⊠ 응용논문

A Study on Simultaneous Optimization of Multiple Quality Characteristics for Robust Design

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Abstract

Robust design in industry is an approach to reducing performance variation of quality characteristic values in products and processes. In the Taguchi type robust design, the product array approach using orthogonal arrays is mainly used. However, it often requires an excessive number of experiments. In this paper, for the combined array approach to assign control and noise factors, we propose how to simultaneously optimize multiple quality characteristics. Two examples are illustrated to show the difference between the product-array approach and the combined-array approach.

1. Introduction

Products and their manufacturing processes are influenced both by control factors that can be controlled by designers and by noise factors that are difficult or expensive to control such as environmental conditions, properties of raw materials, and product aging. The basic idea of robust design is to identify, through exploiting interactions between control factors and noise factors, appropriate settings of control factors that make the system's performance robust to changes in the noise factors. Robust design(or Parameter design in a narrow sense) is a quality improvement technique proposed by the Japanese quality expert Taguchi (1978), which was described by Taguchi (1986, 1987), Kackar (1985), and others.

A large number of experimental trials in Taguchi's product array may be required because the noise array is repeated for every row in the control array. There have been efforts for integrating Taguchi's important notion of heterogeneous variability with the standard experimental design and modeling technology provided by response surface methodology(RSM). They combined control and noise factors in a single design matrix, which we call a "combined array".

Welch, Yu, Kang, and Sacks (1990) first proposed the combined array approach. The initial motivation of the combined array is the run-size saving. Related approaches were discussed by Vining and Myers (1990), Box and Jones (1992), Shoemaker, Tsui and Wu (1991), and Myers, Khuri and Vining (1992), etc. Treatment of the mean and variance responses via a constrained optimization was discussed in Vining and Myers (1990).

The combined-array approach allows one to provide separate estimates for the mean response(or quality-characteristic) and for the variance(or variation) response. Accordingly, we can apply the primary goal of the Taguchi method which is to minimize the variance while constraining the mean, within a RSM.

2. Simultaneous Optimization of Multiple Quality Characteristics

2.1 Estimated Mean and Variance Models

Box and Jones (1992) modeled the mean and variance separately in a single response. But, we are interested in showing the estimated mean and variance response models in multiple responses(or quality characteristics).

Suppose the response y, depends on control variables (or factors) and noise variables. Let a set of control variables be denoted by $\underline{x}=(x_1,x_2,\cdots,x_l)'$ and a set of noise variables by $\underline{z}=(z_1,z_2,\cdots,z_m)'$. Suppose that all response functions in a multiresponse system depend on the same set of $\underline{x}$ and $\underline{z}$ and that they can be represented by second order models within a certain region of interest. Let N be the number of experimental runs and r be the number of response functions. The ith second order model is

$$y_i(\underline{x},\underline{z})=\beta_{i0}+\underline{x}'\underline{\beta}_i+\underline{x}'B_i\underline{x}+\underline{z}'R_i\underline{z}+\underline{z}'\underline{\gamma}_i+\underline{z}'D_i\underline{x}+\varepsilon_i,\quad i=1,2,\cdots,r, \tag{2.1}$$

where $\underline{\beta}_i$ is $l\times 1$, $\underline{\gamma}_i$ is $m\times 1$, $B_i'=B_i$ is $l\times l$, $R_i'=R_i$ is $m\times m$, D_i is $l\times m$, which are vectors or matrices of unknown regression parameters, and ε_i is a random error associated with the ith response.

Equation (2.1) can be expressed in matrix notation as

$$\underline{y}_i = X\,\underline{\theta}_i + \underline{\varepsilon}_i, \qquad i = 1, 2, \cdots, r, \tag{2.2}$$

in which $\underline{y}_i$ is an $N\times 1$ vector of observations on the ith response, X is an $N\times p$ full column rank matrix of known constants, $\underline{\theta}_i$ is the $p\times 1$ column vector of unknown regression parameters, and $\underline{\varepsilon}_i$ is a vector of random errors associated with the ith response. We also assume that

$$E(\underline{\varepsilon}_i) = \underline{0},\ Var(\underline{\varepsilon}_i) = \sigma_{ii}I_N,\ Cov(\underline{\varepsilon}_i, \underline{\varepsilon}_j) = \sigma_{ij}I_N \quad i, j = 1, 2, \cdots, r,\ i \neq j.$$

The $r\times r$ matrix whose (i, j)th element is σ_{ij} will be denoted by Σ. The r equations given in (2.2) may be written in a compact form

$$\underline{y} = \begin{pmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_r \end{pmatrix} = \begin{pmatrix} X & 0 & \cdots & 0 \\ 0 & X & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X \end{pmatrix} \begin{pmatrix} \underline{\theta}_1 \\ \underline{\theta}_2 \\ \vdots \\ \underline{\theta}_r \end{pmatrix} + \begin{pmatrix} \underline{\varepsilon}_1 \\ \underline{\varepsilon}_2 \\ \vdots \\ \underline{\varepsilon}_r \end{pmatrix} = Z\underline{\theta} + \underline{\varepsilon}, \tag{2.3}$$

where $\underline{y}$ is $rN\times 1$, Z is $rN\times rp$, $\underline{\theta}$ is $rp\times 1$, and $\underline{\varepsilon}$ is $rN\times 1$. The variance-covariance matrix of $\underline{\varepsilon}$ is

$$Var(\underline{\varepsilon}) = \Sigma \otimes I = \Omega,$$

where $\otimes$ is a symbol for the direct (or Kronecker) product of matrices.

The BLUE (best linear unbiased estimator) of $\underline{\theta}$ in (2.3) is

$$\begin{aligned} \widehat{\underline{\theta}} &= (Z'\Omega^{-1}Z)^{-1}(Z'\Omega^{-1}\underline{y}) \\ &= (Z'Z)^{-1}Z'\underline{y}. \\ &= (\widehat{\underline{\theta}_1}', \widehat{\underline{\theta}_2}', \cdots, \widehat{\underline{\theta}_r}')', \end{aligned}$$

where $\widehat{\underline{\theta}_i} = (X'X)^{-1}X'\,\underline{y}_i$ is the least squares estimator of the $p\times 1$ vector of regression coefficients for the ith response model. The prediction equation for the ith response is given by

$$\widehat{y}_i(\underline{x}, \underline{z}) = \underline{g}'(\underline{x}, \underline{z})\,\widehat{\underline{\theta}}_i, \qquad i = 1, 2, \cdots, r, \tag{2.4}$$

where $(\underline{x}', \underline{z}')'$ is the vector of coded input variables, $\underline{g}'(\underline{x}, \underline{z})$ is a vector of the same form as a row of the matrix X evaluated at the point $(\underline{x}, \underline{z})$.

The fitted ith second-order model in (2.4) can be rewritten as

$$\hat{y}_i(\underline{x}, \underline{z}) = b_{i0} + \underline{x}'\underline{b}_i + \underline{x}'\hat{B}_i\underline{x} + \underline{z}'\hat{R}_i\underline{z} + \underline{z}'\underline{r}_i + \underline{z}'\hat{D}_i\underline{x}, \quad i = 1,2,\cdots,r.$$

The noise variables $\underline{z}$ are not controllable and random variables. In the absence of other knowledge, $\underline{z}$ would be usually uniformly distributed over R_z.

Let $\widehat{m}_i(\underline{x})$ be the ith estimated mean response at an $\underline{x}$ averaged over the noise variables

$$\widehat{m}_i(\underline{x}) = \int_{R_z} \hat{y}_i(\underline{x}, \underline{z})p(\underline{z})d\underline{z}, \quad i = 1,2,\cdots,r,$$

where $p(\underline{z})$ is a probability density function of $\underline{z}$, and $\underline{z}$ has a uniform distribution over R_z. Box and Jones (1992) showed that the ith estimated mean becomes

$$\widehat{m}_i(\underline{x}) = b_{i0} + \underline{x}'\underline{b}_i + \underline{x}'\hat{B}_i\underline{x} + \frac{1}{3} tr\, \hat{R}_i, \quad i = 1,2,\cdots,r, \tag{2.5}$$

where $tr\, \hat{R}_i$ is the trace of the matrix $\hat{R}_i$. Let us write $\hat{v}_i(\underline{x})$ for the ith mean square variation about the ith mean response

$$\hat{v}_i(\underline{x}) = \int_{R_z} (\hat{y}_i(\underline{x}, \underline{z}) - \widehat{m}_i(\underline{x}))^2 p(\underline{z})d\underline{z}, \quad i = 1,2,\cdots,r. \tag{2.6}$$

Let us call this measure the ith estimated variance, which becomes

$$\hat{v}_i(\underline{x}) = \frac{1}{3}(\underline{r}_i + \hat{D}_i\underline{x})'(\underline{r}_i + \hat{D}_i\underline{x}) + \hat{A}_i, \quad i = 1,2,\cdots,r, \tag{2.7}$$

where $\hat{A}_i = [4\Sigma_{j=1}^{m}(r^i_{jj})^2 + 5\Sigma_{j=1}^{m-1}\Sigma_{k=j+1}^{m}(r^i_{jk})^2]/45$ and r^i_{jk} is the jth row and kth column element of the matrix $\hat{R}_i$.

2.2 P_V Measure

In this section, we propose the simultaneous-optimization measure of multiple responses(quality characteristics) for robust design in a combined array.

If we have a prior knowledge about the estimated mean response $\widehat{\underline{m}}(\underline{x})$, it is possible to minimize the estimated variance response while constraining the estimated mean response. Let

$$\hat{v}_i^*(\underline{x}) = \frac{\hat{v}_i(\underline{x}) - \min_{\underline{x} \in R_x} \hat{v}_i(\underline{x})}{\max_{\underline{x} \in R_x} \hat{v}_i(\underline{x}) - \min_{\underline{x} \in R_x} \hat{v}_i(\underline{x})}, \quad i = 1,2,\cdots,r,$$

where $\hat{v}_i(\underline{x})$ is the ith mean square variation about the ith mean response which is in (2.6) Note that $\hat{v}_i^*(\underline{x})$ is a "standardized" measure of $\hat{v}_i(\underline{x})$. The proposed simultaneous optimization measure can be written as

$$\underset{\underline{x}\in R_x}{Min}\ P_V(\underline{x}) = \underset{\underline{x}\in R_x}{Min}\ \underline{w}'\ \widehat{\underline{v}}^*(\underline{x}) = \underset{\underline{x}\in R_x}{Min}\ \sum_{i=1}^{r} w_i\ \widehat{v_i}^*(\underline{x}), \qquad i=1,2,\cdots,r,$$

$$\text{subject to}\ \begin{cases} m_{i*}\le \widehat{m}_i(\underline{x})\le m_i^* & :\text{nominal}-\text{is}-\text{best} \\ \widehat{m}_i(\underline{x})\ge m_{i*} & :\text{larger}-\text{the}-\text{better} \\ \widehat{m}_i(\underline{x})\le m_i^* & :\text{smaller}-\text{the}-\text{better} \end{cases}$$

where $\underline{w}=(w_1, w_2, \cdots, w_r)'$, $\widehat{\underline{v}}^*(\underline{x})=(\hat{v}_1^*(\underline{x}), \hat{v}_2^*(\underline{x}), \cdots, \hat{v}_r^*(\underline{x}))'$, $\Sigma_{i=1}^{r} w_i=1$, m_{i*} is the minimum acceptable value of $\widehat{m}_i(\underline{x})$, and m_i^* is the maximum acceptable value of $\widehat{m}_i(\underline{x})$.

The region R determines the set of $\underline{x}$'s that will be considered as possible locations of the optimum operating conditions The user of the simultaneous optimization measure may choose R_x as he wishes. Side conditions on the levels of the responses (such as lower and upper bounds on several responses, that is the acceptable value of $\hat{v}_i(\underline{x})$ or $\widehat{m}_i(\underline{x})$) can be easily incorporated.

3. A Comparative Study : Combined-Array Approach Versus Product-Array Approach

In this section, we will compare the simultaneous-optimization procedure using a Taguchi's product array with using a combined array that was proposed by P_V measure.

3.1 Product-Array Approach

Suppose that the objective is to find the simultaneous optimum conditions for increasing the strength of plastic product and reducing the wear on the plastic product. Suppose there are three control factors A, B, and C which are assigned to an orthogonal array, $L_{18}(2^1\times 3^7)$. Interactions among three factors are partially confounded a little in each of the remaining columns. Therefore, it is not recommanded to use this array for experiments where interactions are necessary. Also suppose there is a noise factor N with three levels(N_0 : good condition, N_1 : normal condition, N_3 : bad condition). The control factors are listed in <Table 3.1>, <Table 3.2> gives a set of hypothetical strength data y_1 and wear data y_2.

<Table 3.1〉 Factors and Levels

Control factor	0 level	1 level	2 level
A : time (min)	120	125	130
B : temperature (℃)	60	70	80
C : stir speed (rpm)	700	800	900

Suppose that quality characteristics for y_1 and y_2 are the "larger-the-better" characteristics and the "smaller-the-better" characteristics, respectively. We can calculate SN ratios from replication at each experimental condition as follows : (1) In the case of larger-the-better : $SN_i = -10\log_{10}(\Sigma_{j=1}^{3} 1/3y_{ij}^2) - 35$, (2) In the case of smaller-the better : $SN_i = -10\log_{10}[\Sigma_{j=1}^{3}(y_{ij}^2/3)] + 25$.

〈Table 3.2〉 Assignment of Source and Data in the Product Array

Source	e	A	B	C	e	e	e	e	y_1				y_2			
Col / Run #	1	2	3	4	5	6	7	8	N_0	N_1	N_2	SN	N_0	N_1	N_2	SN
1	-1	-1	-1	-1	-1	-1	-1	-1	45	49	52	-1.30	30	25	18	-2.90
2	-1	-1	0	0	0	0	0	0	65	64	60	0.97	15	11	10	3.28
3	-1	-1	1	1	1	1	1	1	73	69	75	2.17	29	31	22	-3.82
4	-1	0	-1	-1	0	0	1	1	63	60	69	1.08	8	14	11	3.96
5	-1	0	0	0	1	1	-1	-1	55	56	49	-0.51	9	7	15	4.27
6	-1	0	1	1	-1	-1	0	0	68	72	72	1.97	19	17	12	0.77
7	-1	1	-1	0	-1	1	0	1	62	66	61	0.97	9	12	5	5.79
8	-1	1	0	1	0	-1	1	-1	55	49	56	-0.51	14	20	17	0.30
9	-1	1	1	-1	1	0	-1	0	74	80	74	2.60	8	15	17	2.15
10	1	-1	-1	1	1	0	0	-1	69	55	66	0.91	25	29	28	-3.75
11	1	-1	0	-1	-1	1	1	0	57	52	44	-1.00	19	19	13	0.27
12	1	-1	1	0	0	-1	-1	1	78	76	68	2.34	12	15	14	2.25
13	1	0	-1	0	1	-1	1	0	50	52	56	-0.60	9	12	8	5.16
14	1	0	0	1	-1	0	-1	1	51	45	46	-1.53	15	22	23	-1.16
15	1	0	1	-1	0	1	0	-1	66	75	69	1.87	12	13	8	4.01
16	1	1	-1	1	0	1	-1	0	56	51	59	-0.19	18	25	23	-1.93
17	1	1	0	-1	1	-1	0	1	50	45	48	-1.46	11	19	13	1.64
18	1	1	1	0	-1	0	1	-1	73	67	76	2.11	11	7	10	5.46
	SUM											9.89				25.75

From the analysis of variance(ANOVA) tables, <Table 3.3> and <Table 3.4>, only B(temperature) is very significant for the data y_1, and the main effects of A(time) and C(stir speed) are very significant for the data y_2. One can find the simultaneous optimum conditions, $A_1B_2C_1$(125 min, 80 ℃, 800 rpm) by summarizing the results of all the data as shown in <Table 3.5>.

〈Table 3.3〉 ANOVA (SN for Strength)

Source	S	f	V	F_0
A	0.29	2	0.15	0.20
B	25.84	2	12.92	17.46**
C	1.07	2	0.54	0.73
e	8.10	12	0.74	
T	35.30	17		

〈Table 3.4〉 ANOVA (SN for Wear)

Source	S	f	V	F_0
A	44.96	2	22.50	15.85**
B	1.68	2	0.84	0.59
C	106.88	2	53.44	37.63**
e	15.59	11	1.42	
T	169.14	17		

〈Table 3.5〉 Summarized Table of Factorial Effects

Source	Level	Sum of SN for y_1	Sum of SN for y_2	Optimum Level
A	0 (120 min)	4.09	-4.69	
	1 (125 min)	2.28	17.01	○
	2 (130 min)	3.52	13.41	
B	0 (60 ℃)	0.87	6.33	
	1 (70 ℃)	-4.04	8.60	
	2 (80 ℃)	13.06	10.82	○
C	0 (700 rpm)	1.79	9.13	
	1 (800 rpm)	5.28	26.21	○
	2 (900 rpm)	2.82	-9.59	

3.2 Combined-Array Approach

The combined array consists of three control variables x_1(or A), x_2(or B), and x_3(or C) and one noise variable z (or N) which are assigned in the orthogonal array, $L_{18}(2^1\times3^7)$. In order to compare the product array approach with the combine array approach, the data come from the combination for each level of factors in a product array(see Table 3.2). From the results of ANOVA for the data y_1 and y_2 in the product array approach, we see that interactions between the control factors and the noise factor are not significant. Therefore, we do not consider interactions between the control factors and the noise factor in the combined array approach. The combined array $L_{18}(2^1 \times 3^7)$ is not used for experiments where interactions exist. Table 3.6 is the case when the noise variable is arranged in column 6.

〈Table 3.6〉 Assignment of Sources and Data in the Combined Array

Source	e	x_1	x_2	x_3	e	z	e	e	Data	
Run # \ Col	1	2	3	4	5	6	7	8	y_1	y_2
1	-1	-1	-1	-1	-1	-1	-1	-1	45	30
2	-1	-1	0	0	0	0	0	0	64	11
3	-1	-1	1	1	1	1	1	1	75	22
4	-1	0	-1	-1	0	0	1	1	60	14
5	-1	0	0	0	1	1	-1	-1	49	15
6	-1	0	1	1	-1	-1	0	0	68	19
7	-1	1	-1	0	-1	1	0	1	61	5
8	-1	1	0	1	0	-1	1	-1	55	14
9	-1	1	1	-1	1	0	-1	0	80	15
10	1	-1	-1	1	1	0	0	-1	55	29
11	1	-1	0	-1	-1	1	1	0	44	13
12	1	-1	1	0	0	-1	-1	1	78	12
13	1	0	-1	0	1	-1	1	0	50	9
14	1	0	0	1	-1	0	-1	1	45	22
15	1	0	1	-1	0	1	0	-1	69	8
16	1	1	-1	1	0	1	-1	0	59	23
17	1	1	0	-1	1	-1	0	1	50	11
18	1	1	1	0	-1	0	1	-1	67	7
	SUM								1074	216

The estimated response models by the method of least squares are given by

$$\hat{y}_1(\underline{x}, z) = 53.15 + 1.24x_1 + 7.88x_2 - 0.87x_3 + 6.28x_1^2 + 11.04x_2^2 + 0.05x_3^2 - 7.93x_1x_2 - 3.12x_1x_3 - 0.15x_2x_3 - 1.10z + 2.37zx_1 + 1.46zx_2 - 3.40zx_3 - 7.60z^2$$

$$\hat{y}_2(\underline{x}, z) = 9.36 - 3.78x_1 - 0.32x_2 + 4.25x_3 - 1.03x_1^2 - 1.55x_2^2 + 8.91x_3^2 + 5.77x_1x_2 + 0.22x_1x_3 + 0.31x_2x_3 + 0.77z + 2.98zx_1 - 1.44zx_2 + 4.50zx_3 + 2.89z^2$$

Using equations (2.5) and (2.7), the estimated mean and variance models are given by

$$\widehat{m_1}(\underline{x}) = 1.24x_1 + 7.88x_2 - 0.87x_3 + 6.28x_1^2 + 11.04x_2^2 + 0.05x_3^2 - 7.93x_1x_2 - 3.12x_1x_3 - 0.15x_2x_3 + 48.09,$$

$$\widehat{m_2}(\underline{x}) = 3.78x_1 - 0.32x_2 + 4.25x_3 - 1.03x_1^2 - 1.55x_2^2 + 8.91x_3^2 + 5.77x_1x_2 + 0.22x_1x_3 + 0.31x_2x_3 + 12.30,$$

$$\widehat{v_1}(\underline{x}) = (2.37x_1 + 1.46x_2 - 3.40x_3 - 1.10)^2/3 + 5.13,$$

$$\widehat{v_2}(\underline{x}) = (2.98x_1 - 1.44x_2 + 4.50x_3 + 0.77)^2/3 + 0.74.$$

The region of interest R_x is given by the inequality $-1 \le x_1, x_2, x_3 \le 1$. The ranges for $\widehat{m}_1(\underline{x})$, $\widehat{m}_2(\underline{x})$, $\hat{v}_1(\underline{x})$, and $\hat{v}_2(\underline{x})$ are $47.38 \le \widehat{m}_1(\underline{x}) \le 84.66$, $0.00 \le \widehat{m}_2(\underline{x}) \le 32.22$, $5.13 \le \hat{v}_1(\underline{x}) \le 28.26$, and $0.74 \le \hat{v}_2(\underline{x}) \le 32.04$, respectively.

The results of the simultaneous optimization according to the P_V measure are given in <Table 3.7>, <Table 3.7> indicates that the optimal setting for $\widehat{m}_1(\underline{x}) \ge 75.54$, $\widehat{m}_2(\underline{x}) \le 8.58$, $w_1 = 0.1$, and $w_2 = 0.9$ is $x_1 = -0.72$, $x_2 = 1.0$, and $x_3 = 0.02$ which produces a predicted value of 77.64, 8.55, 5.80, and 3.22 for $\widehat{m}_1(\underline{x})$, $\widehat{m}_2(\underline{x})$, $\hat{v}_1(\underline{x})$, and $\hat{v}_2(\underline{x})$, respectively.

〈Table 3.7〉 Simultaneous Optimization for P_V

Weight		Location of Optima			Simultaneous Optimum Value			
w_1	w_2	x_1	x_2	x_3	$\widehat{m}_1(\underline{x})$	$\widehat{m}_2(\underline{x})$	$\hat{v}_1(\underline{x})$	$\hat{v}_2(\underline{x})$
		Subject to $\widehat{m}_1(\underline{x}) \ge 75.54$, $\widehat{m}_2(\underline{x}) \le 8.58$						
0.1	0.9	-0.72	1.00	0.02	77.64	8.55	5.80	3.22
0.5	0.5	-0.58	1.00	-0.10	75.45	8.58	5.28	3.44
0.9	0.1	-0.52	1.00	-0.22	74.59	8.57	5.14	4.17
		Subject to $\widehat{m}_1(\underline{x}) \ge 81.51$, $\widehat{m}_2(\underline{x}) \le 7.38$						
0.1	0.9	-1.00	1.00	-0.02	77.64	8.55	5.80	3.22
0.5	0.5	-0.96	1.00	-0.06	81.63	7.34	6.11	5.56
0.9	0.1	-1.00	1.00	-0.42	81.64	7.16	5.24	10.97
		Subject to $\widehat{m}_1(\underline{x}) \ge 77.51$, $\widehat{m}_2(\underline{x}) \le 8.58$						
0.1	0.9	-1.00	1.00	-0.02	82.47	7.33	6.39	5.40
0.5	0.5	-0.94	1.00	-0.08	81.22	7.36	5.98	5.63
0.9	0.1	-0.88	1.00	-0.30	79.78	7.37	5.30	7.92

3.3 Comparison of Results

From the results of Sections 3.1 - 3.2, we compare the results of the product array approach with the combined array approach for the case of $w_1 = 0.5$ (equal weights), since the product array approach assumes equal weight for each response. The simultaneous optimum condition of the product array approach is $A_1B_2C_1(x_1 = 0,\ x_2 = 1,\ x_3 = 0)$, that is, 125 min, 80℃, and 800 rpm. In the case of combined-array approach, the simultaneous optimum condition of the P_1 measure is $x_1 = -0.58$, $x_2 = 1.00$ and $x_3 = -0.10$(122.10 min, 80℃, 790 rpm) and so on.

4. A Comparative Study for Two-Level Orthogonal Array Design

In this section, we will study the simultaneous-optimization measure for two level orthogonal array design. Also, in the case that interactions between the control factors and the noise factors exist, we will compare the product-array approach with the combined-array approach.

4.1 Product-Array Approach

We want to find the simultaneous optimum conditions for increasing the strength of plastic product and reducing the wear on the plastic product.

The control factors are listed in <Table 4.1>. Suppose there are five control factors A, B, C, D, and F which are assigned to an orthogonal array, $L_{16}(2^{15})$. Also suppose there is a noise factor N with two levels (N_0 : normal condition, N_1 : bad condition).

⟨Table 4.1⟩ Factors and Levels

Control factor	0 level	1 level
A : plasticity time (min)	120	130
B : plasticity temperature (℃)	70	80
C : cooling temperature (℃)	-20	-15
D : quantity of additive (%)	5	10
F : stir speed (rpm)	800	900

<Table 4.2> gives a set of hypothetical strength data y_1 and wear data y_2. From the results of ANOVA for the data y_1 and y_2, we see that interactions between the control factors and the noise factor, that is, $A \times N$ of y_1 and $B \times N$ of y_2 are significant.

We can calculate SN ratios as follows:

(1) In the case of larger-the-better : $SN_i = -10\log_{10}(\Sigma_{j=1}^{3} 1/3y_{ij}^2) - 30$,

(2) In the case of smaller-the-better : $SN_i = -10\log_{10}[\Sigma_{j=1}^{3}(y_{ij}^2/3)] + 35$.

〈Table 4.2〉 Assignment of Source and Data in the Product Array

Source	e	A	B	C	e	e	e	e	y_1				y_2			
Col / Run #	1	2	3	4	5	6	7	8	N_0	N_1	N_2	SN	N_0	N_1	N_2	SN
1	-1	-1	-1	-1	-1	-1	-1	-1	45	49	52	-1.30	30	25	18	-2.90
2	-1	-1	0	0	0	0	0	0	65	64	60	0.97	15	11	10	3.28
3	-1	-1	1	1	1	1	1	1	73	69	75	2.17	29	31	22	-3.82
4	-1	0	-1	-1	0	0	1	1	63	60	69	1.08	8	14	11	3.96
5	-1	0	0	0	1	1	-1	-1	55	56	49	-0.51	9	7	15	4.27
6	-1	0	1	1	-1	-1	0	0	68	72	72	1.97	19	17	12	0.77
7	-1	1	-1	0	-1	1	0	1	62	66	61	0.97	9	12	5	5.79
8	-1	1	0	1	0	-1	1	-1	55	49	56	-0.51	14	20	17	0.30
9	-1	1	1	-1	1	0	-1	0	74	80	74	2.60	8	15	17	2.15
10	1	-1	-1	1	1	0	0	-1	69	55	66	0.91	25	29	28	-3.75
11	1	-1	0	-1	-1	1	1	0	57	52	44	-1.00	19	19	13	0.27
12	1	-1	1	0	0	-1	-1	1	78	76	68	2.34	12	15	14	2.25
13	1	0	-1	0	1	-1	1	0	50	52	56	-0.60	9	12	8	5.16
14	1	0	0	1	-1	0	-1	1	51	45	46	-1.53	15	22	23	-1.16
15	1	0	1	-1	0	1	0	-1	66	75	69	1.87	12	13	8	4.01
16	1	1	-1	1	0	1	-1	0	56	51	59	-0.19	18	25	23	-1.93
17	1	1	0	-1	1	-1	0	1	50	45	48	-1.46	11	19	13	1.64
18	1	1	1	0	-1	0	1	-1	73	67	76	2.11	11	7	10	5.46
	SUM											9.89				25.75

〈Table 4.3〉 ANOVA (SN for Strength)

Source	S	f	V	F_0
A	3.7830	1	3.7830	13.27**
B	3.6290	1	3.6290	12.73**
C	7.1824	1	7.1824	25.19**
D	0.6400	1	0.6400	2.24
F	0.0240	1	0.0240	0.08
e	2.8514	10	0.2851	
T	18.1148	15		

〈Table 4.4〉 ANOVA (SN for Wear)

Source	S	f	V	F_0
A	0.4658	1	0.4658	1.04
B	2.1830	1	2.1830	4.87*
C	1.7490	1	1.7490	3.90
D	13.7456	1	13.7456	30.63**
F	13.3043	1	13.3043	29.65**
e	4.4805	10	0.4487	
T	34.9342	15		

From <Table 4.3> and <Table 4.4>, we see that in the case of y_1, the main effects of A, B and C are very significant and in the case of y_2, the main effects of D and F are very significant and B is significant. One can find the simultaneous optimum levels, $A_0B_1C_0D_0F_0$(120 min, 80℃, -20℃, 5%, 800 rpm) by summarizing the results of all the data as shown in <Table 4.5>.

〈Table 4.5〉 Summarized Table of Factorial Effects

Source	Level	Sum of SN for y_1	Sum of SN for y_2	Optimum Level
A	0 (120 min)	47.62	47.81	○
	1 (130 min)	39.84	45.08	
B	0 (70 ℃)	39.92	43.49	
	1 (80 ℃)	47.54	49.40	○
C	0 (-20 ℃)	49.09	43.40	○
	1 (-15 ℃)	38.37	49.49	
D	0 (5 %)	42.13	53.86	○
	1 (10%)	45.33	39.03	
F	0 (800 rpm)	43.42	53.74	○
	1 (900 rpm)	44.04	39.15	

4.2 Combined-Array Approach

Suppose that the control variables and noise variables $(\underline{x}, \underline{z})$ can be represented by a first-order model in the control variables and noise variables with, in addition, cross-product terms between the control variables and the noise variables.

The combined array consists of five control variables x_1 (or A), x_2 (or B), x_3 (or C), x_4 (or D), and x_5 (or F) and one noise variable z (or N) which are assigned in the orthogonal array, $L_{16}(2^{15})$. In order to compare the product array approach with the combined array approach, the data come from the combination for each level of factors in a product array (see Tables 4.2). For the case that the noise variable is arranged in column 4, the results of assignment are shown in Table 4.6. In the product array, interactions between the control factors and the noise factor, that is, $A\times N$ of y_1 and $B\times N$ of y_2 are significant. Therefore, we must consider interactions between the control factors and the noise factor in the combined array.

The estimated response models by the method of least squares are given by

$$\hat{y}_1(\underline{x}, z) = 60.00 - 3.25x_1 + 3.13x_2 - 4.25x_3 + 1.38x_4 - 0.13x_5 - 1.00z - 2.25zx_1 - 0.63zx_2 - 2.25zx_3 - 0.88zx_4 + 0.13zx_5 \quad (4.1)$$

$$\hat{y}_2(\underline{x}, z) = 29.31 + 0.94x_1 - 1.06x_2 - 1.19x_3 + 3.06x_4 + 2.94x_5 + 0.06z - 0.81zx_1 - 1.81zx_2 - 0.44zx_3 + 0.31zx_4 + 0.19zx_5 \quad (4.2)$$

⟨Table 4.6⟩ Assignment of Sources and Data in the Combined Array

Run \ Col	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Data y_1	Data y_2
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	59	23
2	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	59	30
3	-1	-1	-1	1	1	1	1	-1	-1	1	-1	1	1	1	1	69	33
4	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	56	29
5	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	69	36
6	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	-1	-1	61	21
7	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	74	30
8	-1	1	1	1	1	-1	-1	1	1	1	-1	-1	-1	1	1	59	25
9	1	-1	1	-1	1	-1	1	-1	1	1	1	-1	1	-1	1	60	30
10	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	51	31
11	1	-1	1	1	-1	1	-1	-1	1	1	1	1	-1	1	-1	57	41
12	1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	44	26
13	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	64	31
14	1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	65	32
15	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	62	20
16	1	1	-1	1	-1	-1	1	1	-1	1	1	-1	1	1	-1	51	31
basic mark	a	b	ab	c	ac	bc	abc	d	ad	bd	abd	cd	acd	bcd	abcd		
Source	x_1	x_2	e	z	x_1z	x_2z	e	x_3	e	e	x_4	e	e	x_5	e		

From (4.1) and (4.2), the estimated mean and variance models are given by

$$\widehat{m_1}(\underline{x}) = -3.25x_1+3.13x_2-4.25x_3+1.38x_4-0.13x_5+60.00,$$
$$\widehat{m_2}(\underline{x}) = 0.94x_1-1.06x_2-1.19x_3+3.06x_4+2.94x_5+29.31,$$
$$\widehat{v_1}(\underline{x}) = (-2.25x_1-0.63x_2-2.25x_3-0.88x_4+0.13x_5-1.00)^2/3,$$
$$\widehat{v_2}(\underline{x}) = (-0.81x_1-1.81x_2-0.44x_3+0.31x_4+0.19x_5+0.06)^2/3.$$

The results of simultaneous optimization according to the P_V measure are given in <Table 4.7>.

⟨Table 4.7⟩ Simultaneous Optimization under P_V

Weight		Location of Optima					Simultaneous Optimum Value			
w_1	w_2	x_1	x_2	x_3	x_4	x_5	$\widehat{m}_1(\underline{x})$	$\widehat{m}_2(\underline{x})$	$\widehat{v}_1(\underline{x})$	$\widehat{v}_2(\underline{x})$
		Subject to $\widehat{m}_1(\underline{x}) \geq 68.00$, $\widehat{m}_2(\underline{x}) \leq 24.00$								
0.1	0.9	-1.0	0.6	-0.9	-0.7	-1.0	68.12	23.72	3.82	0.02
0.3	0.7	-1.0	0.8	-0.7	-0.5	-1.0	68.17	23.89	2.31	0.13
0.5	0.5	-1.0	0.9	-0.6	-0.4	-1.0	68.20	23.97	1.70	0.22
0.7	0.3	-1.0	1.0	-0.5	-0.4	-1.0	68.08	23.74	1.29	0.36
0.9	0.1	-1.0	1.0	-0.5	-0.4	-1.0	68.08	23.74	1.29	0.36
		Subject to $\widehat{m}_1(\underline{x}) \geq 69.00$, $\widehat{m}_2(\underline{x}) \leq 23.00$								
0.1	0.9	-1.0	0.9	-1.0	-0.9	-1.0	69.21	22.91	4.31	0.21
0.3	0.7	-1.0	0.9	-1.0	-0.9	-1.0	69.21	22.91	4.31	0.21
0.5	0.5	-1.0	1.0	-0.9	-0.8	-1.0	69.23	22.99	3.45	0.32
0.7	0.3	-1.0	1.0	-0.9	-0.8	-1.0	69.23	22.99	3.45	0.32
0.9	0.1	-1.0	1.0	-0.9	-0.8	-1.0	69.23	22.99	3.45	0.32
		Subject to $\widehat{m}_1(\underline{x}) \geq 66.00$, $\widehat{m}_2(\underline{x}) \leq 26.00$								
0.1	0.9	-1.0	0.5	-0.2	0.2	-1.0	66.07	25.75	0.39	0.00
0.3	0.7	-1.0	0.6	-0.1	0.3	-1.0	66.10	28.83	0.17	0.02
0.5	0.5	-1.0	0.6	-0.1	0.3	-1.0	66.10	28.83	0.17	0.02
0.7	0.3	-1.0	0.7	0.0	0.4	-1.0	66.12	25.91	0.04	0.07
0.9	0.1	-1.0	0.8	0.1	0.5	-1.0	66.15	25.99	0.00	0.14

4.3 Comparison of Results

From the results of Sections 4.1 and 4.2, for $w_1 = 0.5$ (equal weights) the simultaneous optimum condition of the product array approach is $A_0B_1C_0D_0F_0$

($x_1=-1.0$, $x_2=1.0$, $x_3=-1.0$, $x_4=-1.0$, $x_5=-1.0$), that is, 120 min, 80℃ -20℃, 5%, and 800 rpm (see Table 4.5). In the case of combined array approach the simultaneous optimum conditions of the P_V measure are almost the same as the case of product array approach, and so on (see <Table 4.7>).

5. Concluding Remarks

The product array approach does not consider empirical modeling between a response variable and several control factors, and it selects only the simultaneous optimal levels of the concerned factors. However, the combined array approach considers empirical modeling, usually a second order response model, and it selects the simultaneous optimal conditions of the concerned control factors in the region of interest through the fitted empirical model. When there are multiple quality-characteristics(responses), it is obvious that selecting optimal conditions of the control factors in their region of interest is more desirable than selecting optimal levels among given levels of experiment.

In the combined array approach, an experimental design and analysis can be regarded as a stage of a sequential investigation where the investigator uses the information gained from each stage of experimentation. However, the product array approach does not usually consider the sequential experiment. In these points of view, we can judge that the combined array approach is better than the product array approach.

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