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RANDOM COMPLETELY GENERALIZED NONLINEAR VARIATIONAL INCLUSIONS WITH NON-COMPACT VALUED RANDOM MAPPINGS

NAN-JING HUANG, XIANG LONG AND YEOL JE CHO

ABSTRACT. In this paper, we introduce and study a new class of random completely generalized nonlinear variational inclusions with non-compact valued random mappings and construct some new iterative algorithms. We prove the existence of random solutions for this class of random variational inclusions and the convergence of random iterative sequences generated by the algorithms.

1. Introduction

Variational inequalities, introduced by Hartman and Stampacchia [12] in the early sixties, are a very powerful tool of the current mathematical technology. These have been extended and generalized to study a wide class of problems arising in mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium and engineering sciences, etc. Quasivariational inequalities are generalized forms of variational inequalities in which the constraint set depend on the solution. These were introduced and studied by Bensoussan, Goursat and Lions [3]. For further details we refer to [1, 2, 4, 6, 20, 24] and the references therein.

In 1991, Chang and Huang [7, 8] introduced and studied some new class of complementarity problems and variational inequalities for set-valued mappings with compact values in Hilbert spaces. In the recent paper [13], Hassouni and Moudafi introduced and studied a new class of

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variational inclusions, which included many variational and quasivariational inequalities considered by Noor [21-23], Isac [19], Siddiqi and Ansari [25, 26] as special cases. In 1996, Huang [17] has introduced and studied a new class of generalized nonlinear variational inclusions with non-compact valued mappings in Hilbert spaces.

On the other hand, the random variational inequality and random quasi-variational inequality problems have been introduced and studied by Chang [6], Chang and Huang [9, 10], Chang and Zhu [11], Huang [15, 16], Husain, Tarafdar and Yuan [18], Tan, Tarafdar and Yuan [28], Tan [27] and Yuan [29].

The main purpose of this work is to extend their ideas to more general problems. We introduce and study a new class of random completely generalized nonlinear variational inclusions with non-compact valued random mappings and construct some new iterative algorithms. We also prove the existence of random solutions for this class of random variational inclusions and the convergence of random iterative sequences generated by the algorithms.

2. Preliminaries Throughout this paper, let $(\Omega, \mathcal{A})$ be a measure space and H be a separable real Hilbert space endowed with the norm $\|\cdot\|$ and inner product $(\cdot, \cdot)$. We denote by $\mathcal{B}(H)$, 2^H, $CB(H)$ and $H(\cdot, \cdot)$ the class of Borel σ-fields in H, the family of all nonempty subsets of H, the family of all nonempty closed bounded subsets of H and the Hausdorff metric on $CB(H)$, respectively.

Definition 2.1. A mapping $x : \Omega \to H$ is said to be *measurable* if for any $B \in \mathcal{B}(H)$, $\{t \in \Omega : x(t) \in B\} \in \mathcal{A}$.

Definition 2.2. A mapping $T : \Omega \times H \to H$ is called a *random operator* if for any $x \in H$, $T(t, x) = x(t)$ is measurable. A random operator T is said to be continuous if for any $t \in \Omega$, the mapping $T(t, \cdot) : H \to H$ is continuous.

Definition 2.3. A set-valued mapping $V : \Omega \to 2^H$ is said to be measurable if for any $B \in \mathcal{B}$, $V^{-1}(B) = \{t \in \Omega : V(t) \cap B \neq \emptyset\} \in \mathcal{A}$.

Definition 2.4. A mapping $u : \Omega \to H$ is called a *measurable selection of a set-valued measurable mapping* $V : \Omega \to 2^H$ if u is measurable and for any $t \in \Omega, u(t) \in V(t)$.

DEFINITION 2.5. A mapping $V : \Omega \to 2^H$ is called a *random set-valued mapping* if for any $x \in H$, $V(\cdot, x)$ is measurable. A random set-valued mapping $V : \Omega \times H \to CB(H)$ is said to be H-continuous if for any $t \in \Omega$, $V(t, \cdot)$ is continuous in the Hausdorff metric.

Given random set-valued mappings $T, A, g : \Omega \times H \to 2^H$ and random operators $f, p : \Omega \times H \to H$ with $Im\, g \bigcap dom(\partial\varphi) \neq \emptyset$, we consider the following problem:

Find measurable mappings $u, w, y, z : \Omega \to H$ such that, for all $t \in \Omega$, $v \in H$,

(2.1)
$$\begin{cases} w(t) \in T(t, u(t)), \quad y(t) \in A(t, u(t)), \quad z(t) \in g(t, u(t)) \bigcap dom(\partial\varphi), \\ (f(t, w(t)) - p(t, y(t)), v - z(t)) \geq \varphi(z(t)) - \varphi(v), \end{cases}$$

where $\partial\varphi$ denotes the subdifferential of a proper, convex and lower semicontinuous function $\varphi : H \to R \cup \{+\infty\}$. This problem is called a *random completely generalized nonlinear variational inclusion* with non-compact valued random mappings.

If $g : \Omega \times H \to H$ is a random operator with $Im\, g \bigcap dom(\partial\varphi) \neq \emptyset$, then the random nonlinear variational inclusion (2.1) is equivalent to the following problem:

Find measurable mappings $u, w, y : \Omega \to H$ such that, for all $t \in \Omega$, $v \in H$,

(2.2)
$$\begin{cases} w(t) \in T(t, u(t)), \quad y(t) \in A(t, u(t)), \quad g(t, u(t)) \bigcap dom(\partial\varphi) \neq \emptyset, \\ (f(t, w(t)) - p(t, y(t)), v - g(t, u(t))) \geq \varphi(g(t, u(t))) - \varphi(v), \end{cases}$$

which is called a *random generalized nonlinear variational inclusion* with non-compact valued random mappings.

It is clear that the random completely generalized nonlinear variational inclusion (2.1) and the random generalized nonlinear variational inclusion (2.2) include many kinds of variational inequalities and quasivariational inequalities of [6-10, 13, 15-17, 19, 20, 21-23, 25, 26, 30] as special cases.

3. Random Iterative Algorithms

We first give the following lemmas for our main results.

LEMMA 3.1. [5] *Let $V : \Omega \times H \to CB(H)$ be a H-continuous random set-valued mapping. Then for any measurable mapping $u : \Omega \to H$, the set-valued mapping $V(\cdot, u(\cdot)) : \Omega \to CB(H)$ is measurable.*

LEMMA 3.2. [5] *Let V, $W : \Omega \to CB(H)$ be two measurable set-valued mappings, $\epsilon > 0$ be constant and $u : \Omega \to H$ be a measurable selection of V. Then there exists a measurable selection $v : \Omega \to H$ of W such that for all $t \in \Omega$,*

$$\|u(t) - v(t)\| \leq (1 + \epsilon)H(V(t), W(t)).$$

LEMMA 3.3. *Measurable mappings u, w, $y : \Omega \to H$ are solution of the problem (2.1) if and only if for all $t \in \Omega$, $w(t) \in T(t, u(t))$, $y(t) \in A(t, u(t))$, $z(t) \in g(t, u(t))$ and*

$$z(t) = J^{\varphi}_{\alpha(t)}(z(t) - \alpha(t)(f(t, w(t)) - p(t, y(t)))), \tag{3.1}$$

where $\alpha : \Omega \to (0, \infty)$ is a measurable function and $J^{\varphi}_{\alpha(t)} = (I + \alpha(t)\partial\varphi)^{-1}$ is the so-called proximal mapping on H.

Proof. From the definition of $J^{\varphi}_{\alpha(t)}$, it follows that

$$z(t) - \alpha(t)(f(t, w(t)) - p(t, y(t))) \in z(t) + \alpha(t)\partial\varphi(z(t)),$$

for all $t \in \Omega$ and hence

$$p(t, y(t)) - f(t, w(t)) \in \partial\varphi(z(t)).$$

From definition of $\partial\varphi$, we have

$$\varphi(v) \geq \varphi(z(t)) + (p(t, y(t)) - f(t, w(t)), v - z(t))$$

for all $v \in H$ and $t \in \Omega$. Thus u, w, y and z are solutions of (2.1). □

To obtain an approximate solution of (2.1) we can apply a successive approximation method to the problem of solving

$$u(t) \in F(t, u(t)) \tag{3.2}$$

for all $t \in \Omega$, where

$$\begin{aligned} F(t,u(t)) = u(t) - g(t,u(t)) + J^{\varphi}_{\alpha(t)}(g(t,u(t)) \\ - \alpha(t)(f(t,T(t,u(t))) - p(t,A(t,u(t))))). \end{aligned}$$

Based on (3.1) and (3.2), we proceed our algorithms.

Let T, A, $g : \Omega \times H \to CB(H)$ be H-continuous random set-valued mappings, and f, $p : \Omega \times H \to H$ be continuous random operators. For any given measurable mapping $u_0 : \Omega \to H$, the set-valued mappings $T(\cdot, u_0(\cdot))$, $A(\cdot, u_0(\cdot))$, $g(\cdot, u_0(\cdot)) : \Omega \to CB(H)$ are measurable by Lemma 3.1. Hence there exist measurable selection $w_0 : \Omega \to H$ of $T(\cdot, u_0(\cdot))$, measurable selection $y_0 : \Omega \to H$ of $A(\cdot, u_0(\cdot))$ and measurable selection $z_0 : \Omega \to H$ of $g(\cdot, u_0(\cdot))$ by Himmelberg [14]. Let

$$u_1(t) = u_0(t) - z_0(t) + J^{\varphi}_{\alpha(t)}(z_0(t) - \alpha(t)(f(t, w_0(t)) - p(t, y_0(t)))).$$

It is easy to see that $u_1 : \Omega \to H$ is measurable. By Lemma 3.2, there exist measurable selections $w_1 : \Omega \to H$ of $T(t, u_1(t))$, measurable selection $y_1 : \Omega \to H$ of $A(t, u_1(t))$ and measurable selection $z_1 : \Omega \to H$ of $g(t, u_1(t))$ such that, for all $t \in \Omega$,

$$\begin{aligned} \|w_1(t) - w_0(t)\| \le (1+1)H(T(t,u_1(t)), T(t,u_0(t))), \\ \|y_1(t) - y_0(t)\| \le (1+1)H(A(t,u_1(t)), A(t,u_0(t))) \end{aligned}$$

and

$$\|z_1(t) - z_0(t)\| \le (1+1)H(g(t,u_1(t)), g(t,u_0(t))).$$

Letting

$$u_2(t) = u_1(t) - z_1(t) + J^{\varphi}_{\alpha(t)}(z_1(t) - \alpha(t)(f(t, w_1(t)) - p(t, y_1(t)))),$$

then $u_2 : \Omega \to H$ is measurable. By induction, we can obtain our algorithm for the random completely generalized nonlinear variational inclusion (2.1) as follows:

ALGORITHM 3.1. *Let T, A, $g : \Omega \times H \to CB(H)$ be H-continuous random set-valued mappings and f, $p : \Omega \times H \to H$ be continuous random operators. For given measurable $u_0 : \Omega \to H$, we have*

(3.3)
$$\begin{cases} u_{n+1}(t) = u_n(t) - z_n(t) \\ \qquad\qquad + J^{\varphi}_{\alpha(t)}(z_n(t) - \alpha(t)(f(t, w_n(t)) - p(t, y_n(t)))), \\ \|w_{n+1}(t) - w_n(t)\| \\ \le (1 + (1+n)^{-1})H(T(t, u_{n+1}(t)), T(t, u_n(t))), \ w_n(t) \in T(t, u_n(t)), \\ \|y_{n+1}(t) - y_n(t)\| \\ \le (1 + (1+n)^{-1})H(A(t, u_{n+1}(t)), A(t, u_n(t))), \ y_n(t) \in A(t, u_n(t)), \\ \|z_{n+1}(t) - z_n(t)\| \\ \le (1 + (1+n)^{-1})H(g(t, u_{n+1}(t)), g(t, u_n(t))), \ z_n(t) \in g(t, u_n(t)) \end{cases}$$

for any $t \in \Omega$ and $n = 0, 1, 2, \cdots$.

From Algorithm 3.1, we can get the algorithm for the random generalized nonlinear variational inclusion (2.2) as follows:

ALGORITHM 3.2. *Let T, $A : \Omega \times H \to CB(H)$ be two H-continuous random set-valued mappings and f, p, $g : \Omega \times H \to H$ be continuous random operators. For given measurable $u_0 : \Omega \to H$, we have*

(3.4)
$$\begin{cases} u_{n+1}(t) = u_n(t) - g(t, u_n(t)) \\ \qquad\qquad + J^{\varphi}_{\alpha(t)}(g(t, u_n(t)) - \alpha(t)(f(t, w_n(t)) - p(t, y_n(t)))), \\ \|w_{n+1}(t) - w_n(t)\| \\ \le (1 + (1+n)^{-1})H(T(t, u_{n+1}(t)), T(t, u_n(t))), \ w_n(t) \in T(t, u_n(t)), \\ \|y_{n+1}(t) - y_n(t)\| \\ \le (1 + (1+n)^{-1})H(A(t, u_{n+1}(t)), A(t, u_n(t))), \ y_n(t) \in A(t, u_n(t)) \end{cases}$$

for any $t \in \Omega$ and $n = 0, 1, 2, \cdots$.

REMARK 3.1. Algorithms 3.1 and 3.2 include several known algorithms of [6-10, 15-17, 21-23, 25, 26, 30] as special cases.

4. Existence and Convergence

DEFINITION 4.1. A random operator $g : \Omega \times H \to H$ is said to be

(i) *strongly monotone* if there exists a measurable function $\delta : \Omega \to (0, \infty)$ such that

$$(g(t, u_1) - g(t, u_2), u_1 - u_2) \geq \delta(t)\|u_1 - u_2\|^2$$

for all $u_i \in H$, $i = 1, 2$, and $t \in \Omega$,

(ii) *Lipschitz continuous* if there exists a measurable function $\sigma : \Omega \to (0, \infty)$ such that

$$\|g(t, u_1) - g(t, u_2)\| \leq \sigma(t)\|u_1 - u_2\|$$

for all $u_i \in H$, $i = 1, 2$, and $t \in \Omega$.

DEFINITION 4.2. A random set-valued mapping $T : \Omega \times H \to CB(H)$ is said to be

(i) *strongly monotone with respect to a random operator* $f : \Omega \times H \to H$ if there exists a measurable function $\beta : \Omega \to (0, \infty)$ such that

$$(f(t, w_1) - f(t, w_2), u_1 - u_2) \geq \beta(t)\|u_1 - u_2\|^2$$

for all $t \in \Omega$, $u_i \in H$ and $w_i \in T(t, u_i)$, $i = 1, 2$,

(ii) *H-Lipschitz continuous* if there exists some measurable function $\gamma : \Omega \to (0, \infty)$ such that

$$H(T(t, u_1), T(t, u_2)) \leq \gamma(t)\|u_1 - u_2\|$$

for all $u_i \in H$, $i = 1, 2$.

(iii) *strongly monotone* if there exists a measurable function $\xi : \Omega \to (0, \infty)$ such that

$$(w_1 - w_2, u_1 - u_2) \geq \xi(t)\|u_1 - u_2\|^2$$

for all $u_i \in \Omega$, $u_i \in H$ and $w_i \in T(t, u_i)$, $i = 1, 2$.

THEOREM 4.1. *Let $g : \Omega \times H \to CB(H)$ be strongly monotone and H-Lipschitz continuous random set-valued mapping, $f, p : \Omega \times H \to H$ be Lipschitz continuous random operators, T, $A : \Omega \times H \to CB(H)$ be H-Lipschitz continuous random set-valued mappings and T be strongly monotone with respect to f. If the following conditions hold:*

$$\begin{aligned} &\left|\alpha(t) - \frac{\beta(t) + \epsilon(t)\mu(t)(k(t)-1)}{\eta(t)^2\gamma(t)^2 - \epsilon(t)^2\mu(t)^2}\right| \\ &< \frac{\sqrt{(\beta(t) + (k(t)-1)\epsilon(t)\mu(t))^2 - l(t)}}{\eta(t)^2\gamma(t)^2 - \epsilon(t)^2\mu(t)^2}, \end{aligned} \tag{4.1}$$

$$\beta(t) > (1 - k(t))\epsilon(t)\mu(t) + \sqrt{l(t)}, \quad \eta(t)\gamma(t) > \epsilon(t)\mu(t), \tag{4.2}$$

$$\begin{gathered} l(t) = (\eta(t)^2\gamma(t)^2 - \epsilon(t)^2\mu(t)^2)k(t)(2 - k(t)), \\ \alpha(t)\mu(t)\epsilon(t) < 1 - k(t), \end{gathered} \tag{4.3}$$

$$k(t) = 2\sqrt{1 - 2\delta(t) + \sigma(t)^2}, \quad k(t) < 1 \tag{4.4}$$

for all $t \in \Omega$, where $\beta(t)$ and $\delta(t)$ are strongly monotone coefficients of T and g, respectively, $\sigma(t)$, $\gamma(t)$ and $\mu(t)$ are H-Lipschitz coefficients of g, T and A, respectively, $\eta(t)$ and $\epsilon(t)$ are the Lipschitz coefficients of f and p, respectively, then there exist measurable mappings u, w, y, $z : \Omega \to H$ such that (2.1) holds. Moreover,

$$u_n(t) \to u(t), \quad w_n(t) \to w(t), \quad y_n(t) \to y(t), \quad z_n(t) \to z(t), \quad n \to \infty,$$

where $\{u_n(t)\}$, $\{w_n(t)\}$, $\{y_n(t)\}$ and $\{z_n(t)\}$ are generated by Algorithm 3.1.

Proof. From (3.3), for any $t \in \Omega$, we have

$$\begin{aligned} \|u_{n+1}(t) - u_n(t)\| = \|&u_n(t) - u_{n-1}(t) - (z_n(t) - z_{n-1}(t)) \\ &+ J^{\varphi}_{\alpha(t)}(h(t, u_n(t))) - J^{\varphi}_{\alpha(t)}(h(t, u_{n-1}(t)))\|, \end{aligned}$$

where

$$h(t, u_n(t)) = z_n(t) - \alpha(t)(f(t, w_n(t)) - p(t, y_n(t))).$$

Also we have, for all $t \in \Omega$,

$$\begin{aligned}
&\|J^{\varphi}_{\alpha(t)}(h(t, u_n(t))) - J^{\varphi}_{\alpha(t)}(h(t, u_{n-1}(t)))\| \\
&\leq \|h(t, u_n(t)) - h(t, u_{n-1}(t))\| \\
&\leq \|u_n(t) - u_{n-1}(t) - \alpha(t)(f(t, w_n(t)) - f(t, w_{n-1}(t)))\| \\
&\quad + \|u_n(t) - u_{n-1}(t) - (z_n(t) - z_{n-1}(t))\| \\
&\quad + \alpha(t)\|p(t, y_n(t)) - p(t, y_{n-1}(t))\|,
\end{aligned}$$

that is,

$$\begin{aligned}
&\|u_{n+1}(t) - u_n(t)\| \\
&\leq 2\|u_n(t) - u_{n-1}(t) - (z_n(t) - z_{n-1}(t))\| \\
&\quad + \|u_n(t) - u_{n-1}(t) - \alpha(t)(f(t, w_n(t)) - f(t, w_{n-1}(t)))\| \\
&\quad + \alpha(t)\|p(t, y_n(t)) - p(t, y_{n-1}(t))\|.
\end{aligned} \tag{4.5}$$

By H-Lipschitz continuity and strongly monotonicity of g, we obtain

$$\begin{aligned}
&\|u_n(t) - u_{n-1}(t) - (z_n(t) - z_{n-1}(t))\|^2 \\
&\leq (1 - 2\delta(t) + (1 + n^{-1})^2\sigma(t)^2)\|u_n(t) - u_{n-1}(t)\|^2
\end{aligned} \tag{4.6}$$

for all $t \in \Omega$. Also from H-Lipschitz continuity and strongly monotonicity with respect to f of T and Lipschitz continuity of f, we have

(4.7)

$$\begin{aligned}
&\|u_n(t) - u_{n-1}(t) - \alpha(t)(f(t, w_n(t)) - f(t, w_{n-1}(t)))\|^2 \\
&\leq (1 - 2\beta(t)\alpha(t) + \alpha(t)^2\eta(t)^2(1 + n^{-1})^2\gamma(t)^2)\|u_n(t) - u_{n-1}(t)\|^2
\end{aligned}$$

By H-Lipschitz continuity of A, Lipschitz continuity of p and (3.3), it follows that

(4.8)

$$\alpha(t)\|p(t, y_n(t)) - p(t, y_{n-1}(t))\| \leq \alpha(t)\epsilon(t)(1 + n^{-1})\mu(t)\|u_n(t) - u_{n-1}(t)\|$$

for all $t \in \Omega$. So by combining (4.5)-(4.8) and denoting

$$\theta_n(t) := 2\sqrt{1 - 2\delta(t) + (1 + n^{-1})^2\sigma(t)^2} + \alpha(t)\epsilon(t)(1 + n^{-1})\mu(t)$$
$$+ \sqrt{1 - 2\beta(t)\alpha(t) + \alpha(t)^2\eta(t)^2(1 + n^{-1})^2\gamma(t)^2},$$

we get, for all $\in \Omega$,

$$\|u_{n+1}(t) - u_n(t)\| \leq \theta_n(t)\|u_n(t) - u_{n-1}(t)\|.$$

Letting

$$\theta(t) := 2\sqrt{1 - 2\delta(t) + \sigma(t)^2}$$
$$+ \sqrt{1 - 2\beta(t)\alpha(t) + \alpha(t)^2\eta(t)^2\gamma(t)^2} + \alpha(t)\epsilon(t)\mu(t),$$

for all $t \in \Omega$, we know that $\theta_n(t) \searrow \theta(t)$ for all $t \in \Omega$. It follows from (4.1)-(4.4) that $\theta(t) < 1$ for all $t \in \Omega$. Hence, for any $t \in \Omega$, $\theta_n(t) < 1$, for n sufficiently large. Therefore $\{u_n(t)\}$ is a Cauchy sequence and we can suppose that $u_n(t) \to u(t)$ for all $t \in \Omega$.

From (3.3), we get

$$\|w_n(t) - w_{n-1}(t)\| \leq (1 + n^{-1})\gamma(t)\|u_n(t) - u_{n-1}(t)\|,$$
$$\|y_n(t) - y_{n-1}(t)\| \leq (1 + n^{-1})\mu(t)\|u_n(t) - u_{n-1}(t)\|,$$
$$\|z_n(t) - z_{n-1}(t)\| \leq (1 + n^{-1})\sigma(t)\|u_n(t) - u_{n-1}(t)\|$$

for all $t \in \Omega$, i.e., $\{w_n(t)\}$, $\{y_n(t)\}$ and $\{z_n(t)\}$ are Cauchy sequences. Let $w_n(t) \to w(t)$, $y_n \to y(t)$ and $z_n(t) \to z(t)$. Since $\{u_n(t)\}, \{w_n(t)\}$, $\{y_n(t)\}$ and $\{z_n(t)\}$ are sequences of measurable mappings, we know that mappings u, w, y, $z : \Omega \to H$ are measurable. Further, for any $t \in \Omega$, we have

$$\begin{aligned} d(w(t), T(t, u(t))) &= \inf\{\|w(t) - z\| : z \in T(t, u(t))\} \\ &\leq \|w(t) - w_n(t)\| + d(w_n(t), T(t, u(t))) \\ &\leq \|w(t) - w_n(t)\| + H(T(t, u_n(t)), T(t, u(t))) \\ &\leq \|w(t) - w_n(t)\| + \gamma(t)\|u_n(t) - u(t)\| \\ &\to 0. \end{aligned}$$

Hence, $w(t) \in T(t, u(t))$ for all $t \in \Omega$. Similarly, we have $y(t) \in A(t, u(t))$ and $z(t) \in g(t, u(t))$ for all $t \in \Omega$. This completes the proof of Theorem 4.1. □

From Theorem 4.1, we can get the following:

THEOREM 4.2. *Let $g : \Omega \times H \to H$ be strongly monotone and Lipschitz continuous random operator, f, $p : \Omega \times H \to H$ be Lipschitz continuous random operators, T, $A : \Omega \times H \to CB(H)$ be H-Lipschitz continuous random set-valued mappings and T be strongly monotone with respect to f. Suppose that $\beta(t)$ and $\delta(t)$ are strongly monotone coefficients of T and g, respectively, $\gamma(t)$ and $\mu(t)$ are H-Lipschitz coefficients of T and A, respectively, $\sigma(t)$, $\eta(t)$ and $\epsilon(t)$ are the Lipschitz coefficients of g, f and p, respectively. If for all $t \in \Omega$ the conditions* (4.1)-(4.4) *in Theorem* 4.1 *hold, then there exist measurable mappings u, w, $y : \Omega \to H$ such that* (2.2) *holds. Moreover,*

$$u_n(t) \to u(t), \quad w_n(t) \to w(t), \quad y_n(t) \to y(t), \quad n \to \infty,$$

where $\{u_n(t)\}$, $\{w_n(t)\}$ and $\{y_n(t)\}$ are generated by Algorithm 3.2.

REMARK 4.1. For a suitable choice of the mappings g, T, A, f, p and the function φ, we can obtain several known results of [6-10, 15-17, 19, 21-23, 25, 26, 30] as special cases of Theorems 4.1 and 4.2.

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DEPARTMENT OF MATHEMATICS, SICHUAN UNION UNIVERSITY, CHENGDU, SICHUAN 610064, P. R. CHINA

DEPARTMENT OF MATHEMATICS, GYEONGSANG NATIONAL UNIVERSITY, CHINJU 660-701, KOREA