□ 論 文 □

A Study on the Fuzzy System for Freeway Incident Duration Analysis

고속도로 사고존속시간 분석을 위한 퍼지시스템에 관한 연구

최 회 균
(협성대학교 도시계획공학과 교수)

목 차

요 약

도시고속도로의 원활한 운영을 위해서는 신속하고 정확한 교통사고관리체계가 요구된다. 교통사고의 동적인 특질과 이에 관련된 불확실성은 교통사고 운영자(관리자) 판단에 의한 해결을 필요로 하고 있다. 퍼지시스템은 인간의 전문가적 의견에 적응시키려는 시도와 운영자의 결정을 내리는 능력을 반복적으로 할 수 있도록 설계된다. 퍼지시스템은 복합적인 교통정보를 처리하고, 그 정보를 단순화하여 이해할 수 있는 형태로 교통운영자에게 전달한다.

이 논문에서 퍼지규칙들은 미국 Los Angeles에 있는 Santa Monica 고속도로의 실제 교통사고로 부터 조사된 자료에 근거하여 개발되었다. 이러한 퍼지규칙들은 언어학에 기초를 두었기 때문에 사용자가 편리하게 이용할 수 있다. 언어학적 모형에 의한 사고존속시간과 실제 사고존속시간 조건을 비교한 결과 서로 신뢰할 수 있는 일치를 보았다. 이 모형은 고속도로 교통사고 존속시간을 거의 확실하게 예측(예보)하는 것을 가능하게 한다. 이는 고속도로 교통사고 응답시스템 및 급파(급송)체계 개발에도 적용이 가능할 것이다.

I. Introduction

Traffic congestion is no longer limited to a peak-period and particular roads. It has spreading over large metropolitan areas (Gordon and Richardson, 1994). Traffic congestion is a daily phenomenon in most metropolitan areas, and has negative effects on traffic safety, mobility, and productivity of the transportation system. Congestion is creating bottlenecks on the freeway system. Freeway congestion poses serious problems in urban areas.

Freeway congestion is made up of two components: recurring congestion and non-recurring congestion. Recurring congestion is predictable and occurs in locations where the traffic volumes routinely exceed capacity. Non-recurring congestion is very unpredictable and is caused by incidents. Incidents can be defined as traffic accidents, disabled vehicles, spilled loads, and other random events (i.e., the occurrence of incidents) that reduce the freeway capacity at a specific location.

At least 60% of urban freeway congestion in the U.S. is non-recurring congestion caused by incidents (Lindley, 1987). About 80% of freeway incidents are minor incidents-vehicle disablements, and about 10% incidents are really vehicle accidents (Giuliano, 1989 and Grenzeback and Woodle, 1992). In large metropolitan areas such as Los Angeles, New York, and San Francisco, over half of all traffic delays are caused by freeway incidents rather than recurring congestion. (U.S. DOT, 1994)

Incident management is a very significant element of traffic management systems. The impact of incidents is the most obvious factor affecting freeway operation. Even for short duration incidents, effective traffic management will reduce the impact of the incident and provide a safer environment for involved in the incident and those responding to the incident. Advanced Traffic Management Systems (ATMS) provide travelers with real-time traffic information about incidents. ATMS integrates management of various roadway functions, such as monitoring traffic conditions, adjusting traffic operations, and respond to incidents.

Traveler information is defined as information that allows travelers to make informed travel decisions. Traveler information is broader in scope than traffic management. Advanced Traveler Information Systems (ATIS) includes a variety of traveler information, such as locations of traffic incidents, optimal routes, weather conditions, and recommended speeds. ATIS will have a significant impact on congested urban networks.

ITS (Intelligent Transportation Systems) is a monitoring and communication system that gives real-time traffic information to travelers. ATMS and ATIS are major components of the ITS program. For example, the immediate reporting of any accident or breakdown will bring a tow truck or freeway service patrol. Therefore, both freeway incident management and ITS involved in a similar technological opportunities.

The management of incidents is one of the major challenges in freeway traffic operations, requiring constant attention and considerable investment. Several methods are currently employed for incident management, automatic techniques are becoming increasingly important for reducing freeway system delay time. Incident management requires important expertise and judgment to restore the affected freeway and its surrounding network to normal traffic operation quickly. Solutions to incident management problems will not be found solely in numerical algorithms, but rather in the application of algorithmic tools guided by human experts using their knowledge and experience. Hence, the evaluation of fuzzy systems emerge as suitable for the incident management problem. Fuzzy systems approach may also be used to study detailed traffic control operator behavior in the context of an incident management environment.

The purpose of this study is to develop methods to enhance existing incident management programs intended reduce to incident related traffic congestion and delay. The main objective is to provide an improved incident management framework making use of a fuzzy system. The model will advance the field of incident management modeling by introducing an evaluation of fuzzy systems. This study describes the fuzzy logic application in freeway incident duration and proposes a methodology for developing fuzzy systems to assist in freeway incident management. This study demonstrates a fuzzy incident response strategy for freeway incident duration. This approach may also be used to study detailed incident operator behavior and develop intuition for the interaction-operator-incident management system while contributing to the human factors aspects of ITS environment.

II. Background Overview

1. Conceptualization of Freeway Incident Management

As economic and social activities of people become more complex, traffic demand is increasing rapidly, particularly in metropolitan areas. Recently, the attention in traffic operation has turned to research of how to more effectively control of existing urban road networks' facilities and capacity. This operation is requires a traffic control system. A distinction is made between urban street control and freeway control, although the two are functionally related in urban areas. This research is concerned about freeway incident management based on incident duration.

Over the past several decades, trans-portation researchers have devoted con-siderable effort to developing freeway incident detection algorithms and responsive control management. Incident management falls into two broad categories, as detection and response. Incident detection offers a means by which the problems associated with incidents are identified, and provides the appropriate response to reduce overall incident-related traffic delay. The time saved by an incident management program depends on how well the common stages or elements (detection, verification, response, clearance, traffic management and traveler information) of an incident are managed. Improvements in these stages combine to improve the efficiency of incident coordination.

Incident detection is the determination that an incident has occurred. An incident can be detected by several information resources, for example, electronic detectors, closed circuit TV (CCTV), service patrols, maintenance crews, call boxes, cellular mobile telephone, and media. Most freeway incident management programs in the U.S. include a detection element. Detection measures combine verification in a single stage. If an incident is detected, the incident is immediately verified with respect to its nature and location information. Incident verification refers to establishing that an incident exists at a given location.

Effective incident detection and verification is not dependent on electronic surveillance systems, nor is any single detection and verification technique adequate by itself. It requires a systematic approach to tap all available sources.

The incident response stage involves all of those activities associated with getting appropriate resources to the nature of the incident. It is the phase of the incident management program that offers the greatest opportunities to reduce overall incident-related traffic delay. Preplanning, coordination, communication, and traffic management of the appropriate personnel and equipment necessary to clear incident. Clearance is the safe and timely removal or termination of the incident. It also includes actually clearing the incident to restore normal traffic operations. Response activities overlap with the clearance stage. It is beneficial to consider these together. Service patrols are one of the most effective incident management techniques available (Judycki and Robinson, 1988). They not only reduce response and clearance times, but can reduce detection time as well. The travelers can also play a significant role in reducing the duration of incidents and the number of incidents. Most importantly, the overall duration of an incident is controlled by the essential incident response services, such as traffic management and traveler information.

The most effective traffic management strategy is fast incident detection and removal. For example, CCTV has been used to verify and monitor removal of incidents from a remote operations center. The ATMS promoted by ITS initiatives is likely to significantly increase the area of the urban network covered by CCTV, broadening its utility for incident verification and removal. For response incidents, the traffic management plan is to ensure that responders, police, service patrols, and private wreckers are equipped with proper traffic control devices, and that they understand traffic management requirements.

Overlaying each of the stages is the need to inform travelers of the potential or actual incident condition and to provide the information needed to reduce the impact of the incident condition. Traveler information may begin concurrently with the incident detection stage and continue for a period after the incident response stage to assure travelers that the condition no longer exists. The information can also be used in real-time to cause travelers to change to less congested routes and thus avoid becoming part of the problem by avoiding the queue. As advanced traveler information systems (ATIS) penetrate the automobile market, the information can be provided in voice, text, or graphic modes and can include specific information on alternative routes. Depending on the final architecture of the ATIS, the in-vehicle equipment may serve as traffic probes and feed real-time travel information to the control center, thus improving the quality of traveler information and the decisions that are made based on that information.

Incident management cannot be done one stage, it must address each of the stages in a well-organized incident management program. A successful incident management program must include a coordination or communication centers. These centers support center for the incident response and clearance activities, and have been incorporated into existing traffic operation centers for the purpose of other traffic management and traveler information.

2. Fuzzy Incident Response Strategy

The terms of the fuzziness are the uncertainty or ambiguity that can be found in many fields, such as in manufacturing, in informational systems, in control engineering, and others. In these areas, human judgment, decision-making, and evaluation processes are very significant. Fuzzy set theory provides a mathematical framework that represents fuzziness in the concept of sets. As fuzzy set theory is a generalization of an ordinary (crisp) set theory by defining graded membership, and fuzzy logic is an extension of ordinary (crisp) logic. There are correspondences between fuzzy set theory and fuzzy logic. Fuzzy system is comprised of the four components: a fuzzification interface, a fuzzy rule base, a fuzzy inference engine, and a defuzzification interface.

In summary, the function of fuzzification converts crisp input data into appropriate linguistic values of fuzzy sets through predefined input membership functions. They then go through a set of fuzzy logic rules in the conditional of the form "IF-THEN" to describe how fuzzy system performs. The fuzzy inference engine performs fuzzy implication and approximate reasoning to determine a mapping from the fuzzy sets in the input space to the fuzzy sets in the output space. Finally, the defuzzification interface performs changing fuzzy output back into crisp (numerical) values for fuzzy control system action.

Freeway incident management deals with the various activities (e.g., detection, verification, response, and clearance) of multiple agencies (e.g., Caltrans, CHP, LACM-TA, and LADOT). This procedure is performed by human operators who consider many environmental conditions around the freeway. The feature of the incident operators' judgment is to be able to determine suitable incident management. The uncertainties in incident management are considered in this section, and describe the fuzzy incident response strategy and freeway traffic management system.

Many kinds of traffic data are collected during actual freeway incidents. For example, pavement detectors are installed near ramps and along freeway mainlines at 0.5 mile intervals on passing lanes of the mainline. Generally, the detectors can measure traffic volume, occupancy, and speed. Therefore, data of the queue length is obtained at 0.5 mile intervals. If the actual traffic queue length is 2.2 miles, for example, the incident operator can only get traffic information that queue length is from 2 miles to 2.5 miles long, namely, approximately 2 miles linguistically. The other data such as traffic volume, occupancy, speed, queue lengths, travel times, etc. are similarly subject to fuzziness, In addition, the case of incident is very complicated types (e.g., disablement, injury accident, noninjury accident, detector malfunction, ramp signal malfunction, and others).

The incident management process should be improved to generate a more effective situation of the urban freeway systems. For example, when an incident is detected, the incident operator must complete various procedures to response incident. It is very difficult to improve the process of the incident management because actual process of the incident operators' judgment is not described clearly. In order to solve these problems, it becomes necessary to formulate the incident operators' judgment and to develop an automatic decision-making system in place of incident operators.

Incident detection and verification require a freeway traffic surveillance system. In general, this system may depend upon traffic volume, occupancy, speed, or CCTV measurements. This method is in use on many arterials and freeways. In this study, incident detection is considered as the time interval between the incident occurrence and its reporting to a traffic operation center. The freeway incidents can be detected by passing travelers (motorists), CHP or FSP patrols, FSP trucks, or Caltrans maintenance crews. If an incident is detected by these responders, the incident is instantly verified with respect to its location and description. For example, if the incident has been reported by motorists, patrols, or any another authorized person at the incident site, the incident operator is available to rapidly verify this report. There is no need to screen out false alarms.

After the two steps (detection and verification) have been accomplished, incident response is under the control of the fuzzy system. It shows fuzzy freeway incident response strategies with several elements (see Appendix B). These questions are asked by an incident operator, and the responses are selected and recommended by the fuzzy system.

The improvement of incident response strategies affect incident time interval from occurrence to clearance (Figure 1). The overall incident duration is controlled by essential incident response services. The fuzzy system describes more effective freeway incident management by getting appropriate responses to the scene of an incident.

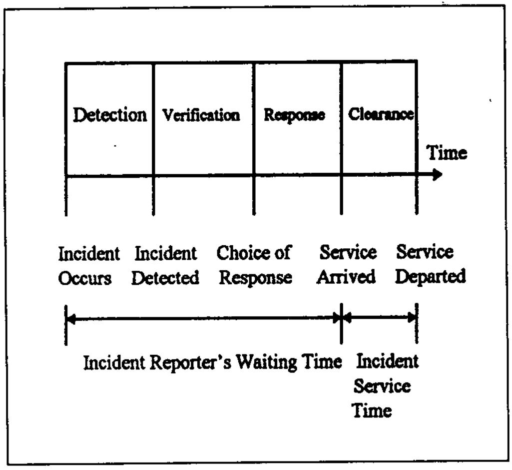

Figure 1 : Components of Incident Duration

III. Fuzzy System Model for Freeway Incident Duration

1. Operating Framework

The framework for a model incorporating fuzzy incident duration of freeway incident management consists of three stages.(Figure 2)

Stage 1 : The data related to the problem of vehicle, type of incident, and location of vehicle are transformed to input variables for fuzzy algorithm.

Stage 2 : The fuzzy algorithm process consisting of

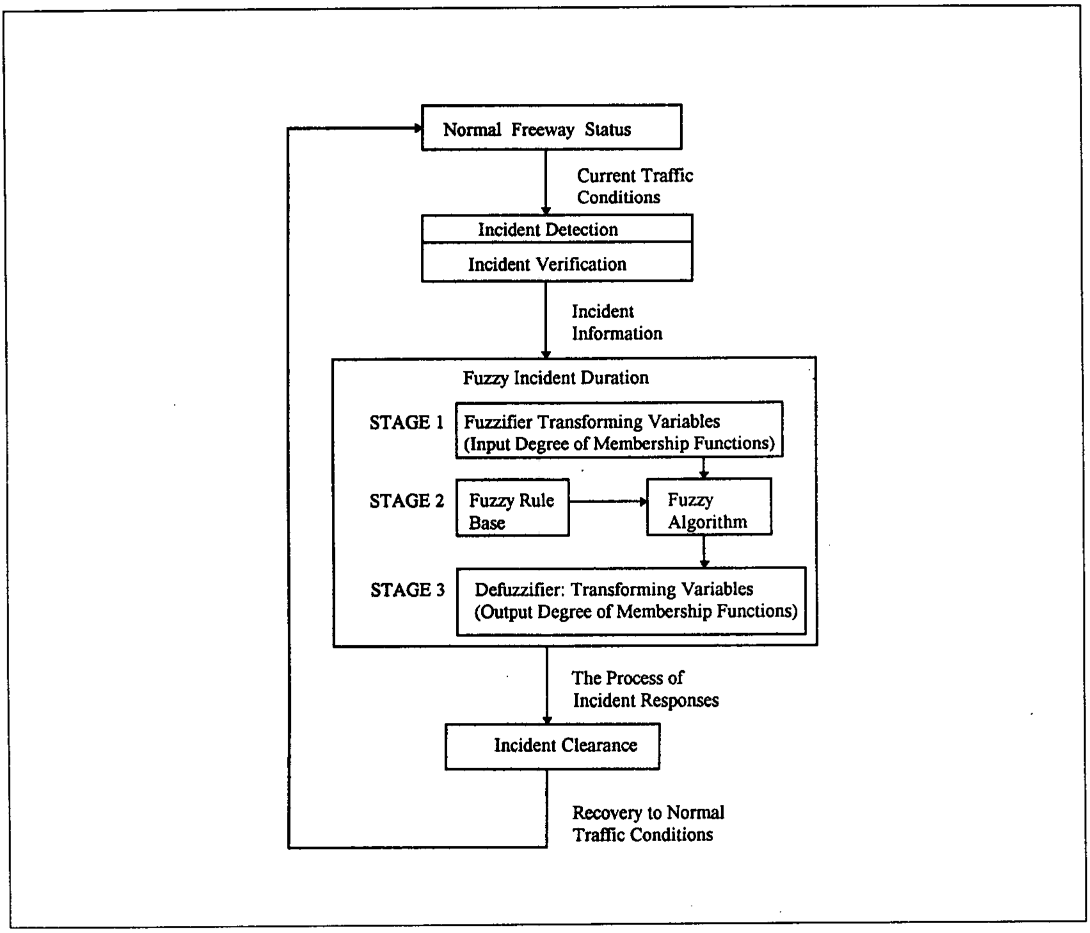

Figure 2 : Operating Framework of Fuzzy Incident Duration

some fuzzy rules put out the result of incident operator' s judgment and decision.

Stage 3 : The fuzzy outputs that show the time of incident duration (incident occurrence to clearance) is transformed to the needs of the actual freeway incident management.

As freeway traffic conditions change throughout the time of the incident duration, the duration of such incidents depends on the severity of the incident, as well as the placement of emergency units. The importance of simulating traffic incidents is evident, since the benefits for incident operator' s response will be greatest during the occurrence of such events.

2. Structure of Fuzzy System Model

A fuzzy system model is based on the input, process structure, and output flow concept. Fuzzy system models fundamentally fall into two important categories[1] which differ basically in their ability to represent different types of information. One of the main directions in fuzzy systems is the linguistic approach, based on linguistically described models. The linguistic model depends on the existence of a rule-base and the theory of approximate reasoning. In this study, the linguistic model is extended to the multiple-input, sgle-output (MISO) form as a tool for complex fuzzy systems.

1) Input/Output Linguistic Variables

There are three inputs in the fuzzy system model : (i) vehicle problems (out of gas, electrical problem, debris removal, over heated, vehicle fire, mechanical problem, flat tire, locked out, other, accident, abandoned, and unknown) ; (ii) the type of vehicle assisted (auto, van, pickup, truck less than 1 ton, truck more than 1 ton, motorcycle, big rig, no assist due to oversize and other) ; and (iii) the disabled vehicle' s location on the guideway (in freeway lanes, on left shoulder, on right shoulder, on a ramp, other and unable to locate). In this study it is assumed that an incident operator considers all the volumes of the inputs. For example, the twelve vehicle problems may be classified separately into three categories (Small, Medium, and Big) by each rate of the vehicle problems.(Table 1) These input variables are represented by the corresponding three categories as membership functions.

The outputs of the fuzzy system model decides the time of incident manager' s response and the time of incident duration. The time of incident duration is treated in this model as fuzzy variable. Five categories (Very Short, Short, Medium, Long, and Very Long) are represented by appropriate fuzzy sets. The membership function gives the membership degree (and represents a value from 0 to 1. The membership functions for inputs (SM = Small, ME = Medium and BI = Big) and output (VS = Very Short, SH = Short, ME = Medium, LO = Long, and VL = Very Long) may be used to predict the freeway incident duration.

2) Fuzzy Rule Base

The fuzzy system is a kind of expert knowledge-based

1) The first includes Linguistic Models that are based on collections of IF-THEN rules with vague predicates and use fuzzy reasoning.(Tong, 1979 and Pedrycz, 1989) In these models fuzzy quantities are associated with linguistic labels, and the fuzzy model is essentially a qualitative expression of the system. The second category of fuzzy models is based on the Takagi-Sugeno-Kang method of reasoning that was proposed by Sugeno and his co-workers.(Takagi and sugeno, 1983 and 1985; Sugeno and Kang, 1986) These models are formed by logical rules have a fuzzy *antecedent part and functional consequent*; essentially they are a combination of fuzzy and nonfuzzy models.

system that contains the control algorithm in a simple rule-base. The fuzzy rule base maps the combination of the inputs to the outputs to decide whether and how to respond to the incident. In the fuzzy system encoded knowledge is expressed by IF-THEN statements. The number of rules is equal to the number of input combinations derived from the number of membership functions per input. For instance, if there are three inputs each having three membership functions, then the number of fuzzy rules would equal twenty-seven ($3 \times 3 \times 3$), as given in Appendix A.

In this study, the term sets of the input variables of the vehicles' problem ratings, vehicles' assisted type ratings, and disabled vehicles' location ratings, include the linguistic labels "Small" (SM), "Medium" (ME) and "Large" (LA). In a similar way the term sets of the output variables of the time of incident duration D include the linguistic labels "Very Short" (VS), "Short" (SH), "Medium" (ME), "Long" (LO) and "Very Long" (VL). This condition is the analysis and synthesis of a multivariable fuzzy model of a system with three inputs and one output. It can be formed by the multiple-input, single-output system.

3) Fuzzy Algorithm

Zadeh (1968, 1971 and 1973) developed the idea of formulating fuzzy control algorithms by logical rules. Mamdani and Assilian (1975) and Mamdani (1976) discussed Zadeh's concept that logical rules with vague predicates can be used to derive inference from vaguely formulated data. A fuzzy control algorithm for multivariable systems proposed by Sanchez (1977 and 1979). Gupta et al. (1986 and 1987) suggested a solution of multivariable fuzzy control systems.

The analysis and synthesis of a multivariable structure is an important problem in fuzzy control systems. In the multiple-input, single-output systems the encoded knowledge can be expressed by IF-THEN rules. This system has three inputs and one output. The fuzzy system can be described by the following linguistic specification :

IF U_1 is $A_{(1)1}$ and U_2 is $A_{(1)2}$ and U_3 is $A_{(1)3}$
THEN V_1 is $B_{(1)1}$
ALSO
...
ALSO
IF U_1 is $A_{(i)1}$ and U_2 is $A_{(i)2}$ and U_3 is $A_{(i)3}$
THEN V_1 is $B_{(i)1}$
ALSO
...
ALSO
IF U_1 is $A_{(n)1}$ and U_2 is $A_{(n)2}$ and U_3 is $A_{(n)3}$
THEN V_1 is $B_{(n)1}$ (1)

where U_1, U_2 and U_3 are the input variables and V_1 is the output variable of the fuzzy control process. $A_{(i)1}$, $A_{(i)2}$, $A_{(i)3}$ and $B_{(i)1}$, where i denotes the rule number $i=(1, ..., n)$ are linguistic values (levels) represented as fuzzy subsets of the respective universe of discourse X_1, X_2, X_3 and Y_1.

In the case of single-input and single-output fuzzy rule (IF U is A_i THEN V is B_i), the fuzzy relation R_i is interpreted as a fuzzy intersection of the fuzzy sets A_i and B_i:

$$R_i = A_i \cap B_i \tag{2}$$

R_i is defined on the Cartesian product space $X \times Y$, and is characterized by a membership function μ_R :

$$R_i(x, y) = A_i(x) \wedge B_i(y)$$

and,

$$\mu_R \times Y \rightarrow [0,1] \tag{3}$$

where ($\wedge$) is the min-operator (or intersection operator).

Fuzzy relation R_i associated with the individual relations are aggregated using fuzzy union :

$$R = \bigcup_{i=1}^{n} R_i \tag{4}$$

The membership function of the fuzzy relation R is :

$$\mu_{R(x,y)} = \vee_i R_i(x,y) = \bigvee_{i=1}^{n} R_i(x,y) = \bigvee_{i=1}^{n} (A_i(x) \wedge B_i(y)) \tag{5}$$

where ($\vee$) is the max-operator. Therefore, for a given a fuzzy relation R from U to V and for given fuzzy values of the input A, the fuzzy output B is defined by the max-min compositional rule of inference with union operators :

$$B = A_o R = A_o \left(\bigcup_{i=1}^{n} R_i \right) = \bigcup_{i=1}^{n} (A_o R_i)$$

or,

$$\mu_B(y) = \max_x \{\min(\mu_A(x), \mu_R(x,y))\} \tag{6}$$

where the symbol 'o' represents a general method for max-min composition of fuzzy relations.

This idea will be extended to multiple-input, single-output fuzzy systems. The multivariable linguistic description (1) is expressed as a fuzzy relation which is interpreted as the conjuction of the respective reference fuzzy sets :

$$R_i^j = A_{ij} \cap B_i^k \tag{7}$$

where i denotes the rule number $i = (1, ..., n)$, j denotes the input variables $j = (1,2,3)$, and k denotes the output variable $k = 1$.

By applying the rule of inference (6) to each of the subsystems of the three-input one-output system, the output B^1 is obtained as follows :

$$B^l = \bigcup_{i=1} (A_1, A_2, A_3)_o R_i^j \tag{8}$$

In analogue to the theory of linear systems, B^1 can be expressed in the following form of fuzzy equation

$$B^l = A_{1o} R_i^1 \wedge A_{2o} R_i^2 \wedge A_{3o} R_i^3 \tag{9}$$

Here R is the three-dimensional fuzzy matrix ; B^1 is the one-dimensional fuzzy output ; and these are decomposed into three one-dimensional fuzzy matrices (i.e., R_i^1, R_i^2, R_i^3) and one-dimensional fuzzy output B.

Using the vector-matrix notation, the set of fuzzy equations in (10) permits a simplified decomposed expression for the individual output B of the multiple variable (three-input, one-output) fuzzy control system :

$$[B] = \bigcup_{i=1}^{n} [A_1 A_2 A_3] * \begin{bmatrix} R_i^l \\ R_i^2 \\ R_i^3 \end{bmatrix} \tag{10}$$

where the symbol (*) is the operator (o, $\wedge$)

Generally, the membership function of B can be calculated by the max-min operation. Then the maximum of B determines the final output B. In this case, we can assume that the first and subsequent rules can be decomposed into j separate sub-relations. (see 7)

To obtain the overall jth sub-rule, we can unite all contributions (see 4) :

$$R^j = \bigcup_{i=1}^{n} R_i^j \tag{11}$$

or, by replacing the union operators with fuzzy max operators :

$$R^j = \max\{R_1^j(x_j, y), R_2^j(x_j, y), \cdots, R_n^j(x_j, y)\} \tag{12}$$

where $x \in A$ and $y \in B$.

In this research, the final output B related to set of three inputs (A_1, A_2, A_3) can be obtained using the max-min operation of all three (j = 1,2,3) relations among the three inputs and the relations R^1, R^2 and R^3.

$$B_i = A_{1o} R_i^1, \ A_{2o} R_i^2, \ A_{3o} R_i^3 \tag{13}$$

'o' denotes the usual max-min composition.

In a more explicit equation form,

$$\begin{aligned} B &= \max [\min\{A_1, R^1(x_1, y)\}, \\ &\quad \min\{A_2, R^2(x_2, y)\}, \min\{A_3, R^3(x_3, y)\}] \end{aligned} \tag{14}$$

where $B = \max[B_i]$.

Therefore, the corresponding membership function is defined as follows :

$$\mu_B(y) = \max \min\{\mu_{A1}(x_1) \times \mu_{A2}(x_2) \times \mu_{A3}(x_3), \mu_R(x_1, x_2, x_3, y)\} \tag{15}$$

$x_1 \in A_1$, $x_2 \in A_2$, $x_3 \in A_3$,

Let us denote this fuzzy clause by R. The Membership function of fuzzy clause R is given by :

$$\mu_R = \max \{\mu_{B_1}(x_1, x_2, x_3, y),\ \mu_{B_2}(x_1, x_2, x_3, y), \ldots, \mu_{B_{27}}(x_1, x_2, x_3, y)\} \tag{16}$$

Then the maximum of B_i ($i = 1,\ldots,27$) determines B, which is calculated as a union :

$$B = \bigcup_{i=1}^{27} B_i = \max(B_1, B_2, \ldots, B_{27}) \tag{17}$$

Finally, defuzzification the output is an operation that produces a nonfuzzy output action, a single crisp value B^*, that adequately represents the membership function μ_{agg} (B) of an aggregated fuzzy output. In this study, we describe the defuzzification method called mean of maximum (MOM) method which is simple to apply. We define to be the midpoint of the incident duration interval $[T_1, T_2]$, that is,

$$B^* = \frac{T_1 + T_2}{2} \tag{18}$$

The three-input, one-output fuzzy system shows how to evaluate the contribution of each component to the overall performance of the system. The block diagram allows a readily visual examination as compared to the linguistic system. In the next chapter, the experimental freeway application of the model is discussed within this framework.

IV. Fuzzy System Application

This part presents the test results of freeway traffic incident duration algorithm using fuzzy logic. The test results show that the output of the fuzzy system for reasonableness and to develop further insight into the incident management problem.

In this study, Freeway Service Patrol (FSP) tow truck drivers' reports provided the incident data sources. The Los Angeles FSP is the largest dedicated truck patrol program in this country. The FSP is a joint program of the California Department of Transportation (Caltrans), the California Highway Patrol (CHP), and the Los Angeles Metropolitan Transportation Authority (MTA). The FSP has 164 tow trucks provided by 20 towing contractors patrolling 40 beats with a coverage of 393 centerline miles of freeway in Los Angeles county.(Caltrans, 1995)

The study area covers 16 miles of the Santa Monica Freeway (I-10), between the Bundy Drive on the west and the Route 60 @ 3rd street on the east. Two north-south freeways, the San Diego Freeway (I-405) and the Harbor Freeway (I-110), cross the study area. This area nearly coincides with the Smart Corridor project area.

The data was collected for 62 weekdays (between January 3, 1995 and March 31, 1995, except holidays), generally from 6:00 to 10:00 a.m. and from 2:30 to 7:00 p.m. Scantron cards (motorist assist form) of 2,981 incident cases were considered. Twelve items were used to evaluate the incident databases.(see Appendix B) For this study, the three important characteristics of the data were selected(Table 1) : (1) vehicle problems (out of gas, electrical problem, debris removal, over heated, vehicle fire, mechanical problem, flat tire, locked out, other, accident, abandoned and unknown) ; (2) vehicle assisted types (auto, van, pickup, truck less than 1 ton, truck more than 1 ton, motorcycle, big rig, no assist due to oversize and other) ; and (3) disabled vehicle locations.(in freeway lanes, on left shoulder, on right shoulder, on a ramp, other and unable to locate)

In this study, an incident database was used to arrive at the figures of incident activity on the Santa Monica(I-10) Freeway. Table 2 shows the three key incident characteristics for vehicle problems, vehicle assisted types and disabled vehicle locations. These factors are input variables. Most vehicle problems are due to three reasons.(i.e., mechanical problem, flat tire, out of gas) Seventy-one percent of the types of vehicle assisted are auto, and seventy-four percent of location of disabled vehicles are on the right shoulder of the freeway. The average incident durations for each of the variables are 21, 22.2 and 21.8 minutes respectively.

Table 3 shows incident percentage, average waiting time, average service time, and average incident duration by categorization of input variables using the fuzzy terms, . small, medium and large.(See Table 1)

It shows the average for each category. The lowest average waiting time (small label) is 5 minutes for the vehicle problems' group ; (large label), is 7.3 minutes for both the highest average waiting time vehicle problems' and vehicle assisted types' groups. With respect to location of disabled vehicles category, vehicles in freeway lanes represent the highest average service time and incident duration. (20.3 and 26.2 minutes respectively)

Figure 3 shows fuzzy variable ranges for each of (a) vehicles' problem rating (b) Vehicles' assisted type rating and (c) disabled vehicles' location rating categories, and gives the crisp range of outputs for each of the fuzzy variables --small, medium and large. The membership functions vary between 0 and 1.

The crisp range for incidents' duration fuzzy variables --very short, short, medium, long, very longe--are shown in Table 4. If incident duration is interpreted as linguistic variable, then its term set D(incident duration) could be D = (Very Short, Short, Medium, Long, Very Long) where each term in D(incident duration) is characterized by a fuzzy set in a universe of discourse U(time) = [0,50]. We might interpret "very short" as "a time below about 5 minutes," "short" as "a time close to 15 minutes," "medium" as "a time close to 25 minutes," "long" as "a time close to 35 minutes," and "very long" as "a time above about 45 minutes." Similarly, vehicle problems, vehicle assisted types, and disabled vehicle locations are considered by a fuzzy set in a universe of discourse U(percent) = [0,100]. These terms can be characterized as fuzzy sets whose membership functions are shown in Figure 4.

Table 1 : Categories of Input Variables for Fuzzy Terms

Input Variables		Fuzzy Terms
Vehicles' Problem Rating	Mechanical Probem, Flat Tire, Out of Gas	Large
	Electrical Problem, Abandoned, Over Heated, Accident, Other, Unknown	Medium
	Debris Removal, Vehicle Fire, Locked Out	Small
Vehicles' Assisted Type Rating	Auto	Large
	Pickup, Van	Medium
	Truck more than 1 ton, Other, Big Rig, Truck less than 1 ton, Motorcycle, No assist due to oversize	Small
Disabled Vehicles' Location Rating	On Right Should	Large
	In Freeway Lanes, On Left Shoulder	Medium
	On A Ramp. Other, Unable to Locate	Small

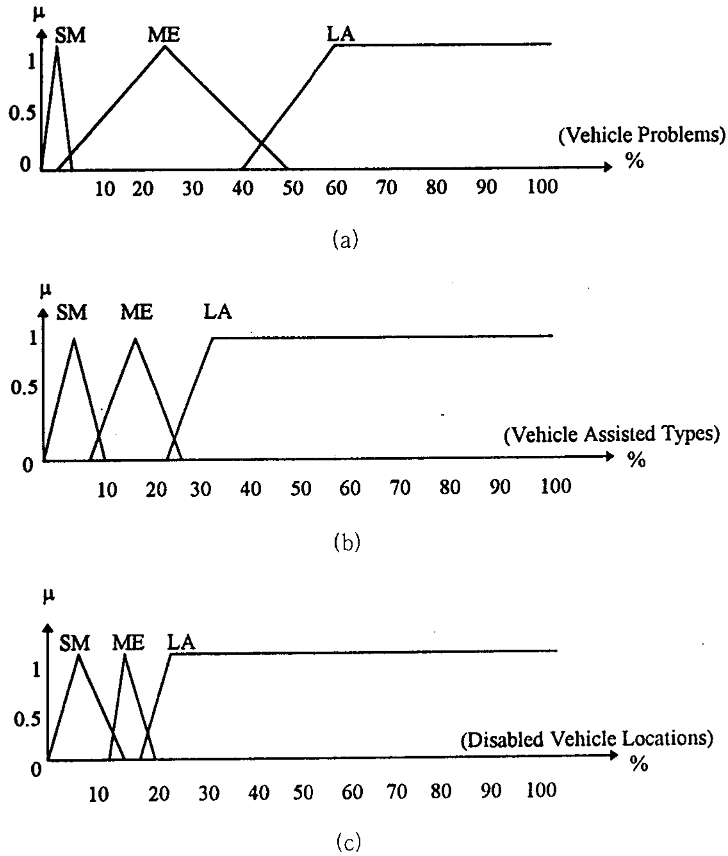

Figure 3 : Fuzzy Membership Functions for
(a)Vehicles' Problem Rating, (b)Vehicles' Assisted Type Rating, (c)Disabled Vehicles' Location Rating

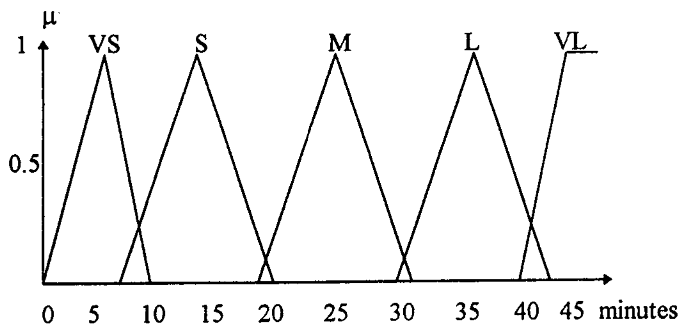

Figure 4 : Fuzzy Membership Functions for Incident Duration

Table 2 : Three Key Incident Characteristics

(1) Vehicles' Problem	Number of Incidents	Percentage (%)	Avg. Waiting Time (min.)	Avg. Service Time (min.)	Avg. Incident Duration (min.)
Out of Gas	423	14.2	6.7	11.1	17.8
Electrical Problem	337	11.3	9.1	17.7	26.8
Debris Removal	18	0.6	2.9	9.7	12.6
Over Heated	215	7.2	7.6	16.4	24
Vehicle Fire	9	0.3	5.3	16.9	22.2
Mechanical Problem	617	20.7	8.1	20	28.1
Flat Tire	527	17.7	7.2	14.6	21.8
Locked Out	3	0.1	6.7	20	26.7
Other	203	6.8	4.9	14.8	19.7
Accident	212	7.1	7.1	19.3	26.4
Abandoned	226	7.6	2	9.6	11.6
Unknown	191	6.4	2	12.8	14.8
Avg., Total	Total (2,981)	Total (100)	Avg. (5.8)	Avg. (15.2)	Avg. (21)
(2) Type of vehicle Assisted	**Number of Incidents**	**Percentage (%)**	**Avg. Waiting Time (min.)**	**Avg. Service Time (min.)**	**Avg. Incident Duration (min.)**
Auto	2,149	72.1	6.2	15.5	21.7
Van	277	9.3	6.5	15.1	21.6
Pickup	391	13.1	5.9	15.3	21.2
Truck less than 1 ton	27	0.9	8.9	17.2	26.1
Truck more than 1 ton	63	2.1	6.8	15.9	22.7
Motorcycle	12	0.4	6.3	9.3	15.6
Big Rig	18	0.6	7.5	20.8	28.3
No assist due to oversize	3	0.1	10	11	21
Other	41	1.4	4.5	16.5	21
Avg., Total	Total (2,981)	Total (100)	Avg. (7)	Avg. (15.2)	Avg. (22.2)
(3) Location of Disabled Vehicle	**Number of Incidents**	**Percentage (%)**	**Avg. Waiting Time (min.)**	**Avg. Service Time (min.)**	**Avg. Incident Duration (min.)**
In Freeway Lanes	426	14.3	5.9	20.3	26.2
On Left Shoulder	128	4.3	8.4	19	27.4
On Right Shoulder	2,248	75.4	6.2	14.4	20.6
On A Ramp	104	3.5	6.1	15.3	21.4
Other	54	1.8	6.3	20.5	26.8
Unable to Locate	21	0.7	2.1	6.4	8.5
Avg., Total	Total (2,981)	Total (100)	Avg. (5.8)	Avg. (16)	Avg. (21.8)
Total Avg.	2,981	100	6.2	15.5	21.7

Table 3 : Percentage of Incidents and Average Times (Waiting, Service, and Duration) Categorized by 'Large', 'Medium' and 'Small' Labels*

Categories of Input Variables		Label	%	Avg. Waiting Time (min.)	Avg. Service Time (min.)	Avg. Incident Time (min.)
Vehicles' Problem	Mechanical Problem, Flat Tire, Out of Gas 〈1〉	Large	52.6	7.3	15.2	22.5
	Electrical Problem, 〈2〉 Abandoned, Over Heated, Accident, Other, Unknown	Medium	46.4	5.5	15.1	20.6
	Debris Removal, Vehicle Fire, Locked Out 〈3〉	Small	1	5	15.5	20.5
Vehicles' Assisted Type	Auto 〈4〉	Large	72.1	6.2	15.5	21.7
	Pickup, Van 〈5〉	Medium	22.4	6.2	15.2	21.4
	Truck more than 1 ton, Other, Truck less than 1 ton, Big Rig, Motorcycle, No assist due to Oversize 〈6〉	Small	5.5	7.3	15.1	22.4
Disabled Vehicles' Location	On Right Shoulder 〈7〉	Large	75.4	6.2	14.4	20.6
	In Freeway Lanes 〈8〉	Medium	14.3	5.9	20.3	26.2
	On Left Shoulder, On A Ramp, Other, Unable to Locate 〈9〉	Small	10.3	5.7	15.3	21

* Categories of input variables divided by fuzzy terms (see Table 1)

Table 4 : Fuzzy Variable Ranges for Incident Duration

Fuzzy Variables	Crisp Range of Incident Duration
Very Short	0 ~ 10 minutes
Short	8 ~ 22 minutes
Medium	20 ~ 32 minutes
Long	30 ~ 42 minutes
Very Long	Over 40 minutes

In this application, since each fuzzy rule is represented by a four-dimensional array, the fuzzy algorithm employed for the three incident characteristics to decide the incident duration is represented by the 27 fuzzy rules. (see Appendix Afor complete rules) For the initial rule matrix we define the rules to be the intuitive rules resulting from the trivial mapping between the three linguistic label(i.e., Small, Medium and Large) inputs and five linguistic label (i.e., Very Short, Short, Medium, Long and Very Long) outputs.

The establishment of the fuzzy relationship is important. In this study, the time of incident duration is defined according to vehicle problem (VP), vehicle assisted type (VA) and disabled vehicle location.(VL)

The input given by the user consists of several linguistic variables for some criteria, for example "IF 'vehicles' problem rating is small and vehicles' assisted type rating is small and disabled vehicles' location rating is small, then the time of incident duration is medium" is a fuzzy linguistic model, which has three input and five output linguistic variables values, that is, input (small, medium and big) and output (very short, short, medium, long and very long).

The first fuzzy rule (see appendix A) is

IF VP (vehicles' problem rating) = small

and VA (vehicles' assisted type rating) = small

and VL (disabled vehicles' location rating) = small

THEN ID (the incident duration) = medium

From Table 3 and Table 6 we have

IF $\mu_{small}(1) = 0$

$\mu_{small}(5.5) = 0$

$\mu_{small}(10.3) = 0.5$

THEN $\mu_{small}(25) = 0.9.$

Thus

$\mu_{R_1}(1, 5.5, 10.3, 25)$

$= \min\{\mu_{small}(1), \mu_{small}(5.5), \mu_{small}(10.3), \mu_{small}(25)\}$

$= \min\{0, 0, 0.5, 0.9\} = 0$

and we decide the maximum value of 27 fuzzy rules as a union.(see equation 17)

$$B = \max\{0,0,0,0,0,0,0,0,0,0.4,0.4,0.4,0.4,0.4,0.4,0.4,0.4,0.4,0,0,0,0,0,0,0,0,0\} = 0.4$$

Similarly total nine variables (small vehicles' problem rating, medium vehicles' problem rating, large vehicles' problem rating, small vehicles' assisted type rating, medium vehicles' assisted type rating, large vehicles' assisted type rating, small disabled vehicles' location rating, medium disabled vehicles' location rating, and large disabled vehicles' location rating) are calculated and formulated by membership functions.

In the final step, the output is operation defuzzification that produces a non fuzzy output action using mean of maximum (MOM) method. The MOM method calculates the arithmetic mean of all values with maximum membership.(see equation 18)

A total of 9 maximum values are applied to fuzzy membership functions for incident duration, having determined the membership functions corresponding to the incident duration for each maximum value. They are points of intersection of the lines $\mu = 0.2$, $\mu = 0.4$, $\mu = 0.5$, $\mu = 0.8$, and $\mu = 0.9$ and the fuzzy variables. (small, medium, lareg)

Solving together equations

(1) μ=0.4 (small vehicles' problem rating)

(2) $\mu = 0.8$ (medium vehicles' problem rating)

(3) $\mu = 0.9$ (large vehicles' problem rating)

(4) $\mu = 0.2$ (small vehicles' assisted type rating)

(5) $\mu = 0.4$ (medium vehicles' assisted type rating)

(6) $\mu = 0.9$ (large vehicles' assisted type rating)

(7) $\mu = 0.5$ (small disabled vehicles' location rating)

(8) $\mu = 0.9$ (medium disabled vehicles' location rating)

(9) $\mu = 0.9$ (large disabled vehicles' location rating)

we get correspondingly

(1) (5+16+27+37+44)/5 = 25.8 minutes

(2) (5+16+27+37+46)/5 = 26.2 minutes
(3) (5+16+27+37+46.5)/5 = 26.3 minutes
(4) (5+16+27+37+43)/5 = 25.6 minutes
(5) (5+16+27+37+44)/5 = 25.8 minutes
(6) (5+16+27+37+46.5)/5 = 26.3 minutes
(7) (5+16+27+37+44.5)/5 = 25.9 minutes
(8) (5+16+27+37+46.5)/5 = 26.3 minutes
(9) (5+16+27+37+46.5)/5 = 26.3 minutes

V. Conclusion

The development of an effective incident management on urban freeways has become an important part of the transportation system operation. The objective of a freeway incident management strategy is to focus on minimizing congestion caused by incidents

For this study, the main inputs to the model are incident data such as FSP (Freeway Service Patrol) tow truck drivers' reports. Incident data were selected : (1) twelve vehicle problems (out of gas, electrical problem, debris removal, over heated, vehicle fire, mechanical problem, flat tire, locked out, other, accident, abandoned and unknown) ; (2) nine vehicle assisted types (auto, van, pick-up, truck less than 1 ton, truck more than 1 ton, motorcycle, big rig, no assist due to oversize, and other) ; and (3) six disabled vehicle locations (in freeway lanes, on left shoulder, on right shoulder, on a ramp, other and unable to locate). This study has developed a fuzzy logic model for incident duration in freeway traffic after an incident occurs. Thus, the IF-THEN fuzzy rules for incident verification (incident characteristics) and response(incident duration) were developed.

The key elements of the model are : (1) the structure of the three inputs and one output linguistic variables ; (2) the characterization of simple relation between variables treated by the 27 fuzzy rules (conditional fuzzy statements) ; (3) the estimation of complex relations by the fuzzy algorithm. This linguistic model based on collections of IF-THEN rules using approximate (fuzzy) reasoning.

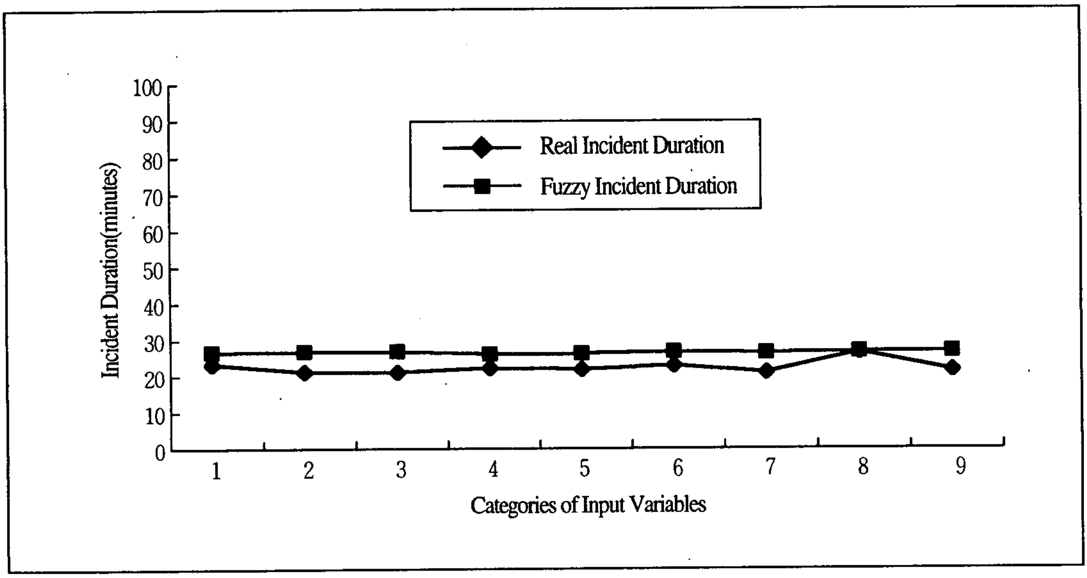

Figure 5 : Comparison of Real and Fuzzy Incident Duration

The linguistic model is a new attempt to explain the phenomenon of freeway incident duration when there are alternative incident characteristics. It is based on fuzzy logic, due to the fact that the selection process in transportation is very often characterized by uncertainty and ambiguity.

The model output includes measures of freeway incident duration. It has achieved using a membership function which alternatively calculated the times of incident duration on the freeway and compared them with real incident duration.(Figure 5) The result obtained using the fuzzy logic were good agreement with real incident duration. These results are reasonable predictions of the incident duration in a freeway network, when the input to the model is comprised of current and past information regarding incident conditions on the freeway.

The linguistic fuzzy model has following advantages:

1. Unlike other models, the linguistic fuzzy model has highly interactive capability with the operator makes it more friendly than other models.
2. Unlike block-box programming, this model can explain its reasoning process through why or how queries from the operator.

Conclusively, this study shows that fuzzy logic approach could be applied to other problems regarding dispatch system in transportation.

APPENDIX A : Fuzzy Rules

Rule 1

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Medium
ALSO

Rule 2

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Long
ALSO

Rule 3

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Short
ALSO

Rule 4

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Very Short
ALSO

Rule 5

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Short
ALSO

Rule 6

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Very Short
ALSO

Rule 7

IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Short
ALSO

Rule 8
IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Medium
ALSO

Rule 9
IF Vehicles' Problem Rating is Small
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Very Short
ALSO

Rule 10
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Long
ALSO

Rule 11
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Very Long
ALSO

Rule 12
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Medium
ALSO

Rule 13
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Short
ALSO

Rule 14
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Medium
ALSO

Rule 15
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Very Short
ALSO

Rule 16
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Medium
ALSO

Rule 17
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Big

and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Long
ALSO

Rule 18
IF Vehicles' Problem Rating is Medium
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Short
ALSO

Rule 19
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Very Long
ALSO

Rule 20
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Very Long
ALSO

Rule 21
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Small
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Long
ALSO

Rule 22
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Medium
ALSO

Rule 23
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Long
ALSO

Rule 24
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Medium
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Short
ALSO

Rule 25
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Small
THEN the Time of Incident Duration is Long
ALSO

Rule 26
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Medium
THEN the Time of Incident Duration is Very Long
ALSO

Rule 27
IF Vehicles' Problem Rating is Big
and Vehicles' Assisted Type Rating is Big
and Disabled Vehicles' Location Rating is Big
THEN the Time of Incident Duration is Medium

APPENDIX B

Freeway Service Patrol SurveyData (Motorist Assist Form) : Santa Monica Freeway (I-10) in the Los Angeles

[1] Incident service arrived time (hours)
[2] Incident service arrived time (minutes)
[3] Incident service departed time (hours)
[4] Incident service departed time (minutes)
[5] How long did motorist wait for the Freeway ServicePatrol? (minutes)
[6] Did you tow vehicle to :
(1) Shoulder (2) Off Freeway (3) No Tow
[7] Did the motorist need additional assistance?
(1) Yes (2) No
[8] At what speed was traffic traveling prior to this assist?
(1) under 20 mph (2) 21 to 30 mph
(3) 31-40 mph (4) above 40 mph
[9] Problem with the vehicle :
(1) Out of Gas (2) Electrical Problem
(3) Debris Removal (4) Over Heated
(5) Vehicle Fire (6) Mechanical Problem
(7) Flat Tire (8) Locked Out
(9) Other (10) Accident
(11) Abandoned (12) Unknown
[10] Type of vehicle assisted :
(1) Auto (2) Van
(3) Pickup (4) Truck less than 1 ton
(5) Truck more than 1 ton
(6) Motorcycle (7) Big Rig
(8) No assist due to oversize
(9) Other
[11] Vehicle location was :
(1) Found By You (2) Dispatched By CHP
(3) Dispatched By Caltrans
[12] Disabled vehicle was :
(1) In Freeway Lanes (2) On Left Shoulder
(3) On Right Shoulder (4) On A Ramp
(5) Other (6) Unable to Locate

BIBLIOGRAPHY

California Department of Trans portation, Los Angeles County Metro Freeway Service Patrol : Scantron Report, California Department of Transportation, 1995.

Giuliano, G., "Incident characteristics, frequency, and duration on a high volume urban freeway," Transportation Research A, 23A(5):387-396, 1989.

Gordon, P. and Richardson, H. W., "Congestion trends in metropolitan areas," Curbing Gridlock : Peak-Period Fees to Relieve Traffic Congestion, In Transportation Research Board Special Report 242, Volume 2, Washington,D.C.: National Academy Press, 1994.

Grenseback, L. and Woodle, C., "The true costs of highway congestion," ITE Journal, 62(3):16-20, 1992.

Gupta, M. M., Kiszka, J. B., and Trojan, G. M., "Multivariable Structure of Fuzzy Control Systems," IEEE Transactions on Systems, Man, and Cybernetics, 16(5):638-656, 1986.

Gupta, M. M. and Yamakawa, T. (Eds.), Fuzzy Logic in Knowledge-based Systems, Decision and control, Amsterdam: North-Holland Publishing Company, 1987.

Judycki, D. C. and Robinson, J., "Freeway incident manage ment," Compendium of Technical Papers, Washington,D.C.: Insti tute of Transportation Engineers, pp. 161-165, 1988.

Kosko, B., Fuzzy Thinking, New York: Hyperion, 1993.

Lindley, J. A., "Urban freeway congestion : quantification of the problem and effectiveness of potential solutions," ITE Journal, 57:27-32, 1987.

Mamdani, E. H., "Advances in the linguistic synthesis of fuzzy controllers," International Journal of Man-Machine Studies, 8(6):669-678, 1976.

Mamdani, E. H. and Assilian, S., "An experiment in Linguistic Synthesis with a fuzzy logic controller," International Journal of Man-Machine Studies, 7(1):1-13, 1975.

Pedrycz, W., Fuzzy Control and Fuzzy Systems, New York: John Wiley & Sons Inc., 1989.

Sanchez, E., "Solutions in composite fuzzy relation equations," In Gupta, M. M., Saridis, G. N., and Gaines, B. R., (Eds.), Fuzzy Automata and Decision Processes, New York: Elsevier North-Holland, 1977.

Sanchez, E., "Composion of fuzzy relations," In Gupta, M. M., Ragade, R. K., and Yager, R. R., (Eds.), Advances in Fuzzy Set Theory and Applications, Amsterdam: North-Holland, pp. 421-433, 1979.

Sugeno, M. and Kang, G. T., "Fuzzy modeling and control of multilayer incinerator," Fuzzy Sets and Systems, 18:329-341986.Takagi, T. and Sugeno, M., "Derivation of fuzzy control rules from human operators actions," Proceedings of the IFAC Symposium on Fuzzy Information, Marseille, pp. 50-60, 1983.

Takagi, T. and Sugeno, M., "Fuzzy identification of systems and its application to modeling and control," IEEE Transactions on Systems, Man, and Cybernetics, 15:116-132, 1985.

Tong, R. M., "The construction and evaluation of fuzzy models," In Gupta, M. M., Ragade, R. K., and Yager, R. R. (Eds.), Fuzzy Set Theory and Applications, Amsterdam: North-Holland, pp. 559-576, 1979.

U.S. Department of Transportation, Transportation Statistics Annual Report, 1994, Washington, D.C.: Bureau of Transportation Statistics, 1994.

Zadeh, L. A., "Fuzzy Algorithm," Information and Control, 12:94-102, 1968.

Zadeh, L. A., "Similarity relations and fuzzy orderings," Information Science, 3:177-200, 1971.

Zadeh, L. A., "Outline of a new approach to the analysis complex systems and decision processes," IEEE Transactions on Systems, Man, and Cybernetics, SMC-3:28-44, 1973.

Zadeh, L. A., " The concept of a linguistic variable and its application to approximate reasoning: Part 1," Information Sciences, 8:199-249, 1975.