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A Heuristic Approach to Steiner Ring Problem

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Abstract

Optical fiber systems play an essential role in today's telecommunications networks. The recently standardized SONET technology has made a ring structure the preferred architecture for inter-city communication networks. In designing a SONET with ring structure, we consider inserting optional cites, which are not necessary in constructing the SONET, but cost-effective in connecting essential nodes in the ring. This problem is modeled as Steiner ring problem. Efficient heuristic procedures are developed based on the procedures for the traveling salesman problem. Computational results show that the proposed algorithm is excellent compared to the optimal solution. The error bound by the proposed method is 2 - 6% in experimented problems.

I. INTRODUCTION

In the recently standardized Synchronous Optical Network (SONET) technology, to economically utilize the high capacity of fiber optic cables, networks are often organized as a *hubbing* structure based on a single-homing concept. Such a hubbing network architecture, however, is inherently vulnerable to single cable cuts and major hub failures. Strategies such as *diverse protection* and *dual homing* have been designed to provide protection for cable cuts and hub failures, respectively. These designs, however, have been shown to be expensive for interoffice networks. The SONET technology and associated high-speed add/drop multiplexing make a *ring* structure which is the preferred architecture for configuring fiber optic networks.

In constructing a SONET with ring structure, including some sites which are not originally considered in forming the SONET can cut down the construction cost. This happens, for example, when we construct a

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SONET in a terrain with many "blocks". This problem is modeled as Steiner ring problem (SRP).

In SRP, the nodes which must be included in the ring are called as *special nodes* and the nodes which are included only when their inclusion causes cost cutdown of the ring are called as *steiner nodes*. Steiner ring problem is to find the cheapest ring that contains all the special nodes.

The name steiner comes from the well-known Steiner tree problem [4], where we find a minimum spanning tree in the existence of steiner points. Due to the NP-hardness [7] of the problem, many heuristic procedures have been developed [11,12,13]. Most of those procedures make use of the fact that Steiner tree problem has a tree topology, which can be handled easily by the procedure for minimum spanning tree problem. However, the solution of SRP has a ring topology which includes all special nodes and selective steiner nodes. Thus, procedures used for the traveling salesman problem (TSP) can be employed for the SRP.

Recent research on the survivability of communication networks [8,9,10] can be considered as special cases of the SRP. In the survivability issue, problems usually deal with minimizing the sum of link costs under the assumption of triangular inequality. The SRP in this paper considers both the link and node construction costs. The assumption on the triangular inequality is relaxed. In other words, any two special nodes may be connected via one or more steiner nodes, even if the direct connection is the shortest route. Such a case occurs when the construction of the link is impossible or when the construction cost is too expensive.

In this paper procedures based on traveling salesman problem (TSP) heuristic is proposed to solve SRP. An integer programming (IP) formulation of SRP and a method to obtain a lower bound are proposed in section II. In section III we develop heuristic procedures for the SRP. Computational experiments are given in section IV. Finally, conclusions are given in section V.

II. FORMULATION OF THE SRP

2.1 Integer Programming Formulation of SRP

An IP formulation of SRP is presented in [2]. To formulate the problem precisely, we define the following notations.

$G=(V,E)$: undirected graph

$N \subseteq V$: set of special nodes

$\overline{N} \equiv V-N$: set of steiner nodes

c_{ij}: link cost to connect node i and node j

w_k: steiner node weight

$x_{ij}=1$ if node i and node j are connected to each other

$y_k=1$ if steiner node k is included in the ring

Then SRP can be formulated as follows:

SRP: *Minimize* $\sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} + \sum_{k \in \overline{N}} w_k y_k$

Subject to

$$\sum_{j \in V} x_{ij} = 2 \qquad \forall i \in N \qquad (1)$$

$$\sum_{j \in V} x_{kj} = 2y_k \qquad \forall k \in \overline{N} \qquad (2)$$

$$\sum_{i \in S} \sum_{j \in \overline{S}} x_{ij} \geq 2 \forall S \subset V, S \cap N \neq \overline{S} \cap N \neq \phi \qquad (3)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in V \qquad (4)$$

$$y_k \in \{0, 1\} \qquad \forall k \in \overline{N} \qquad (5)$$

The first term in the objective function is the total link costs to connect all the special nodes and selected steiner nodes. The second term is the installation costs of those selected steiner nodes. It is assumed that special nodes are established for the ring construction. Constraints (1) and (2) respectively represent the connectivity of special and steiner nodes. Constraint (3) guarantees the single ring structure of the solution.

2.2 Lower Bound

2.2.1 Distribution of the weight of steiner node to adjacent links

As stated previously, if a steiner node decreases the cost of the ring, it can be included in the ring. Unlike the special nodes, however, the inclusion of steiner nodes causes some additional cost such that installation cost of necessary equipment in that node. Therefore, this installation cost is specially treated in most problems with steiner nodes.

Now we propose a method with which we need not consider the node weight of each steiner node. If a steiner node is included in the construction of a ring, it is always connected to other two nodes which can be a special node or another steiner node. Thus, by modifying the link costs we can exclude the installation costs from the consideration. Provided all the steiner nodes have the same installation cost, the link costs are modified as follows:

1. For each link between a steiner node and a special node, add to the link cost half of the installation cost of the steiner node.

2. For each link between two steiner nodes, add to the link cost the installation cost of the steiner node.

If the installation cost of a steiner node is different from another, the link cost between two steiner nodes is modified as the sum of its original link cost and half of the installation cost of each steiner node. This idea is used in obtaining a lower bound for SRP.

2.2.2 Lower bound

Since the installation cost of each steiner node is included into the adjacent links, if we provide the shortest length between each pair of special nodes, the SRP becomes a well-known traveling salesman problem. Solving this traveling salesman problem provides us with a lower bound for SRP. This solution may be infeasible because the shortest path length of each pair of special nodes may visit steiner nodes which are used in other pair of special nodes.

The computing time required to obtain lower bound increases exponentially with the number of special nodes. However, we can use this lower bound to check the solution quality when the increasing number of steiner nodes makes problems hard to solve, since lower bound considers the network with special nodes.

Ⅲ. HEURISTIC SOLUTIONS

Steiner ring problem can be considered as traveling salesman Problem (TSP) with steiner nodes which may or may not be visited in the tour. First we obtain a ring that contains all special nodes. Then using the initial ring, we apply a TSP heuristic. When the TSP heuristic reaches a local optimal solution, steiner nodes are considered to reduce the cost of the ring. The addition of steiner nodes leads to a new ring. Thus the tour needs to be modified by applying the TSP heuristic. Again steiner nodes are included as far as their addition reduces the cost of the ring. This procedure is repeated until one of the following two termination conditions is satisfied:

1. No steiner node reduces the cost of ring when included.

2. No cost reduction is obtained when the TSP heuristic is applied to the ring.

3.1 Basic Algorithm

Now we specify the procedure mentioned above. To get an initial ring the nearest neighbor algorithm is employed. Let n be the number of nodes to form a ring. Nearest neighbor algorithm is stated as follows:

[Nearest Neighbor Algorithm]

Step 1. Start with a partial tour consisting of a single, arbitrarily chosen node a_1.

Step 2. Let the current partial tour be $a_1, \cdots, a_k$, where $k<n$ and a_i is i-th node in the current partial tour. Let a_{k+1} be the node, not currently in the tour, which can be connected to a_k with the cheapest cost. Add a_{k+1} to the end of the tour.

Step 3. Halt when the current tour contains all the nodes.

After obtaining an initial ring composed of all special nodes, we apply a TSP heuristic to the ring. As a TSP heuristic, ThreeOpt [3,5] is used which is well-known edge exchange procedure. In ThreeOpt, all exchanges of three edges are tested until there is no feasible exchange that improves the current solution. Among many TSP heuristics available, we use ThreeOpt since it gives relatively better solutions. Results of the comparison between TwoOpt and ThreeOpt are summarized in Table 1.

When the TSP heuristic reaches its local optimal solution, steiner nodes are considered to reduce the cost of the ring. A steiner node is included into a pair of special nodes that guarantees the greatest cost reduction compared with other pairs of nodes. After all the steiner nodes are checked for inclusion a new ring is produced. We use this ring as an initial

Table 1. Comparison of TwoOpt and ThreeOpt

Number of Nodes	Optimal	TwoOpt	ThreeOpt
10	1052	1052	1052
20	1846	1887	1846
30	2148	2197	2148
40	2518	2558	2527
50	2953	3071	2987
60	3211	3290	3211
70	3375	3647	3406

ring for the TSP heuristic. Basic algorithm is summarized as follows:

Step 1. Obtain an initial ring by nearest neighbor method over the network with special nodes.

Step 2. Improve the solution with ThreeOpt.

Step 3. Consider each steiner node to include into a pair of special nodes.

Step 4. Repeat Step 2 and Step 3 until there is no cost cutdown.

3.2 Modified Algorithm I

In the basic algorithm, the ring is completed by inserting steiner nodes into the initial tour composed of special nodes. Each steiner node is included when it decreases the connection cost of a pair of special nodes. In the algorithm, however, the insertion of two or more consecutive steiner nodes between two special nodes is not considered.

To construct a ring which includes all special nodes and some cost effective steiner nodes a modified algorithm is examined. In the algorithm the initial ring is constructed by considering all special and steiner nodes in the network.

In the heuristic the nearest neighbor method is implemented over all the nodes. That is, if a steiner node is the nearest to a special node, that steiner node is connected to the special node. Then the procedure searches the nearest node from that steiner node. After connecting all special nodes, if the procedure selects the start node as a nearest one, an initial ring is completed.

Modified algorithm I is summarized as follows:

Step 1. Obtain an initial ring by nearest neighbor method over all nodes. When all the special nodes are included in the tour and the starting node is the nearest neighbor node, then complete the initial ring.

Step 2. Improve the solution by applying ThreeOpt.

Step 3. Consider each steiner node to include into a pair of special nodes.

Step 4. Repeat Step 2 and Step 3 until there is no cost cutdown.

3.3 Modified Algorithm II

Additional steps are considered by examining the shortest route and the current route of each pair of special nodes. The pair of special nodes whose difference of the two routes is maximum is selected. Then by replacing the current route with the shortest route a modified ring is constructed. The TSP heuristic is applied to the ring and this procedure is repeated.

Modified algorithm II is summarized as follows:

Step 1. Obtain an initial ring by nearest neighbor method over all nodes. When all the special nodes are included in the tour and the starting node is the nearest neighbor node, then complete the ring.

Step 2. Improve the solution by applying ThreeOpt.

Step 3. Consider each steiner node to include into a pair of special nodes.

Step 4. Repeat Step 2 and Step 3 until there is no cost cutdown.

Step 5. Assign steiner nodes to the link with maximum cost cutdown.

Step 6. Repeat Step 2 to Step 5 until there is no cost cutdown.

Ⅳ. COMPUTATIONAL RESULTS

To test the proposed heuristics, various networks are generated. Nodes are randomly generated in the 500 by 500 plane. The cost of each link is calculated directly proportional to the length. Between each pair of two special nodes a 'blocking factor' is added to the link cost. This factor represents the difficulty of connecting the two nodes. That is, the larger the blocking factor between two special nodes, the more difficult to connect them. All steiner nodes are assumed to have the same node weights, which does not affect the NP-completeness of SRP.

The heuristics are implemented in C language and run on HP 9000/827S. The CPLEX optimizer [1] is used to obtain optimal solutions and lower bounds.

Three cases of problems are considered depending on the number of special nodes (10, 20 and 40) in the ring. For each case three different number of steiner nodes are generated as in Table 2. For each case, 10 problems are experimented and average costs and CPU seconds are presented in the table.

The table shows that the lower bound gives good approximation to the optimal solution. The bound becomes tight as the problem size increases. Modified Algorithm II demonstrates better solution quality than the Modified Algorithm I. The rate of improvement is increasing as the number of steiner nodes increases. The error bound of the Modified Algorithm II is within 4% in the generated problems with less than 60 nodes. The bound is increased to 6% in problems with 40 special and 40 steiner nodes. In problems which have more than 80 nodes, the optimal solutions could not be obtained due to the increased computational burden. However, it is clear from the table the solution by the Modified Algorithm II does not exceed 110 % of its corresponding lower bound. Figures 1, 2 and 3 summarize the computational results of the proposed algorithms. Clearly, the Modified

Table 2. Summary of Computational Results

Problem Size		Lower Bound	Optimal Solution	Basic Algorithm	Modified Algorithm I	Modified Algorithm II
(10,5)	Cost	3153.5 (93.13)	3350. 2 (100)	3764. 9 (112.38)	3364. 8 (100.44)	3364. 8 (100.44)
	CPU seconds	0.05	0.36	0.00	0.00	0.19
(10,10)	Cost	2881. 9 (94.00)	3065.7 (100)	3951. 1 (128.88)	3296. 8 (106.66)	3149. 1 (102.72)
	CPU seconds	0.05	0.67	0.00	0.15	0.41
(10,20)	Cost	2687. 3 (97.55)	2754.9 (100)	3805. 4 (138.13)	2968. 9 (107.77)	2816. 5 (102.24)
	CPU seconds	0.03	34.83	0.01	0.72	1.23
(20,10)	Cost	4677. 8 (95.08)	4919.8 (100)	5966. 1 (121.27)	5083. 4 (103.33)	5034. 4 (102.33)
	CPU seconds	0.54	1.57	0.23	1.05	2.22
(20,20)	Cost	4467. 8 (96.11)	4648.5 (100)	6100. 1 (131.23)	4962. 3 (106.75)	4830. 3 (103.91)
	CPU seconds	0.42	28.59	0.24	3.22	6.13
(20,40)	Cost	4119. 1 (98.11)	4198.4 (100)	5857. 5 (139.52)	4703. 3 (112.03)	4353. 8 (103.70)
	CPU seconds	0.70	892.58	0.28	11.39	16.21
(40,20)	Cost	7740. 9 (96.12)	8053.3 (100)	9364. 9 (116.29)	8427. 8 (104.65)	8364. 7 (103.87)
	CPU seconds	23.36	35.54	3.76	20.26	34.18
(40,40)	Cost	7006. 3 (97.34)	7197.5 (100)	9048. 2 (125.71)	7869. 8 (109.34)	7635. 4 (106.08)
	CPU seconds	11.12	968.63	3.36	53.89	91.78
(40,80)	Cost	6590.0	-	8896.9	7481.9	7197.7
	CPU seconds	6.34	-	3.29	222.49	254.98

1. In the first column, *(n,m)* implies *n* special nodes and *m* Steiner nodes.
2. Each case is averaged over ten problems.
3. Numbers in the parentheses represent relative magnitude.
4. The zero in time row implies the algorithm solves the problem in 100 milliseconds.

Algorithm II outperforms other algorithms. The error bound is 2 to 6 % of the optimal solutions in the generated problems.

V. CONCLUSIONS

In designing a SONET with ring structure, we considered inserting optional nodes which are not necessary in the ring, but cost-effective

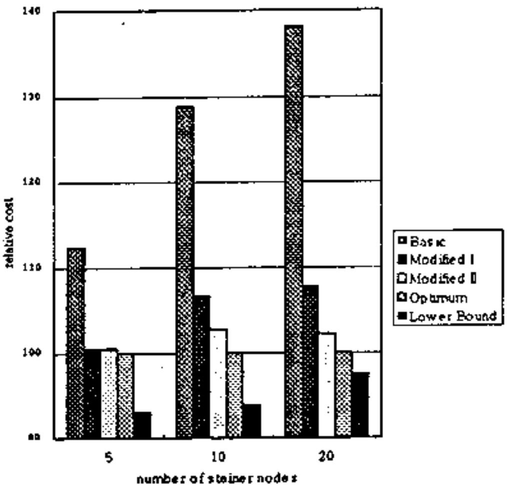

Figure 1. Relative magnitude of costs (number of special nodes=10)

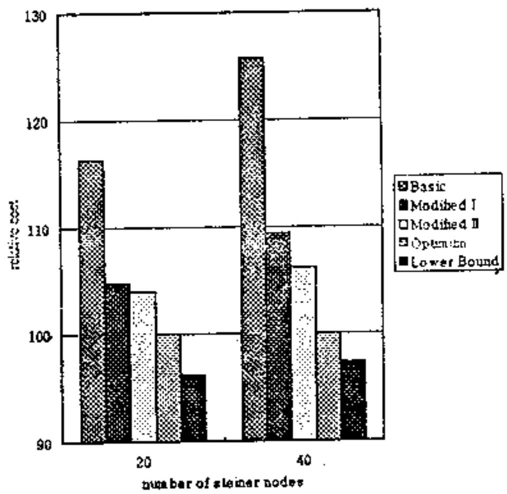

Figure 2. Relative magnitude of costs (number of special nodes=20)

to include in the connection of two essential nodes. The problem is modeled as Steiner ring problem (SRP). Algorithms based on TSP heuristic are proposed. Modified algorithm I which constructs initial ring with all special and optional steiner nodes gives better solution

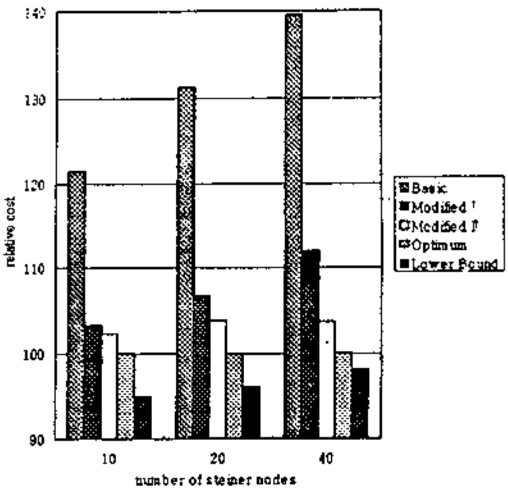

Figure 3. Relative magnitude of costs (number of special nodes=40)

than the basic algorithm which starts with only special nodes. The effect of the shortest route between a pair of special nodes which allows multiple steiner nodes is promising as the problem size increases. Modified algorithm II provides an excellent near-optimal solution the error bound of which is 2 - 6% in the problems experimented.

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