbb{R}^n$ is measurable. Since $f_0(t) \in F(t, \frac{1}{2}(|h_0(t)| + h_0(t)))$ and $g_0(t) \in G(t, \frac{1}{2}(|h_0(t)| + h_0(t)))$, by Lemma 3.2, there exist measurable selections $f_1 : \Omega \to \mathbb{R}^n$ of $F(t, \frac{1}{2}(|h_1(t)| + h_1(t)))$ and $g_1 : \Omega \to \mathbb{R}^n$ of $G(t, \frac{1}{2}(|h_1(t)| + h_1(t)))$ such that

$$
\|f_0(t) - f_1(t)\| \leq H(F(t, \frac{1}{2}(|h_0(t)| + h_0(t))), F(t, \frac{1}{2}(|h_1(t)| + h_1(t))))
$$
$$
\text{for all } t \in \Omega,
$$

$$
\|g_0(t) - g_1(t)\| \leq H(G(t, \frac{1}{2}(|h_0(t)| + h_0(t))), G(t, \frac{1}{2}(|h_1(t)| + h_1(t))))
$$
$$
\text{for all } t \in \Omega.
$$

Letting

$$
\begin{aligned}
h_2(t) = {} & \frac{1}{2}(1 - \lambda(t))(|h_1(t)| + h_1(t)) \\
& + \frac{1}{2}\lambda(t)(|h_1(t)| + h_1(t) - \rho(t)(f_1(t) + g_1(t))),
\end{aligned}
$$

then $h_2 : \Omega \to \mathbb{R}^n$ is measurable.

Inductively, we can obtain three sequences $\{f_n\}, \{g_n\}$ and $\{h_n\}$ of

measurable mappings as follows :

$$f_n(t) \in F(t, \frac{1}{2}(|h_n(t)|) + h_n(t))),$$

$$g_n(t) \in G(t, \frac{1}{2}(|h_n(t)|) + h_n(t))),$$

$$\|f_n(t) - f_{n+1}(t)\| \le H(F(t, \frac{1}{2}(|h_n(t)|) + h_n(t))),$$

$$F(t, \frac{1}{2}(|h_{n+1}(t)| + h_{n+1}(t)))),$$

(3.1)

$$\|g_n(t) - g_{n+1}(t)\| \le H(G(t, \frac{1}{2}(|h_n(t)|) + h_n(t))),$$

$$G(t, \frac{1}{2}(|h_{n+1}(t)| + h_{n+1}(t)))),$$

$$h_{n+1}(t) = \frac{1}{2}(1 - \lambda(t))(|h_n(t)| + h_n(t))$$

$$+ \frac{1}{2}\lambda(t)(|h_n(t)| + h_n(t) - \rho(t)(f_n(t) + g_n(t)))$$

for all $t \in \Omega$ and $n = 0, 1, 2, \cdots$, where $\lambda(t)$ and $\rho(t)$ are the same as in (2.5).

Similarly, we can obtain the following algorithms :

ALGORITHM 3.2. Let $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ be a continuous random operator and $F : \Omega \times \mathbb{R}^n \to \mathbb{C}(R^n)$ a H-continuous random set-valued mappings. For any given measurable mapping $h_0 : \Omega \to \mathbb{R}^n$, we can obtain two sequences $\{g_n\}$ and $\{h_n\}$ of measurable mappings as follows:
(3.2)

$$\begin{cases} g_n(t) \in F(t, \frac{1}{2}(|h_n(t)| + h_n(t))), \\ \|g_n(t) - g_{n+1}(t)\| \le H(F(t, \frac{1}{2}(|h_n(t)| + h_n(t))), \\ \qquad F(t, \frac{1}{2}(|h_{n+1}(t)| + h_{n+1}(t)))), \\ h_{n+1}(t) = \frac{1}{2}(1 - \lambda(t))(|h_n(t)| + h_n(t)) + \frac{1}{2}\lambda(t)(|h_n(t)| + h_n(t)) \\ \qquad - \rho(t)(F(t, \frac{1}{2}(|h_n(t)| + h_n(t))) + g_n(t))), \end{cases}$$

for all $t \in \Omega$ and $n = 0, 1, 2, \cdots$, where $\lambda(t)$ and $\rho(t)$ are the same as in (2.5).

ALGORITHM 3.3. Let $T, S : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ be two continuous random operators, then for any given measurable mapping $h_0 : \Omega \to \mathbb{R}^n$ we can obtain a sequence $\{h_n\}$ of measurable mappings such that
(3.3)
$$h_{n+1}(t) = \frac{1}{2}(1 - \lambda(t))(|h_n(t)| + h_n(t)) + \frac{1}{2}\lambda(t)(|h_n(t)| + h_n(t)$$
$$- \rho(t)(T(t, \frac{1}{2}(|h_n(t)| + h_n(t))) + S(t, \frac{1}{2}(|h_n(t)| + h_n(t))))),$$

for all $t \in \Omega$ and $n = 0, 1, 2, \cdots$, where $\lambda(t)$ and $\rho(t)$ are the same as in (2.5).

ALGORITHM 3.4. Let $F, G : \mathbb{R}^n \to C(\mathbb{R}^n)$ be two H-continuous set-vlaued mappings, then for any given $z_0 \in \mathbb{R}^n$, we can obtain three sequences $\{x_n\}, \{y_n\}$ and $\{z_n\}$ in $\mathbb{R}^n$ as follows :

(3.4)
$$\begin{cases} x_n \in F(\frac{1}{2}(|z_n|) + z_n)), y_n \in G(\frac{1}{2}(|z_n|) + z_n)), \\ \|x_n - x_{n+1}\| \leq H(F(\frac{1}{2}(|z_n|) + z_n)), F(\frac{1}{2}(|z_{n+1}| + z_{n+1}))), \\ \|y_n - y_{n+1}\| \leq H(G(\frac{1}{2}(|z_n|) + z_n)), G(\frac{1}{2}(|z_{n+1}| + z_{n+1}))), \\ z_{n+1} = \frac{1}{2}(1 - \lambda)(|z_n| + z_n) + \frac{1}{2}\lambda(|z_n| + z_n - \rho(x_n + y_n)), \end{cases}$$

for $n = 0, 1, 2, \cdots$, where $0 < \lambda < 1$ and $\rho > 0$ are both constants.

4. Existence and convergence

In this section we shall discuss the existence of random solutions for the random generalized strongly set-valued complementarity problem (RGSSCP)(1.1) and the convergence of random sequences constructed by the random algorithm.

DEFINITION 4.1. A random operator $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ is called

(1) *strongly monotone*, if there exists a measurable function $\alpha : \Omega \to (0, \infty)$ such that

$$(T(t, x) - T(t, y), x - y) \geq \alpha(t)\|x - y\|^2 \text{ for all } x, y \in \mathbb{R}^n \text{ and } t \in \Omega;$$

(2) *Lipschitz continuous*, if there exists a measurable function $\beta : \Omega \to (0, \infty)$ such that

$$\|T(t, x) - T(t, y)\| \leq \beta(t)\|x - y\| \text{ for all } x, y \in \mathbb{R}^n \text{ and } t \in \Omega.$$

It is easy to see that $\alpha(t) \leq \beta(t)$ for all $t \in \Omega$.

DEFINITION 4.2. A random set-valued mapping $F : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ is called

(1) *strongly monotone*, if there exists a measurable function $\alpha : \Omega \to (0, \infty)$ such that for all $t \in \Omega$, $\quad x, y \in \mathbb{R}^n$,

$$(u - v, x - y) \geq \alpha(t) \|x - y\|^2 \text{ for all } u \in F(t, x) \text{ and } v \in F(t, y);$$

(2) *H-Lipschitz continuous*, if there exists a measurable function $\eta : \Omega \to (0, \infty)$ such that

$$H(F(t, x), F(t, y)) \leq \eta(t) \|x - y\| \text{ for all } t \in \Omega \text{ and } x, y \in \mathbb{R}^n.$$

THEOREM 4.1. *Suppose that* $F : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ *is a strongly monotone H-Lipschitz continuous random set-valued mapping with coefficients* $\alpha(t)$ *and* $\beta(t)$ *respectively and* $G : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ *is a H-Lipschitz continuous-random set-valued mapping with coefficient* $\eta(t)$, *if for all* $t \in \Omega$,

$$(4.1) \quad 0 < \rho(t) < \frac{4(\alpha(t) - \eta(t))}{\beta^2(t) - \eta^2(t)}, \text{where } \rho(t)\eta(t) < 2 \text{and } \rho(t) < \alpha(t),$$

then there exist measurable mappings $f, g, k : \Omega \to \mathbb{R}^n$, *which are soutions of the random generalized strongly set-valued complementarity problem (RGSSCP) (1.1) and*

$$\frac{1}{2}(|h_n(t)| + h_n(t)) \to k(t).$$
$$f_n(t) \to f(t), \quad g_n(t) \to g(t) \quad (n \to \infty) \text{ for all } t \in \Omega,$$

where $\{f_n\}, \{g_n\}$ *and* $\{h_n\}$ *are three sequences of measurable mappings obtained by Algorithm 3.1.*

Proof. By Algorithm 3.1, we have

$$(4.2)$$
$$\|h_{n+1}(t) - h_n(t)\|$$
$$= \|\frac{1}{2}(1 - \lambda(t))(|h_n(t)| + h_n(t)) + \frac{1}{2}\lambda(t)(|h_n(t)| + h_n(t)$$
$$- \rho(t)(f_n(t) + g_n(t))) - \frac{1}{2}(1 - \lambda(t))(|h_{n-1}(t)| + h_{n-1}(t))$$

$$-\frac{1}{2}\lambda(t)(|h_{n-1}(t)| + h_{n-1}(t) - \rho(t)(f_{n-1}(t) + g_{n-1}(t)))\|$$

$$\leq (1 - \lambda(t))\|h_n(t) - h_{n-1}(t)\| + \frac{1}{2}\lambda(t)\| \mid h_n(t) \mid + h_n(t) - |h_{n-1}(t)|$$

$$- h_{n-1}(t) - \rho(t)(f_n(t) - f_{n-1}(t) + g_n(t) - g_{n-1}(t))\|$$

$$\leq (1 - \lambda(t))\|h_n(t) - h_{n-1}(t)\| + \frac{1}{2}\lambda(t)\rho(t)\|g_n(t) - g_{n-1}(t)\|$$

$$+ \lambda(t)\|\frac{1}{2}(|h_n(t)| + h_n(t)) - \frac{1}{2}(|h_{n-1}(t)| + h_{n-1}(t))$$

$$- \frac{1}{2}\rho(t)(f_n(t) - f_{n-1}(t))\|.$$

Since F and G are H-Lipschitz continuous, from (3.1) we know that
(4.3)
$$\|f_{n-1}(t) - f_n(t)\| \leq H(F(t, \frac{1}{2}(|h_{n-1}(t)| + h_{n-1}(t))), F(t, \frac{1}{2}(|h_n(t)| + h_n(t))))$$
$$\leq \beta(t)\|h_{n-1}(t) - h_n(t)\|.$$

(4.4)
$$\|g_{n-1}(t) - g_n(t)\| \leq H(G(t, \frac{1}{2}(|h_{n-1}(t)| + h_{n-1}(t))), G(t, \frac{1}{2}(|h_n(t)| + h_n(t))))$$
$$\leq \eta(t)\|h_{n-1}(t) - h_n(t)\|.$$

Furthermore, from the strong monotonicity of F and (4.3) we have
(4.5)
$$\|\frac{1}{2}(|h_n(t)| + h_n(t)) - \frac{1}{2}(|h_{n-1}(t)| + h_{n-1}(t))$$

$$- \frac{1}{2}\rho(t)(f_n(t) - f_{n-1}(t))\|^2$$

$$\leq (1 - \alpha(t)\rho(t) + \frac{1}{4}\rho^2(t)\beta^2(t))\|h_n(t) - h_{n-1}(t)\|^2.$$

It follows from (4.2)-(4.5) that
(4.6)
$$\|h_{n+1}(t) - h_n(t)\| \leq 1 - \lambda(t) + \frac{1}{2}\lambda(t)\rho(t)\eta(t) + \lambda(t)(1 - \alpha(t)\rho(t)$$

$$+ \frac{1}{4}\rho^2(t)\beta^2(t))^{\frac{1}{2}}\|h_n(t) - h_{n-1}(t)\|$$

$$= \theta(t)\|h_n(t) - h_{n-1}(t)\|,$$

where

$$\theta(t) = 1 - \lambda(t) + \frac{1}{2}\lambda(t)\rho(t)\eta(t) + \lambda(t)(1 - \alpha(t)\rho(t) + \frac{1}{4}\rho^2(t)\beta^2(t))^{\frac{1}{2}}.$$

In view of (4.1) we know that $0 < \theta(t) < 1$. By (4.6), $\{h_n(t)\}$ is a Cauchy sequence in $\mathbb{R}^n$, say $h_n(t) \to h(t)(n \to \infty)$. It follows from (4.3) and (4.4) that $\{f_n(t)\}$ and $\{g_n(t)\}$ are also Cauchy sequences in $\mathbb{R}^n$. Let $f_n(t) \to f(t)$ and $g_n(t) \to g(t)$. Since $\{f_n\}$, $\{g_n\}$ and $\{h_n\}$ are sequences of measurable mappings, f, g and $h : \Omega \to \mathbb{R}^n$ are measurable. Let $k(t) = \frac{1}{2}(|h(t)| + h(t))$, then

$$\frac{1}{2}(|h_n(t)| + h_n(t)) \to k(t), \quad f_n(t) \to f(t),$$

$$g_n(t) \to g(t) \quad (n \to \infty) \text{ for all } t \in \Omega$$

Now we prove that $f(t) \in F(t, k(t))$. In fact, for all $t \in \Omega$,

$$d(f(t), F(t, k(t))) = \inf\{\|f(t) - z\| : z \in F(t, k(t))\}$$

$$\leq \|f(t) - f_n(t)\| + d(f_n(t), F(t, k(t)))$$

$$\leq \|f(t) - f_n(t)\| + H(F(t, \frac{1}{2}(|h_n(t)| + h_n(t))), F(t, k(t)))$$

$$\leq \|f(t) - f_n(t)\| + \beta(t)\|\frac{1}{2}(|h_n(t)| + h_n(t)) - k(t)\|,$$

hence $d(f(t), F(t, k(t))) = 0$, this implies that $f(t) \in F(t, k(t))$. Similarly, we have $g(t) \in G(t, k(t))$.

By (3.1) and $h_n(t) \to h(t), \quad f_n(t) \to f(t), \quad$ and $\quad g_n(t) \to g(t)$ we know that

$$h(t) = \frac{1}{2}(|h(t)| + h(t) - \frac{1}{2}\lambda(t)\rho(t)(f(t) + g(t)).$$

Therefore, $f, g, k : \Omega \to \mathbb{R}^n$ are solutions of the random generalized strongly set-valued complementarity problem (RGSSCP)(1.1) and

$$\frac{1}{2}(|h_n(t)| + h_n(t)) \to k(t), \quad f_n(t) \to f(t),$$

$$g_n(t) \to g(t) \quad (n \to \infty) \quad \text{for all } t \in \Omega,$$

where $\{f_n\}, \{g_n\}$ and $\{h_n\}$ are sequences of measurable mappings obtained by Algorithm 3.1. This completes the proof. $\quad\square$

From Theorem 4.1, the following results can be obtained immediately.

THEOREM 4.2. *Suppose that* $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ *is a strongly monotone Lipschitz continuous random operator with coefficients* $\alpha(t)$ *and* $\beta(t)$ *respectively,* $G : \Omega \times \mathbb{R}^n \to C(\mathbb{R}^n)$ *is a H-Lipschitz continuous random set-valued mapping with coefficicent* $\eta(t)$, *if for any* $t \in \Omega$, *the condition* (4.1) *in Theorem 4.1 is satisfied, then there exist measurable mappings* $f, g : \Omega \to \mathbb{R}^n$, *which are solutions of the random generalized set-valued complementarity problem* (2.1) *and*

$$\frac{1}{2}(|h_n(t)| + h_n(t)) \to g(t), \quad f_n(t) \to f(t) \quad (n \to \infty) \quad \text{for all } t \in \Omega,$$

where $\{f_n\}$ *and* $\{h_n\}$ *are sequences of measurable mappings obtained by Algorithm 3.2.*

THEOREM 4.3. *Suppose that* $T : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ *is a strongly monotone Lipschitz continuous random operator with coefficients* $\alpha(t)$ *and* $\beta(t)$ *respectively,* $S : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ *is a Lipschitz continuous random operator with coefficicent* $\eta(t)$, *if for any* $t \in \Omega$, *the condition* (4.1) *in Theorem 4.1 is satisfied, then there exists a measurable mapping* $f : \Omega \to \mathbb{R}^n$, *which is a solution of the random generalized complementarity problem* (2.2) *and*

$$\frac{1}{2}(|h_n(t)| + h_n(t)) \to f(t) \quad (n \to \infty) \quad \text{for all } t \in \Omega,$$

where $\{h_n\}$ *is a sequence of measurable mappings obtained by Algorithm 3.3.*

THEOREM 4.4. *Suppose that* $F : \mathbb{R}^n \to C(\mathbb{R}^n)$ *is a strongly monotone H-Lipschitz continuous set-valued mapping with coefficients* α *and* β *respectively,* $G : \mathbb{R}^n \to C(\mathbb{R}^n)$ *is a H-Lipschitz continuous set-valued mapping with a coefficicent* η, *if*

$$0 < \rho < \frac{4(\alpha - \eta)}{\beta^2 - \eta^2}, \text{ where } \rho\eta < 2 \text{ and } \eta < \alpha,$$

then there exist $x, y, z \in \mathbb{R}^n$, *which are solutions of the generalized strongly set-valued complementarity problem* (2.3) *and*

$$\frac{1}{2}(|z_n| + z_n) \to z, \quad x_n \to x, \quad y_n \to y \quad (n \to \infty),$$

where $\{x_n\}, \{y_n\}$ *and* $\{z_n\}$ *are three sequences in* $\mathbb{R}^n$ *obtained by Algorithm 3.4.*

References

1. S. S. Chang, *Variational Inequality and Complementarity Problem Theory with Applications*, Shanghai Scientific and Tech. Literature Publishing House, Shanghai, 1991.

2. S. S. Chang and N. J. Huang, *Random generalized set-valued quasi-complementarity problems*, Acta Math. Appl. Sinica **16** (1993), 396-405.

3. _____, *Generalized random multivalued quasi-compleme- ntarity problems*, Indian J. Math. **35** (1993), 305-320.

4. S. S. Chang and L. Qing, *Random variational inequalities and random saddle point theorems*, Applied Math. and Mech. (special issue) (1993), 132-173.

5. S. S. Chang and Y. G. Zhu, *Random variational inequalities*, J. Math. Research Expo. **9** (1989), 385-393.

6. G. Y. Chen and X. Q. Yang, *The vector complementarity problem and its equivalence with the weak minimal element in ordered sets*, J. Math. Anal. Appl. **153** (1990), 136-158.

7. R. W. Cottle, *Complementarity and variational problems*, Sympos. Math. **19** (1976), 177-208.

8. R. W. Cottle and G. B. Dantzig, *Complementarity pivot theory of mathematical programming*, Linear Algebra Appl. **1** (1968), 163-185.

9. G. M. Lee, B. S. Lee and S. S. Chang, *Random vector variational inequalities and random noncooperative vector equilibrium*, J. Appl. Math. and Stoc. Anal..

10. C. B. Lemke, *Bimatrix equilibrium points and mathematical programming*, Management Sci. **11** (1965), 681-689

11. M. A. Noor, *Fixed point approach for complementarity problems*, J. Math. Anal. Appl. **133** (1988), 437-448.

12. M. A. Noor and E. H. AL-Shemas, *Fixed point methods for nonlinear complementarity problems*, Indian J. Math. **35** (1993), 71-97.

13. N. X. Tan, *Random quasi-variational inequality*, Math. Nachr. **125** (1986), 319-329.

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