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Development and Comparisons of Hybrid and Failure-Free Period Acceptance Sampling Plans for Gamma Lifetime Distributions

정현석*, 박정원**, 염봉진**

Hyun-Seok Jeong*, Jung-Won Park**, Bong-Jin Yum**

Abstract

In this paper, we develop two replacement-type reliability acceptance sampling plans (RASPs) for the gamma lifetime distribution assuming that the shape parameter is known. The two plans are respectively based upon failure-free period and hybrid life tests. We then compare the plans in terms of expected test time to reach a decision, power, etc. Computational results indicate among others that the failure-free period RASP has a shorter expected completion time than the corresponding hybrid RASP when the true scale parameter is 'large'. Finally, sensitivity analyses reveal that the effects of the uncertainties involved in the assumed shape parameter on the producer and the consumer risks are in favorable directions for both parties for both types of plans.

1. Introduction

A reliability acceptance sampling plan(RASP) consists of a set of life test procedures and rules for either accepting or rejecting the item(s) on test based upon the sampled lifetime data. While RASPs for the exponential lifetime distribution have been extensively developed in the literature[1, 4, 5, 6, 8], little work has been done for the gamma lifetime distribution. One exception is Gupta and Groll[9] in which a non-replacement type RASP is developed under hybrid censoring.

A gamma lifetime distribution may arise if

* 삼성데이타시스템

** 한국과학기술원 산업공학과

an item subject to a given environment fails only when it has experienced k shocks which occur at a Poisson rate[3]. According to Kitagawa[11], some pressure valves or switches possess such a property. The gamma distribution was also derived by Birnbaum and Saunders[2] as one of the statistical models for the life-length of materials. An example in their article shows that the lifetime of a certain type of aluminum strips can be adequately described by a gamma distribution. Wilk et al. [13] also describe the lifetimes of a certain type of transistors using a gamma distribution.

Recently, Angus *et al.*[1] proposed a replacement type RASP(for the exponential lifetime distribution) in which the item on test is accepted only if a specified length of failure-free period is observed before a specified number of failures occur, and showed that it has smaller expected test time than the time-truncated replacement test(i.e., replacement test under hybrid censoring) when the true mean lifetime is large relative to the value specified in the null hypothesis. In this article, we develop a similar RASP for the gamma lifetime distribution to test the hypotheses on the scale parameter with the shape parameter assumed known. We also develop an RASP under the assumptions of hybrid censoring and replacement for the gamma lifetime distribution. The hybrid censoring scheme was first proposed by Epstein[5] for the exponential distribution. The two plans are tabulated for various combinations of parameters and compared in terms of expected number of failed items, expected test time required to reach a decision, and power.

For the RASPs developed in this article to be operational, we need information on the shape parameter of a gamma distribution, which may be obtained from the historical data on a variety of similar products, engineering knowlegde, experience, etc. If this is not feasible, a preliminary test could be conducted to estimate the shape parameter. In Section 6, sensitivities of the producer's and the consumer's risks to the uncertainties involved in the shape parameter are also evaluated for both plans.

2. Preliminaries

Notation and Acronyms

- θ scale parameter of a gamma distribution.
- θ_0, θ_1 specified values of the scale parameter under H_0 and H_1, respectively.
- k shape parameter of a gamma distribution (assumed known).
- X random variable which denotes the lifetime of a test item.
- $g(x;\theta, k)$ probability density function of a gamma random variable X.
- $G(x;\theta, k)$ cumulative distribution function of a gamma random variable X.
- r_f preassigned number of failures for a failure-free period RASP.
- r_h preassigned number of failures for a hybrid RASP.

t_f prespecified failure-free period.

t_h censoring time for a hybrid RASP.

$N(t)$ number of failures at or before time t.

R random variable which denotes the number of failures until a decision can be reached.

W random variable which denotes the time to reach a decision.

$E_\theta(R)$ expected value of R when the value of the scale parameter is θ.

$E_\theta(W)$ expected value of W when the value of the scale parameter is θ.

$L(\theta)$ operating characteristic function of θ.

OC operating characteristic.

pdf probability density function.

cdf cumulative distribution function.

pmf probability mass function.

RASP reliability acceptance sampling plan.

In the above, all the time-related variables and parameters are standardized ones with respect to the specified value of the original scale parameter under H_0.

Assumptions

(1) The lifetimes of test items are independent and follow a gamma distribution.

(2) Each failed item is replaced by or repaired as a new one.

Hypothesis Test

Suppose that the lifetime of a test item follows a gamma distribution with *pdf*:

$$g(x';\theta',k) = \begin{cases} [1/\{\Gamma(k)(\theta')^k\}](x')^{k-1}e^{-x'/\theta'}; \\ x'>0,\ k>0,\ \theta'>0 \\ 0,\ otherwise. \end{cases}$$

For a hybrid or a failure-free period life test, an item is drawn at random from the above population and placed on test at time 0. In a hybrid life test, the test item failed before a prespecified censoring time or before a prespecified number of failures occur is replaced by or repaired as a new one. If a failure-free period life test is employed, the item failed before a failure-free period is reached is also restored to a new working condition either by repair or replacement. Then, based upon the failure data from a life test we want to test the following hypotheses on the scale parameter θ', with shape parameter k assumed known.

$$H_0 : \theta' = \theta'_0$$
$$H_1 : \theta' = \theta'_1 (< \theta'_0) \qquad (2.1)$$

where θ'_0 and θ'_1 are prespecified. Since the mean of a gamma distribution is proportional to θ' (with k assumed known), the above test is equivalent to the test on the mean lifetime.

For simplicity, we employ the following transformation :

$$\theta = \theta' / \theta'_0.$$

Under the above transformation, (2.1) is reduced to :

$H_0 : \theta = \theta_0 (=1)$

$H_1 : \theta = \theta_1 (= \theta'_1/\theta'_0 \langle 1).$ (2.2)

All the variables and parameters related to time are also standardized with respect to θ'_0. For instance, $X = X' / \theta'_0$, $t_f = t'_f/\theta'_0$, *etc*. From now on, the time-related variables and parameters are assumed to be standardized and used without a prime.

3. RASP Under Failure-Free Period Life Test

Development of RASP

Under a failure-free period life test, H_0 is accepted if and only if a failure-free period of length t_f is obtained before r_f failures occur ($r_f = 1,2,\cdots$). That is, if the number of failures prior to achieving a failure-free period t_f is less than or equal to r_f-1, H_0 is accepted. If we regard the event that a failure-free period of t_f is achieved as a 'success', then the number of 'failures' prior to a success can be described by a geometric distribution with success probability :

$$p_\theta = \int_{t_f}^{\infty} g(x;\theta,k)dx$$

$$= 1\text{-}G(t_f;\theta,k).$$

Then, $L(\theta)$ for the test (2.2) is given by

$$L(\theta) = P(accept H_0 | \theta) = \Sigma_{r=0}^{r_f-1} p_\theta (1-p_\theta)^r$$

$$= 1-(1-p_\theta)^{r_f}.$$

We want to find r_f and t_f such that the producer's and the consumer's risks are satisfied. That is,

$$L(1) = 1-(1-p_1)^{r_f} = 1-\alpha \quad (3.1)$$

$$L(\theta_1) = 1-(1-p_{\theta_1})^{r_f} = \beta. \quad (3.2)$$

where $p_1 = p_\theta$ when $\theta = \theta_0 = 1$ and $p_{\theta_1} = p_\theta$ when $\theta = \theta_1$.

Eqs. (3.1) and (3.2) can be solved iteratively for t_f and r_f given α and β, although it is not generally possible to satisfy both equations exactly due to the discreteness of r_f. Therefore, the smallest r_f and the corresponding t_f are determined such that $L(1) = 1-\alpha$ and $L(\theta_1) \le \beta$. The corresponding procedures are as follows.

Step 1. Specify α, β, k and θ_1.

Step 2. Set the initial value of r_f equal to 10.

Step 3. Determine t_f such that (3.1) is satisfied. If (t_f, r_f) satisfy the consumer's risk, go to Step 5. Otherwise, go to Step 4.

Step 4. Set $r_f = r_f + 10$ and go to Step 3.

Step 5. Set $r_f = r_f$-9.

Step 6. Determine t_f such that (3.1) is satisfied. If (t_f, r_f) satisfy the consumer's risk, stop. The desired (t_f, r_f) are found. Otherwise, go to Step 7.

Step 7. Set $r_f = r_f + 1$ and go to Step 6.

Values of t_f in Steps 3 and 6 can be determined by a numerical search technique.

We used the bisection method[11] utilizing the fact that $L(\theta)$ is a decreasing function of t_f. The *cdf* of a gamma random variable was evaluated using IMSL[10] subroutine DGAMDF. The above procedure was programmed in MS Fortran, and run on a 32-bit personal computer in double precision. Plans are presented in Table 3.1 for the following combinations of parameters.

$$\begin{aligned} &\alpha=0.01,\ 0.05,\ 0.10 \\ &\beta=0.01,\ 0.05,\ 0.10 \\ &k=1/2,\ 2.0,\ 3.0,\ 4.0,\ 5.0,\ 10.0 \\ &\theta_1=1/2,\ 1/3,\ 1/5 \end{aligned} \tag{3.3}$$

The computing time varies from plan to plan with the maximum being 9 sec. for the case where $\alpha=0.01$, $\beta=0.01$, $k=1/2$, and $\theta_1=1/2$.

From Table 3.1, we observe the following. First, as the consumer's and the producer's risks increase, r_f and t_f decrease. Second, as the discrimination ratio $\theta_1(=\theta'_1/\theta'_0)$ decreases, r_f and t_f also decrease. Finally, as the shape parameter increases, r_f decreases while t_f increases.

Related Performance Measures

The waiting time to reach an acceptance of H_0 is a random variable W which is equal to t_f if no failure occurs before a failure-free period of t_f and is equal to $(X_1+\cdots+X_i+t_f)$ if $i(1\leq i\leq r_f-1)$ failures (with $X_i\langle t_f)$ occur before a failure-free period of t_f. Similary, the waiting time W to reach a rejection of H_0 is given by $(X_1+\cdots+X_{r_f})$ where all X's are less than t_f. To calculate $E_\theta(W)$, we need to determine the truncated mean lifetime of X. It is shown in Appendix that

$$\begin{aligned} \mu_\theta &= E_\theta(X\mid X\langle t_f) \\ &= k\theta G(t_f;\theta,k+1)/G(t_f;\theta,k). \end{aligned} \tag{3.4}$$

Let A_i be the event that $i(1\leq i\leq r_f-1)$ failures (with $X_i\langle t_f)$ occur before a failure-free period of t_f, and B be the event that r_f failures occur with all X_i's$(i=1,2,\cdots,r_f)$ less than t_f. Then, $E_\theta(W)$ is given by

$$\begin{aligned} E_\theta(W) &= \textstyle\sum_{i=0}^{r_f-1}E_\theta(W\mid A_i)P(A_i)+E_\theta(W\mid B)P(B) \\ &= \textstyle\sum_{i=0}^{r_f-1}(i\mu_\theta+t_f)(1-p_\theta)^i p_\theta+r_f\mu_\theta(1-p_\theta)^{r_f}. \end{aligned} \tag{3.5}$$

Let R be the number of failures until a decision (either accepting or rejecting H_0) is reached. Then, it is shown in Appendix that

$$E_\theta(R)=\{(1-p_\theta)/p_\theta\}\{1-(1-p_\theta)^{r_f}\}. \tag{3.6}$$

4. RASP Under Hybrid Life Test

Development of RASP

In a hybrid life test with replacement, a test item is placed on test at time 0 and failed items are replaced by or repaired as a new one at once. The decision rule for a hybrid RASP is to reject H_0 if r_h failures are observed before t_h, and accept H_0, otherwise. Values of r_h and t_h are determined such that the producer's and the consumer's risks are satisfied.

Table 3.1. RASPs under failure-free period life test.

α	β	θ_1 \ k	0.5		2.0		3.0		4.0		5.0		10.0	
0.01	0.01	1/2	7237[a]	5.828[b]	517	6.766	205	7.375	107	8.000	65	8.625	15	11.750
		1/3	264	2.828	33	3.531	17	4.000	11	4.500	8	5.000	3	7.375
		1/5	46	1.375	8	1.875	5	2.250	3	2.344	3	3.156	1	4.125
	0.05	1/2	1236	4.203	121	5.094	59	5.719	34	6.281	23	6.875	7	9.750
		1/3	103	2.031	16	2.688	9	3.094	6	3.500	5	4.125	2	6.188
		1/5	26	0.969	5	1.359	3	1.594	3	2.344	2	2.406	1	4.125
	0.10	1/2	563	3.500	66	4.375	34	4.969	21	5.531	15	6.125	5	8.875
		1/3	65	1.656	11	2.250	7	2.719	5	3.188	4	3.688	2	6.188
		1/5	20	0.797	4	1.125	3	1.594	2	1.719	2	2.406	1	4.125
0.05	0.01	1/2	2921	5.391	226	6.320	95	6.938	51	7.547	32	8.188	8	11.250
		1/3	134	2.617	17	3.273	9	3.711	6	4.203	5	4.938	2	7.500
		1/5	25	1.254	5	1.836	3	2.164	2	2.406	2	3.219	1	5.406
	0.05	1/2	493	3.766	54	4.648	27	5.250	16	5.789	11	6.367	4	9.453
		1/3	51	1.809	9	2.516	5	2.875	4	3.539	3	3.984	1	5.406
		1/5	14	0.848	3	1.281	2	1.625	2	2.406	1	1.969	1	5.406
	0.10	1/2	222	3.055	29	3.906	15	4.430	10	5.039	7	5.555	3	8.656
		1/3	32	1.441	6	2.047	4	2.563	3	3.063	2	3.219	1	5.406
		1/5	11	0.691	3	1.281	2	1.625	1	1.359	1	1.969	1	5.406
0.10	0.01	1/2	1691	5.129	137	6.047	59	6.656	33	7.297	21	7.922	6	11.203
		1/3	89	2.492	12	3.172	7	3.727	4	3.977	3	4.477	2	8.250
		1/5	18	1.207	4	1.883	2	1.969	2	2.836	1	2.430	1	6.219
	0.05	1/2	284	3.508	33	4.375	17	4.973	11	5.609	7	6.031	3	9.391
		1/3	33	1.672	6	2.352	4	2.934	3	3.500	2	3.711	1	6.219
		1/5	10	0.801	2	1.141	2	1.969	1	1.742	1	2.430	1	6.219
	0.10	1/2	127	2.797	18	3.656	10	4.234	6	4.641	5	5.422	2	8.250
		1/3	21	1.320	41	1.883	3	2.527	2	2.836	2	3.711	1	6.219
		1/5	8	0.660	2	1.141	1	1.102	1	1.742	1	2.430	1	6.219

k : shape parameter   a : number of failures(r_f)   b : failure-free period(t_f)

$L(\theta)$ for a hybrid RASP can be determined as follows. Let $N(t)$ denote the number of failures at or before time t and $X_j (j=1,2,\cdots)$ denote the jth failure time. Since $S_i=\Sigma_{j=1}^{i} X_j$ is i-convolution of X, S_i follows a gamma distribution with shape parameter ik and scale parameter θ. We note that

$$P(N(t)\geq i)\leftrightarrow P(S_i\leq t).$$

Therefore, the *pmf* of $N(t)$ is given by

$$P(N(t)=i)=P(N(t)\geq i)-P(N(t)\leq i+1)$$
$$=P(S_i\leq t)-P(S_{i+1}\leq t)$$
$$=G(t;\theta,ik)-G(t;\theta,(i+1)k)$$
$$P(N(t)=0)=1-G(t;\theta,k).$$

The OC function is then given by

$$L(\theta)=P(accept\ H_0|\theta)$$
$$=\Sigma_{i=0}^{r_h-1}P(N(t_h)=i|\theta)$$
$$=1-G(t_h;\theta,k)+$$
$$\Sigma_{i=1}^{r_h-1}\{G(t_h;\theta,ik)-G(t_h;\theta,(i+1)k)\}$$

Values of r_h and t_h are determined to satisfy $L(1)=1-\alpha$ and $L(\theta_1)=\beta$. However, it may not be possible to obtain a plan which satisfies α and β risks exactly due to the discreteness of r_h. Therefore, the smallest r_h and the corresponding t_h are determined such that $L(1)=1-\alpha$ and $L(\theta_1)\leq\beta$. The procedures for determining such r_h and t_h for a hybrid RASP are similar to those for determining r_f and t_f for a failure-free period RASP, and therefore, not repeated here.

Plans under hybrid life testing are presented in Table 4.1 for the same combinations of parameters as in (3.3). The case where $\alpha=0.01$, $\beta=0.01$, $k=1/2$, and $\theta_1=1/2$ takes 37 sec. on a 32-bit personal computer, which is the maximum among all the cases considered. From Table 4.1 we observe that as the discrimination ratio decreases, r_h and t_h also decrease; as α and β errors increase, r_h and t_h decrease with several exceptions for t_h; and as k increases, r_h decreases, but t_h either increases or decreases.

Related Performance Measures

The expected number of failures to reach a decision is given by

$$E_\theta(R)=\Sigma_{i=0}^{r_h}iP(R=i)$$

where

$$P(R=i)=P(N(t_h)=i),\ i=0,1,\cdots,r_h.$$

Since we were unable to find a closed form expression for $E_\theta(W)$ for a hybrid RASP, it was estimated based on a simulation of 5000 trials for given α, β, k, and θ_1. Gamma random deviates were generated using IMSL[10] subroutines DRNGAM, RNSET, and RNGET.

5. Comparisons of Failure-Free Period and Hybrid RASPs

The two types of RASPs were first compared in terms of the expected completion time for the following combinations of parameter values.

$$(\alpha,\beta)=(0.01,\ 0.01),\ (0.01,\ 0.10),\ (0.05,\ 0.05),\ (0.10,\ 0.01),\ (0.10,\ 0.10) \quad (5.1)$$
$$k=1/2,\ 2,\ 3,\ 5,\ 10$$
$$\theta_1=1/2,\ 1/5.$$

For each combination, the expected completion time of the corresponding failure-free period RASP was calculated based upon Eq.

Table 4.1. RASPs under hybrid life test.

α	β	k / θ_1	0.5		2.0		3.0		4.0		5.0		10.0	
0.01	0.01	1/2	93[a]	32.000[b]	24	33.250	16	33.250	12	33.250	10	35.000	5	35.000
		1/3	38	10.250	10	11.000	17	11.750	5	11.000	4	11.000	2	11.000
		1/5	18	3.500	5	3.500	3	3.500	3	5.375	2	4.125	1	4.125
	0.05	1/2	71	23.000	18	23.500	12	23.500	9	23.500	8	26.750	4	26.750
		1/3	29	7.125	8	8.125	5	7.438	4	8.125	3	7.438	2	11.000
		1/5	15	2.594	3	2.875	3	3.500	2	2.875	2	4.125	1	4.125
	0.10	1/2	60	18.625	15	18.625	10	18.625	8	20.250	6	18.625	3	18.625
		1/3	25	5.750	7	6.750	5	7.438	4	8.125	3	7.438	2	11.000
		1/5	13	2.031	4	1.875	3	3.500	2	2.875	2	4.125	1	4.125
0.05	0.01	1/2	65	23.688	17	25.000	11	24.125	9	26.688	7	25.844	4	30.188
		1/3	26	7.688	7	8.453	5	9.234	4	10.031	3	9.234	2	13.250
		1/5	13	2.938	4	3.969	3	4.688	2	3.969	2	5.406	1	5.406
	0.05	1/2	47	16.125	12	16.531	8	16.531	6	16.531	5	17.375	3	21.563
		1/3	19	5.047	5	5.406	4	6.906	3	6.906	2	5.406	1	5.406
		1/5	10	1.969	3	2.609	2	2.609	2	3.969	1	1.969	1	5.406
	0.10	1/2	38	12.438	10	13.250	7	14.063	5	13.250	4	13.250	2	13.250
		1/3	16	3.969	4	3.969	3	6.688	2	3.969	2	5.406	1	5.406
		1/5	8	1.359	2	1.359	2	2.609	1	1.359	1	1.969	1	5.406
0.10	0.01	1/2	52	19.703	13	19.703	9	20.578	7	21.453	6	23.219	3	23.219
		1/3	21	6.609	6	7.828	4	7.828	3	7.828	3	10.297	2	14.516
		1/5	10	2.430	3	3.148	2	3.148	2	4.648	1	2.430	1	6.219
	0.05	1/2	36	12.813	9	12.813	6	12.813	5	14.516	4	14.516	2	14.516
		1/3	15	4.266	4	4.648	3	5.422	2	4.648	2	6.219	1	6.219
		1/5	7	1.414	2	1.742	2	3.148	1	1.742	1	2.430	1	6.219
	0.10	1/2	29	9.875	8	11.125	5	10.297	4	11.125	3	10.297	2	14.516
		1/3	12	3.148	3	3.148	2	3.148	2	4.648	2	6.219	1	6.219
		1/5	6	1.102	2	1.742	1	1.102	1	1.742	1	2.430	1	6.219

κ: shape parameter a: number of failures(r_h) b: censoring time(t_h)

(3.5) for some selected values of θ over the interval $[\theta_1, 2]$. For a hybrid RASP we estimated the expected completion time based upon a Monte Carlo simulation of 5000 trials.

Table 5.1 shows the approximate threshold values θ^* above which the failure-free period RASP has a shorter expected completion time than the hybrid RASP, and *vice versa* (see also, Figures 5.1 and 5.2 for typical cases).

For some combinations of parameter values, the corresponding θ^* lies outside the interval $[\theta_1, 2]$, and these cases are indicated in Table

Table 5.1. Threshold Values θ^* above which failure-free period RASP's yield smaller expected completion times.

α	β	θ_1	k=0.5	2	3	5	10
0.01	0.01	1/2	>2.0	1.8	1.5	1.3	1.1
		1/5	1.3	0.8	0.9	0.9	—*
0.01	0.10	1/2	1.7	1.2	1.1	1.0	0.85
		1/5	0.7	<0.2	<0.2	<0.2	—
0.05	0.05	1/2	>2.0	1.4	1.3	1.1	0.8
		1/5	0.8	<0.2	<0.2	—	—
0.10	0.01	1/2	>2.0	1.9	1.6	1.3	1.1
		1/5	1.6	0.9	0.3	—	—
0.10	0.10	1/2	1.9	1.2	1.2	1.1	<0.5
		1/5	1.2	<0.2	—	—	—

* denotes the cases where the two plans yield identical E_θ (W)

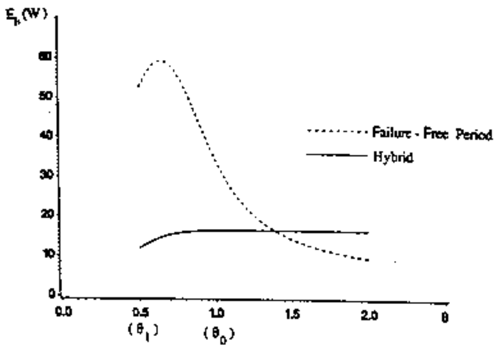

Figure 5.1 Expected completion time when $\alpha=\beta=0.05$, $\theta_1=0.5$, $k=2$

5.1 with appropriately directed inequalities. Note that θ^* tends to decrease as k increases and/or θ_1 decreases. From these results, we may conclude that the failure-free period RASP has a shorter expected completion time than the hybrid RASP when the true value of θ is 'large', or equivalently, when the true mean lifetime is large.

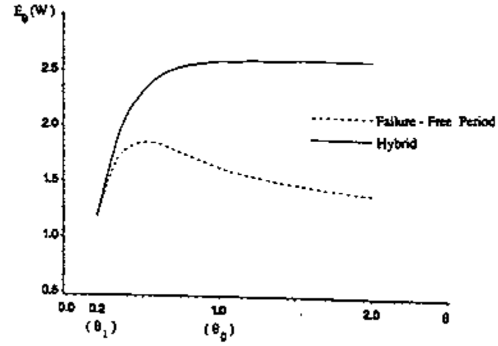

Figure 5.2 Expected completion time when $\alpha=\beta=0.05$, $\theta_1=0.2$, $k=2$

The behavior of $E_\theta(R)$ is similar to $E_\theta(W)$. That is, if true θ is larger than θ^* in Table 5.1, $E_\theta(R)$ of the failure-free period RASP is smaller than that of the corresponding hybrid RASP, and *vice versa*.

As for the power, the failure-free period RASP performs better than the corresponding hybrid RASP if θ^* is not close to θ_1, and *vice versa* (see Figures 5.3 and 5.4).

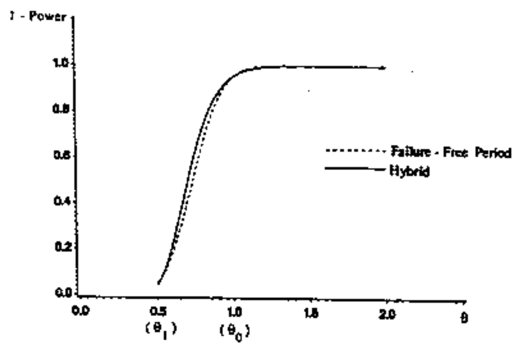

Figure 5.3 Power curves when $\alpha=\beta=0.05$, $\theta_1=0.5$, and $k=2$

6. Sensitivity Analysis

For both types of RASP, the shape parameter k is assumed to be known. To asssess the sensitivities of α and β errors with respect to

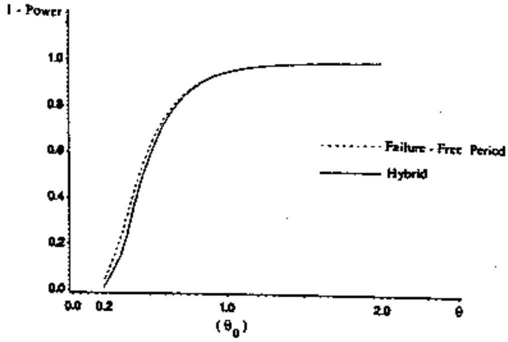

Figure 5.4 Power curves when $\alpha=\beta=0.05$, $\theta_1=0.5$, and $k=2$

the uncertainties in k, we calculated actual values of α and β errors when (true k)/(assumed k) = $(1+\delta)$, $\delta=-0.3$, -0.1, 0.1, 0.3. Then, for the combinations of parameters in (5.1) we calculated

$\Delta\alpha(\Delta\beta)$ = actual $\alpha(\beta)$ error when the true value of k is $(1+\delta)$ times the assumed value $-\alpha(\beta)$ error when the true value of k is equal to the assumed value.

Tables 6.1 and 6.2 respectively show $\Delta\alpha$ and $\Delta\beta$ values when $\alpha=\beta=0.05$. These tables and other computaional results indicate the following.

1. For both RASPs, $\Delta\alpha < 0$ and $\Delta\beta > 0$ for $\delta > 0$, and *vice versa.*
2. As θ_1 decreases, α and β errors become less sensitive to the changes in k for both RASPs.
3. As for α errors, the failure-free period RASP is less sensitive than the corresponding hybrid RASP. As for β errors, the above also holds except the cases where θ^* in Table 5.1 is close to θ_1.

In item 1, $\delta > 0$ ($\delta < 0$) means that the true value of k is larger(smaller) than the assumed value, or equivalently, that the true reliability of the items is better(worse) than the assumed. The findings in item 1 then imply that the producer is protected more(less) and the consumer is protected less (more) if the true reliability is better(worse) than the assumed.

7. Conclusion

In this article, we developed hybrid and failure-free period RASPs for testing the

Table 6.1. $\Delta\alpha$ **when** α**=0.05,** β**=0.05**

θ_1	k	0.5				2				10			
	RASP \ δ	-0.3	-0.1	0.1	0.3	-0.3	-0.1	0.1	0.3	-0.3	-0.1	0.1	0.3
$\frac{1}{2}$	F	0.141	0.032	-0.021	-0.042	0.259	0.055	-0.029	-0.048	0.428	0.082	-0.035	-0.049
	H	0.451	0.082	-0.034	-0.049	0.457	0.083	-0.034	-0.049	0.528	0.095	-0.036	-0.049
$\frac{1}{5}$	F	0.101	0.024	-0.017	-0.036	0.139	0.031	-0.020	-0.040	0.250	0.049	-0.026	-0.045
	H	0.163	0.035	-0.022	-0.042	0.181	0.038	-0.023	-0.043	0.250	0.049	-0.026	-0.045

F : failurre-free period H : hybrid

Table 6.2. $\Delta\beta$ when α=0.05, β=0.05

θ_1	k	0.5				2				10			
	RASP \ δ	-0.3	-0.1	0.1	0.3	-0.3	-0.1	0.1	0.3	-0.3	-0.1	0.1	0.3
$\frac{1}{2}$	F	-0.024	-0.009	0.011	0.036	-0.036	-0.016	0.022	0.086	-0.035	-0.021	0.038	0.192
	H	-0.045	-0.030	0.063	0.313	-0.042	-0.028	0.061	0.307	-0.015	-0.011	0.033	0.230
$\frac{1}{5}$	F	-0.022	-0.008	0.009	0.030	-0.024	-0.010	0.013	0.047	≃0	≃0	≃0	≃0
	H	-0.026	-0.012	0.017	0.071	-0.009	-0.005	0.008	0.036	≃0	≃0	≃0	≃0

hypotheses on the scale parameter of a gamma lifetime distribution with the shape parameter assumed known.

Comparisons of the two types of plans indicate among others that the failure-free period RASP has a shorter expected completion time than the corresponding hybrid RASP when the true scale parameter is large(or equivalently, when the true mean lifetime is large). As pointed out by Angus *et al.*[1], a great deal of effort is often expended in practice, prior to taking an RASP, to obtain assurance that the test will be passed, i.e., to assure that $\theta \geq \theta_0$. Thus, in such cases the failure-free period RASP compares favorably with the hybrid RASP. Otherwise, the failure-free period RASP could require a substantially longer tesing time than the hybrid RASP as shown in Fig.5.1.

The uncertainties involved in the assumed shape parameter result in different α and β errors from the specified values. However, computational results show that these changes are in favorable directions for the producer and the consumer for both types of plans.

Finally, for a given specific situation the tables and figures provided in this article may not be sufficient for selecting a plan and/or for conducting a sensitivity analysis with respect to the uncertainties in k. In such a situation, we recommend detailed analyses using a computer program, which is available from the authors upon request.

Appendix

Derivation of Eq.(3.4)

The truncated *pdf* of X is given by

$$f(x|X<t_f) = g(x;\theta,k)/P(X<t_f),$$
$$0<x<t_f.$$

Then,

$$\begin{aligned}\mu_\theta &= E_\theta(X|X<t_f) \\ &= \int_0^{t_f} x f(x|X<t_f)\,dx \\ &= \int_0^{t_f} x g(x;\theta,k)\,dx / P(X<t_f).\end{aligned}$$

On the other hand,

$$xg(x;\theta,k)=x^k exp(-x/\theta)/\{\Gamma(k)\theta^k\}$$

$$=k\theta[x^{(k+1)-1}exp(-x/\theta)/\{\Gamma(k+1)\theta^{k+1}\}]$$
$$=k\theta g(x;\theta,k+1).$$

Therefore,

$$\mu_\theta=k\theta G(t_f;\theta,k+1)/G(t_f;\theta,k).$$

Derivation of Eq.(3.6)

For a failure-free period RASP, the expected number of failures until a decision is reached is given by

$$E_\theta(R)=\Sigma_{i=0}^{r_f} iP(R=i)$$

$$=\Sigma_{i=0}^{r_f-1} iP(R=i)+r_fP(R=r_f). \quad (A.1)$$

We first determine $P(R=r_f)$ as follows.

$$P(R=r_f)=1-\Sigma_{i=0}^{r_f-1}P(R=i)$$

$$\Sigma_{i=r_f}^{\infty}p_\theta(1-p_\theta)^i$$

$$=\Sigma_{j=0}^{\infty}p_\theta(1-p_\theta)^{j+r_f}$$

$$=(1-p_\theta)^{r_f}\Sigma_{j=0}^{\infty}p_\theta(1-p_\theta)^j$$

$$=(1-p_\theta)^{r_f}. \quad (A.2)$$

Next, the first term in (A.1) can be written as

$$\Sigma_{i=0}^{r_f-1}ip_\theta(1-p_\theta)^i=p_\theta\Sigma_{i=0}^{r_f-1}i(1-p_\theta)^i=p_\theta S.$$

Then

$$S-(1-p_\theta)S$$
$$=p_\theta S$$
$$=(1-p_\theta)+(1-p_\theta)^2+\cdots+(1-p_\theta)^{r_f-1}$$
$$-(r_f-1)(1-p_\theta)^{r_f}$$
$$=(1-p_\theta)\{1-(1-p_\theta)^{r_f-1}\}/$$
$$p_\theta-(r_f-1)(1-p_\theta)^{r_f}. \quad (A.3)$$

Inserting (A.2) and (A.3) into (A.1), we obtain (3.6).

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