⊠ 응용논문

A Literature Survey on Warranty Policies

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Abstract

Estimation of warranty costs is an important application area of reliability engineering. On the manufacturer's point of view, exact estimation of warranty costs helps decisions on budgeting, product pricing, and A/S capacity planning, etc. We reviewed the state-of-the-art of existing models on warranty policies and suggested some future research directions.

1. Introduction

Warranty is a service contract between manufacturer/seller and consumer. Under the warranty contract, the manufacturer promises to repair or replace the failed product over a pre-specified period of time. Manufacturers use warranty policies to achieve various goals such as customer satisfaction, sales promotion, quality assurance, legal obligation, etc. In recent years, many of the manufacturing companies recognize warranty as essential to the manufacturer's competitive edge. For example, electric home-appliance companies and motor companies of Korea extended warranty period, and advertised it extensively. Moreover a boiler-manufacturing company, a PC-manufacturing/selling company, and a construction company of Korea adopted lifetime warranty recently.

During the last 20 years, many authors studied on warranty policies. Most of the studies were dedicated on the estimation of warranty costs. On the manufacturer's point of view, exact estimation of warranty costs is necessary because the decisions on budgeting, product pricing, and A/S capacity planning depend on the measures. On the other hand, consumers purchase a product which provided better warranty services than the other because they prefer a product having lower maintenance costs among the products of same price and performance.

Generally, the previous studies can be classified by the following three types of warranty policies:

(i) The free warranty policy - manufacturer pays all the costs of repair or replacement when the product failure occurs in pre-specified warranty period.

(ii) The pro-rata warranty policy - manufacturer pays a fraction of warranty costs, which depends on the age of the product at the time of failure.

(iii) The hybrid warranty policy - hybrid form of the free and pro-rata policies, in which an initial free warranty period is followed by a pro-rata period.

In the above-stated three policies, there are many kinds of studies according that the product have a specified failure time distribution, the product failures are repairable or irrepairable, and the warranty period is renewed or maintained by the repair or replacement. There are also some researches to estimate the present value at the sales time considering the time value of money.

The purpose of this study is to review the existing models on warranty policies, and to suggest some future research directions.

2. Literature Review

In this section, we explained briefly and summarized the previous studies.

Menke(1969) first introduced the estimation of warranty reserve funds for non-repairable items in the case of linear pro-rata rebate and lump sum rebate plans under the assumption that the failure time distribution is exponential. Amato and Anderson(1976) extended Menke's model to present value of future warranty costs considering the time value of money, and Patankar and Worm(1981) considered the continuous discount rate and prediction intervals of warranty reserves.

For the free warranty case, Glickman and Berger(1976) determined the product price and warranty period maximizing manufacturer's profit under the assumption of good-as-new repair. Karmarkar(1978), Blischke and Scheuer(1981), and Frees(1986) calculated warranty costs using renewal theory for non-repairable products. Nguyen and Murthy(1984a) considered various repair types and analyzed warranty costs for repairable products, and Park and Yee(1984) estimated present-worth of warranty cost for Weibull failure time case. Wells(1987) proposed a lifetime warranty model for phase-type life distribution. Moskowitz and Chun(1994), Yun(1994) and Murthy et al.(1995) discussed two dimensional warranty policies such that the product age and mileage of an automobile are considered simultaneously in warranty policy. Park(1985), Chun and Lee(1992), Jack and Dagpunar(1994), Dagpunar and Jack(1994) studied on the optimal replacement time for some warrantied systems.

For the pro-rata warranty policy, Ritchken and Tapiero(1986) suggested a framework in which warranty policies for non-repairable items can be evaluated according to risk preferences of both manufacturers and consumers, and showed that a warranty can induce more sales and greater profitability. Thomas(1989) extended Menke's model to general failure time case. Mitra and Patankar(1988, 1993) proposed a goal programming approach to the warranty policy with multiple goals.

Under free and pro-rata warranty policies, Mamer(1982) obtained the total costs of warranty and ownership over the product life cycle and the expected cost rate of non-repairable products. Nguyen and Murthy(1982) examined the burn-in time to achieve an optimal trade-off between warranty and burn-in test costs. Mamer(1987) studied both the expected costs to producers and benefits to consumers, and Kao and Smith(1993) extended Mamer's model to phase-type failure time case.

Thomas(1983) proposed a hybrid warranty policy for an irrepairable item in which consumers receive renewed warranties when failure occurs during the warranty period. Nguyen and Murthy(1984b)estimated both consumer costs and manufacturer profits, and Frees and Nam(1988) reexamined this model by introducing several approximations of renewal function.

Tapiero and Posner(1988) considered a problem of warranty reserving by introducing a compound Poisson stochastic model for warranty claims and reserves for non-repairable products under free, prorata and hybrid policies respectively.

Previous studies on warranty policies are summarized in <Table 1>.

〈Table 1〉 Summary of previous studies on warranty policies.

Authors	Warranty Policy F: free P: pro-rata H: hybrid			Failure Type R: repairable N: non-repairable		Warranty Renewal Re: renew Mn: maintain		Failure Time Distribution	Viewpoint M: manufacturer C: consumer		Focus $: warranty costs D: warranty duration T: replacement time			Remarks
	F	P	H	R	N	Re	Mn		M	C	$	D	T	
Glickman & Berger(76)	○			○			○	exponential	○		○	○		good-as-new repair
Karmarkar(78)	○			○			○	general	○		○			good-as-new repair & present value
Blischke & Scheuer(81)	○				○		○	〃	○		○			profits of producer
Nguyen & Murthy(84a)	○			○			○	〃	○		○			two-types, imperfect repair
Park & Yee(84)	○			○			○	Weibull	○		○			continuous discount
Frees(86)	○				○		○	general	○		○			linear approximation of renewal function
Wells(87)	○			○			○	〃	○		○			lifetime warranty policy
Moskowitz & Chun(94)	○				○		○	〃	○		○			two-dimensional warranty policy
Murthy, Iskandar & Wilson(95)	○				○		○	〃	○		○			two-dimensional warranty policy
Yun(94)	○			○			○	〃	○		○			two-dimensional warranty policy
Park(85)	○			○			○	〃		○			○	replacement time of warrantied product
Chun & Lee(92)	○			○			○	〃	○				○	imperfect preventive maintenance
Jack & Dagpunar(94)	○			○			○	〃	○		○			imperfect preventive maintenance
Dagpunar & Jack(94)	○			○			○	〃	○		○			extend Jack & Dagpunar(94)'s model
Rao(95)	○				○		○	phase- type	○		○			phase-type failure distribution
Yun & Yoo(95)	○			○			○	general	○		○			free-replacement at initial failure followed by free-repair
Amato & Anderson(76)		○			○	○		exponential	○		○			extend Menke's model to present-worth
Patankar & Worm(81)		○			○	○		〃	○		○			continuous discount rate
Ritchken & Fuh(86)		○			○		○	general		○			○	age-replacement policy under warranty
Ritchken & Tapiero(86)		○			○	○		〃	○	○	○		○	manufacturer's and consumer's risks
Mitra & Patankar(88)		○			○	○		exponential	○		○	○		multiple goals
Thomas(89)		○			○	○		〃	○		○			present-worth

(continued)

Authors	Warranty Policy F: free P: pro-rata H: hybrid			Failure Type R: repairable N: non-repairable		Warranty Renewal Re: renew Mn: maintain		Failure Time Distribution	Viewpoint M: manufacturer C: consumer		Focus $: warranty costs D: warranty duration T: replacement time			Remarks
	F	P	H	R	N	Re	Mn		M	C	$	D	T	
Mitra & Patankar(93)					○	○		exponential			○	○		extent Mitra & Patankar(88)'s model to multi-product
Patankar & Mitra(95)					○	○		Weibull	○		○	○		consideration of consumer behavior
Menke(69)	○				○	○		exponential			○			warranty expires at the first failure
Mamer(82)	○	○			○	○	○	general	○	○	○			free & maintain, pro-rata & renew
Nguyen & Murthy(82)	○	○		○	○		○	〃	○		○			burn-in time
Balcer & Sahin(86)	○	○			○		○	〃		○	○			estimate consumer's costs
Lie & Chun(87)	○			○	○		○	〃	○	○	○			inspection sample to minimize warranty and quality cost
Mamer(87)	○	○			○	○	○	〃	○	○	○			random damage process
Kao & Smith(93)	○	○			○	○	○	phase-type	○	○	○			extend Mamer(87)'s model
Thomas(83)			○		○	○		general	○			○		benefits from warranty increase
Nguyen & Murthy(84b)			○		○	○	○	〃	○	○	○			estimate short-run & long-run costs
Frees & Nam(88)			○		○	○	○		○	○	○			extend Nguyen & Murthy(84b) using by approximation of renewal function
Kim & Kim(91)			○		○	○		〃			○			stepdown warranty policy
Son, Suh & Park(94b)			○	○				Weibull			○		○	present-worth
Ritchken(85)	○		○		○	○		general			○			producer's risk averse
Bai & Chun(88)	○		○	○	○	○	○				○			two-types of failure
Tapiero & Posner(88)	○		○	○			○				○			warranty claims follow compound Poisson process
Kim, Jang & Kim(92)	○				○				○		○		○	stepdown warranty policy
Son, Suh & Park(94a)	○										○			present-worth

3. Expressions for Warranty Costs

Let $p(t)$ be the fraction of producer's payment of warranty costs. Then $p(t)$ can be expressed as

$$p(t) = \begin{cases} 1, & 0 < t \leq W_1 \\ (W-t)/(W-W_1), & W_1 < t \leq W, \\ 0, & W < t \end{cases} \quad (1)$$

where W : total warranty period, W_1: initial free warranty period of hybrid policy. If $W_1 = W$ then the policy become free warranty, and if $W_1 = 0$ then pro-rata warranty.

For non-repairable products, when warranty contract terminates at the first failure, the present worth of expected warranty cost $C(W)$ is expressed as

$$C(W) = \int_0^W c_r p(t) f(t) e^{-rt} dt, \quad (2)$$

where c_r: replacement cost of failed product, $f(t)$: p.d.f. of failure time, r:continuous discount rate. This expression includes Menke(1969), Amato and Anderson(1976), Patankar and Worm(1981), Thomas(1983).

When the warranty contract maintains a given period regardless of the number of warranty replacements in the warranty period, the sequence of failures (and thus replacements) forms a renewal process. Since the probability of product failures in $[t, t+dt]$ is $\sum_{n=1}^{\infty} f_n(t)dt$ [Barlow and Proschan(1965)], the present-worth of expected warranty cost $C(W)$ is expressed as

$$C(W) = \int_0^W c_r p(t) \sum_{n=1}^{\infty} f_n(t) e^{-rt} dt, \quad (3)$$

where $f_n(t)$ denotes the p.d.f. of n-th failure time. This expression includes Glickman and Berger(1976), Blischke and Scheuer(1981), Nguyen and Murthy(1984b), Frees(1986), Frees and Nam(1988), Son, Suh and Park(1994a).

For repairable products, the warranty contract maintains a given period with minimal repair at failure. Then the sequence of failures (and thus repairs) forms a NHPP (non-homogeneous Poisson process) under the assumption of instantaneous repair. Thus the present-worth of expected warranty cost $C(W)$ is expressed as

$$C(W) = \int_0^W c_f p(t) h(t) e^{-rt} dt, \quad (4)$$

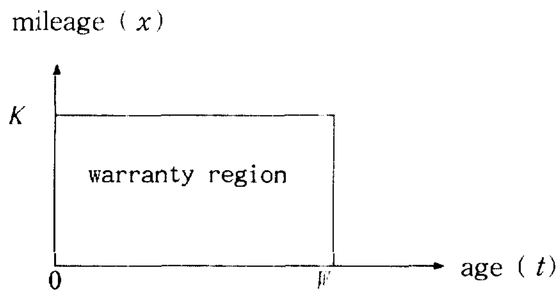
where c_f: repair cost of failed product, $h(t)$: instantaneous failure rate. This expression includes Park and Yee(1984), Nguyen and Murthy(1984a), Son, Suh and Park(1994b).

Wells(1987) proposed a lifetime warranty policy in which all failures throughout the product's lifetime are repaired by producer. He assumed that the useful life of the products will be some finite but random amount of time. This may arise from technological obsolescence, changes in design, critical failure of subsystem, and so on. Yun(1995) expressed the expected warranty cost for free warranty policy as

$$C(W \rightarrow \infty) = \int_0^{\infty} c_f H(t) g(t)\, dt \qquad (5)$$

where the period of product's life is a random variable with p.d.f. $g(t)$ and cumulative hazard $H(t)$.

Moskowitz and Chun(1994), Iskandar and Wilson(1995) discussed two dimensional free warranty policies for non-repairable products, in which the warranty region is characterized by two dimension such as product age and mileage. An example of the policy is depicted in <Figure 1>.



〈Figure 1〉 An example of two dimensional warranty policy.

Then the expected warranty cost $C(W, K)$ is expressed as

$$C(W, K) = c_f M(W, K). \qquad (6)$$

The renewal function $M(t, x)$ of equation (6) is expressed as

$$M(t, x) = F(t, x) + \int_0^x \int_0^t M(t-u, x-v)\, dF(u, v), \qquad (7)$$

where $F(t, x)$ denotes bivariate distribution function of age t and mileage x.

Assuming that the failure of product is affected only by total running miles, Yun(1994) treated two dimensional warranty policy for a repairable product.

4. Further Research Areas

In this paper, we briefly reviewed and classified previous studies on warranty policies for the past two decades, and explained expressions for expected warranty costs. followings will be some further research areas:

(i)The case that consumers' dissatisfaction increases as the number of failures increases, even if all the repair or replacement costs as in free warranty.

(ii)The so-called 'compensation sales', in which the manufacturers count salvage value for a used old model when they sell a new one.

(iii)Service warranty policy, in which nonconformance to service standard such as error, delay, etc. will be regarded as failures.

(iv)Warranty cost estimation model for a general case, in which both repairable and irrepairable failures are included.

(v)The case that consumers may dispose of their products, even if they are usable, during the warranty period when an attractive new product appears. If the challenger product has better function and lower price in comparison with the defender, the disposal of usable products is likely to occur.

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