▨ 연구논문

Optimal Burn-In for a Process with Weak Components

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Abstract

This paper discusses an optimal burn-in procedure to minimize total costs based on the assumption that some of the components are weak for stress and deteriorate faster than the main components. The procedure will define the costs of burn-in errors. An ideal burn-in consists of process in which all weak (substandard) components and no main (standard) components fail. In practice, the burn-in errors could occur for some reasons. For example, it is impossible to eliminate all weak components through burn-in, due to a nonzero proportion of defectives of the components. Probability model and cost function model are formulated to find the optimal burn-in time that minimizes the expected total cost. Several examples are included to show how to use the results.

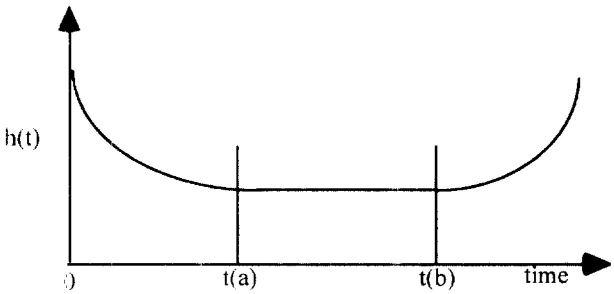
1. Introduction

Components are placed on test, usually at least moderately accelerated over normal conditions, for some fixed time period. The intent, of course, is to discover and replace the weak components in order to improve reliability measures of components/systems such as failure rate ($h(t)$), mean residual life ($m(t)$), and conditional reliability ($R(x \mid t)$). Washburn (1970) presented a mathematical model for optimal burn-in time to minimize total-cost. Subsequently many researchers have presented burn-in procedures for various cases. Way & Kuo (1983) and Leemis & Beneke (1990) gave an excellent review of burn-in problems. Overall, previous work on burn-in procedures and cost models have assumed that the time-to-failure patterns of components follow a single distribution law.

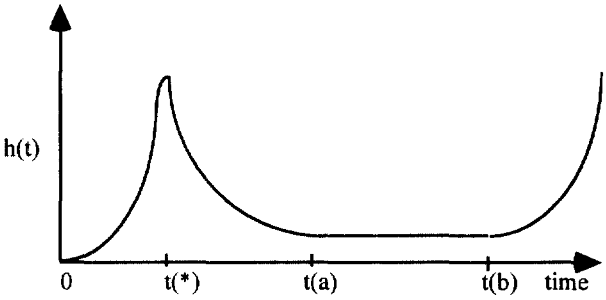
In the past, a distributional life model obtained by a bimodal mixture of two

Weibull distributions has been suggested to describe the time-to-failures of electrical and electro-mechanical devices. We will refer to these models as a two-mixed Weibull distribution. Kao (1959) introduced a two-mixed Weibull distribution to describe the failure time of electronic tubes. Stitch (1975) found that the failure time of microcircuits follows a mixed distribution. Reynolds & Stevens (1978) also found that two-mixed Weibull distribution described the time-to-failure patterns of electronic components. For electro-mechanical device, Boardman and Colvert (1978) found that the failure time of oral irrigators follow a two-mixed Weibull distribution. Other researchers discovered the two-mixed Weibull distribution is to be a good model to describe the time-to-failures of many products. The common bathtub curve for the hazard function can be modeled by a mixed Weibull. <Figure 1> shows the traditional bathtub curve and <Figure 2> presents a modified bathtub curve that was discussed by Jensen and Petersen (1982). In Figures 1-2, $t(a)$ is the time at which the failure rate reaches the useful region (chance failure region), $t(b)$ is the time at which the failure rate reaches the wear-out region in the bathtub curve and $t(^{*})$ is the time at which the failure rate reaches the end of the early failure region under the modified bathtub curve. Kececioglu (1994) stated that "During burn-in tests, generally early and chance failures are encountered. Therefore, a bimodal mixed distribution should be used." However, no previous work has been done for the optimal burn-in procedure for two-mixed Weibull distribution.

In this study, we present an optimal burn-in procedure for two-mixed Weibull distribution. The optimal burn procedure defines the costs of burn-in errors. In practice, the burn-in test errors could occur for various reasons; e.g. it is impossible to eliminate all weak components through burn-in, due to a nonzero proportion of defectives of the components. Probability model and cost function model are formulated for two-mixed Weibull distribution.



< Figure 1 > Traditional Bathtub Curve



< Figure 2 > Modified Bathtub Curve

2. Two-Mixed Weibull Distribution

A two-mixed distribution is composed of two cumulative density functions (CDF). Let $F_1(t)$ be the CDF of weak population (small proportion of susceptible components), $F_2(t)$ be the CDF of main population (the rest of the components), and $F(t)$ be the total CDF for the entire population. Then, $F(t)$ is constructed by taking a weighted average of the CDFs for the weak and main subpopulations. The weights are the proportions of each type of subpopulation. Thus, if the weak population has p proportion, and the main population has $(1-p)$ proportion, then

$$F(t) = pF_1(t) + (1-p)F_2(t) \tag{1}$$

Typically, $F_1(t)$ has a high early failure rate while $F_2(t)$ has a low early failure rate that either stays constant or increases very late in life in the equation (1).

Assuming that $f_i(t)$ is the probability density function (pdf) for $F_i(t)$ where $i = 1, 2$, then, the failure rate of the two-mixed distribution is expressed as

$$h(t) = \frac{pf_1(t)+(1-p)f_2(t)}{1-\{pF_1(t)+(1-p)F_2(t)\}} \tag{2}$$

Now, let us consider the two-mixed Weibull distribution with two parameters for each population. From the equation (1), the CDF of the two-mixed Weibull distribution is as follow:

$$F(t) = 1 - p(\exp[-(t/\eta_1)^{\beta_1}]) - (1-p)(\exp[-(t/\eta_2)^{\beta_2}]) \quad (3)$$

where

$\beta_1 =$ shape parameter of the weak population,

$\beta_2 =$ shape parameter of the main population

$\eta_1 =$ scale parameter of the weak population,

$\eta_2 =$ scale parameter of the main population

$p =$ proportion of the weak population

When the shape parameter is less than one, we observe a decreasing failure rate function. When the shape parameter is equal to one, a constant failure rate is observed. When the shape parameter is greater than one, an increasing failure rate results. i.e., the shape parameter determines in which failure region a product belongs. The scale parameter is also called the characteristic life; the point at which 63.2% of units will have failed. The shape parameters, scale parameters and the proportion of weak population are defined using the following two methods:

Jensen & Petersen method

The method is:

1. Plot the sample data on Weibull Probability Paper and fit a smooth curve by inspection.
2. Determine the place with the smallest slope on the CDF curve (where the curve levels off), and read the corresponding p value from the Y-axis (percentage failures). p represents the mixing weight of the subpopulation (weak population) located at the left.
3. Determine $\hat{\eta}_1$ & $\hat{\eta}_2$ by entering the Y-axis at 0.632 $\hat{p}$ and $\hat{p} + 0.632(1-\hat{p})$ horizontally; intersecting the CDF curve and dropping down,then $\hat{\eta}_1$ & $\hat{\eta}_2$ can be read from the X-axis (time-to-failure).
4. Determine $\hat{\beta}_1$ & $\hat{\beta}_2$ from the slopes of the tangent lines which are drawn at each end of the CDF curve.

Bayesian method

1. Calculate the probability of failure i belonging to the weak and main subpopulations as follow:

Weak population: $\hat{p}_1^i = \dfrac{(f_1 \mid t_i)}{(f_1 \mid t_i) + (f_2 \mid t_i)}$, Main population: $\hat{p}_2^i = 1 - \hat{p}_1^i$

where

$$(f_1 \mid t_i) = \frac{\hat{\beta}_1}{\hat{\eta}_1} \exp\left[-(t_i/\hat{\eta}_1)^{\hat{\beta}_1}\right](t_i/\hat{\eta}_1)^{\hat{\beta}_1 - 1},$$

$$(f_2 \mid t_i) = \frac{\hat{\beta}_2}{\hat{\eta}_2} \exp\left[-(t_i/\hat{\eta}_2)^{\hat{\beta}_2}\right](t_i/\hat{\eta}_2)^{\hat{\beta}_2 - 1}.$$

(the parameter values are estimated by Jensen & Petersen method)

2. Calculate the proportion of weak failures as follow:

$$\hat{p} = \frac{\sum_{i=1}^{r} \hat{p}_1^i}{N}$$

where r is the number of failures and N is the sample size.

3. Burn-In Cost Model

The proposal for an optimal burn-in procedure will need to define the costs of burn-in errors when the components of failures follow a two-mixed Weibull failure law. Previous researchers have assumed that there are no errors of burn-in. They considered an ideal burn-in occurs when all weak components and no main components fail. In practice, the burn-in errors could occur for various reasons; it is impossible to eliminate all weak components through burn-in Components on test during burn-in will either fail or not. In addition, some will be from the weak population while others will be from the main population. A two by two table is constructed to show the probability of the events. <Table 1> has the associated probabilities where

p = probability that a component is initially weak.

p_G = probability that an initially main component fails burn-in. This event can occur when accelerated conditions are too severe and/or the burn-in time is too long under accelerated conditions.

p_B = probability that an initially bad component passes burn-in. This event can occur when the burn-in time is too short or the accelerated conditions are not severe enough.

< Table 1 > Probability Model for Burn-In

Results of Burn-In	Main population	Weak population
Pass	$(1-p)(1-p_G)$	pp_B
Fail	$(1-p)p_G$	$p(1-p_B)$

We see from <Table 1>, for example, the probability that a component survives the burn-in and was from the main portion of the population is $(1-p)(1-p_G)$.

<Table 2> includes the cost function for the situations presented in <Table 1> Thus for a unit which failed the burn-in test and was from the main portion of the population, the total cost are the cost for performing the test, C_B, plus the cost of replacement and lost value of a good component, say C_{GF}. Therefore, the combined cost is the sum $C_B + C_{GF}$. The terms are shown in Table 2 and defined as.

C_B = burn-in cost for a component.

C_{GF} = cost when a main component fails burn-in. The cost includes replacement cost and loosing the value of a main component.

C_{BP} = cost when a weak component passes burn-in. The cost is a field failure cost or repair cost possibly further downstream assembly.

C_{BF} = cost when a weak component fails burn-in. This is the replacement cost

< Table 2 > Cost Function Per Item for Burn-In

Results of Burn-In	Main population	Weak population
Pass	C_B	$C_B + C_{BP}$
Fail	$C_B + C_{GF}$	$C_B + C_{BF}$

In <Table 2>, the costs, $C_B + C_{BP}$ and $C_B + C_{GF}$, are from the errors of burn-in. Under the burn-in cost model in <Table 2> with the associated probabilities of occurrence in Table 1, the average burn-in cost of per component, burn-in time t, is :

$$\begin{aligned} C_{AV} = & (1-p)(1-p_G)C_B + pp_B(C_B + C_{BP}) + (1-p)p_G(C_B + C_{GF}) \\ & + p(1-p_B)(C_B + C_{BF}) \end{aligned} \tag{4}$$

For the two-mixed Weibull distribution,

$$p_G = 1 - \exp[-(t/\eta_2)^{\beta_2}] \text{ and } p_B = \exp[-(t/\eta_1)^{\beta_1}] \tag{5}$$

Then, the average burn-in cost for a component in the equation (4) can be expressed as

$$\begin{aligned} C_{AV} = & (1-p)\{\exp[-(t/\eta_2)^{\beta_2}]\}C_B + p\{\exp[-(t/\eta_1)^{\beta_1}]\}(C_B + C_{BP}) \\ & + (1-p)\{1-\exp[-(t/\eta_2)^{\beta_2}]\}(C_B + C_{GF}) \\ & + p\{1-\exp[-(t/\eta_1)^{\beta_1}]\}(C_B + C_{BF}) \end{aligned} \tag{6}$$

The optimal burn-in time is determined to minimize the average burn-in cost defined in equation (6). In the next section, a method, to find the optimal burn-in time with minimizing the average burn-in cost, will be presented.

4. Minimizing Total Cost

Basic factors in determining the burn-in time are the failure pattern of components , burn-in cost, and costs incurred because of failures during and after the burn-in. Therefore, the optimal burn-in time is determined by minimizing the expected total burn-in cost.

If replacing and repair costs for components are higher than field or downstream repair costs, an unlikely situation, then $\partial C_{AV}/\partial t > 0$ and $\partial^2 C_{AV}/\partial t^2 < 0$. Thus, C_{AV} is an increasing function of t and it is concave This implies that the minimum average cost is achieved at the minimum burn-in time; i.e., burn-in time, $t=0$. Therefore, no burn-in is needed.

If replacing and repair costs for components are much lower than field or downstream repair costs, then the minimum average cost is obtained at the optimal burn-in time; i.e., $t=t(a)$ in <Figure 1> and <Figure 2>.

Other than these extreme cases, optimal burn-in time with the minimum average burn-in costs can be achieved by following method:

Obtain the first derivative of C_{AV} as

$$\frac{\partial C_{AV}}{\partial t} = \frac{\beta_1}{\eta_1}\left\{\frac{t}{\eta_1}\right\}^{\beta_1-1}\exp\left[-\left(\frac{t}{\eta_1}\right)^{\beta_1}\right][p(C_{BF}-C_{BP})] + \frac{\beta_2}{\eta_2}\left(\frac{t}{\eta_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{t}{\eta_2}\right)^{\beta_2}\right][C_{GF}(1-p)] \tag{7}$$

Then, minimal average cost is attained at t such that

$$\frac{\partial C_{AV}}{\partial t} = 0. \tag{8}$$

From equation (7) and (8), we have the following formula:

$$\begin{aligned}&\beta_1\ln t-\beta_2\ln t+(t/\eta_2)^{\beta_2}-(t/\eta_1)^{\beta_1}\\&=\ln\beta_2-\ln\beta_1+\beta_1\ln\eta_1-\beta_2\ln\eta_2+\ln(1-p)-\ln p+\ln C_{GF}-\ln(C_{BP}-C_{BF})\end{aligned} \tag{9}$$

From equation (8), we have the results for determining the optimal burn-in time of two-mixed Weibull distributions. In order to simplify equation (9), without much loss of generality, it is assumed that $C_{GF}=2C_{BF}$ and the characteristic life of weak population is shorter than the characteristic life of main population. These assumptions lead to

1. If the shape parameters of weak and main populations are the same ($\beta_1=\beta_2=\beta$, this is the special case in practice), then the optimal burn-in time t can be calculated from the equation (9) as following:

$$t=\left\{\frac{\eta_1^\beta\eta_2^\beta}{\eta_2^\beta-\eta_1^\beta}\left[\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)+\ln\left(\frac{p\eta_2^\beta}{(1-p)\,\eta_1^\beta}\right)\right]\right\}^{1/\beta} \tag{10}$$

2. If the shape parameters of weak and main populations are different (this case is common in practice), then the equation (9) can be simplified as following.

$$(\beta_2-\beta_1)\ln t+\left(\frac{t}{\eta_1}\right)^{\beta_1}-\left(\frac{t}{\eta_2}\right)^{\beta_2}=\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)+\ln\left|\frac{p\beta_1\eta_2^{\beta_2}}{(1-p)\beta_2\eta_1^{\beta_1}}\right| \tag{11}$$

The optimal burn-in time t is defined by the solution to equation (11).

Note: For a two-mixed Exponential distribution (a special case of Weibull distribution with $\beta_1=\beta_2=1$), we have the formula for minimum burn-in time from equation (9) as follow:

$$t=\frac{\eta_1\eta_2}{\eta_2-\eta_1}\left[\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)+\ln\left(\frac{p\eta_2}{(1-p)\eta_1}\right)\right] \tag{12}$$

In equations (10) - (12), The optimal burn-in time t is defined under normal stress conditions.

5. Acceleration of Burn-In

The optimal burn-in time t in equation (10)-(12) is in terms of "ordinary use" times. The optimal burn-in time t may be too long to perform in a factory. In that case, accelerated burn-in should be considered. The accelerated stress test conditions are intended to hasten the times to remove weak components or systems. The two most important failure accelerating mechanisms are temperature and bias voltage. Failure data from accelerated tests will be useful if a relationship can be established between the time to failure at normal operating conditions and the time to failure at accelerated operating conditions. Suppose that "typical life" of a failure mode is t_1 at a designed normal condition and t_2 at an accelerated test condition. Then the acceleration factor A for those two conditions is $t_1=At_2$. For example, if A = 100, then 1000 hours of burn-in for a particular component at the normal condition corresponds to 10 hours of burn-in under accelerated condition. The value of A varies with type of components and the degree of stress under the accelerated conditions.

One of the most useful models relating component lifetime with temperature has been the Arrhenius model. Arrhenius lifetime relationship is (Jensen & Petersen 1982)

$$t = C\exp(E_a/kT) \tag{13}$$

where t is the temperature-lifetime and C is a (temperature-independent) constant. E_a is the activation energy (eV) which is determined experimentally by observing the times-to-failure of different batches of components at different temperatures. For semiconductor devices, a commonly cited value for E_a is 1.0 eV (refer to MIL-STD-883B, Test Methods and Procedures for Microelectronics). k is Boltzmann's constant (electron-volts per ℃) and T is the absolute Kelvin temperature (^oK) (it equals the Centigrade temperature plus 273.16 degrees).

The arrhenius acceleration factor between lifetime t_1 at designed normal temperature T_1 and lifetime t_2 at accelerated temperature T_2 is defined by equation (13) as follow:

$$A = t_1/t_2 = \exp\{(E_a/k)[(1/T_1) - (1/T_2)]\} \tag{14}$$

Example: For a microprocessor, assume that the accelerated test temperature for burn-in is 125℃ and the designed normal temperature for burn-in is 55℃. Then $T_1 = 55 + 273.16 = 328.16\ ^oK$ and $T_2 = 125 + 273.16 = 398.16\ ^oK$. The activation energy is assumed to be $E_a = 1.0eV$. Then the corresponding acceleration factor is found from equation (14) as

$$A = \exp\{(1.0/8.6171\times10^{-5})[(1/328.16) - (1/398.16)]\} = 501.$$

Since the acceleration factor is 501, burn-in for 2 hours under the accelerated test temperature 125℃ corresponds to burn-in about 1000 hours at normal temperature of 55℃.

If a manufacturer felt that the estimated optimal burn-in time under the normal conditions is too long, burn-in time under accelerated stress condition using equation (14) could be considered. We develop a table of acceleration factors for different values of normal temperature and accelerated temperature when E_a assumed to be 1.0 (see <Table 3>).

< Table 3 > Arrhenius Acceleration Factors for $E_a = 1.0\,eV$

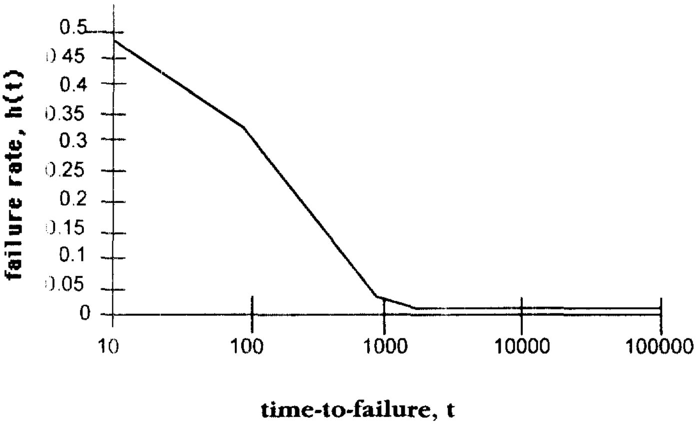
The Accelerated Temperature (℃)

℃	125	135	145	155	165	175	185	195	205	215
25	17594	35933	70923.6	135608	251731	454565	799930	1374110	2307614	3793854
35	4975.6	10161	20056.9	38349	71188	128549	226217	388593	652584	1072887
45	1523.3	3111.2	6140.72	11741	21795	39357	69259.6	118973	199798	328480
55	501.28	1023.7	2020.70	3863.6	7172.1	12951	22791.0	39150	65746	108091
65	176.16	359.78	710.12	1357.7	2520.4	4551.3	8009.35	13758	23105	37986
75	65.74	134.26	265.01	506.70	940.60	1698.4	2988.96	5134.4	8622.4	14175
85	25.92	52.94	104.49	199.79	370.87	669.71	1178.54	2024.4	3399.8	5589.5
95	10.75	21.95	43.33	82.86	153.82	277.76	488.79	839.65	1410.1	2318.2
105	4.67	9.54	18.83	36.00	66.83	120.69	212.38	364.83	612.68	1007.3
115	2.11	4.32	8.54	16.33	30.31	54.74	96.33	165.48	277.90	456.88
125		2.04	4.03	7.70	14.30	25.83	45.46	78.09	131.15	215.62
135			1.97	3.77	7.01	12.65	22.26	38.24	64.21	105.57
145				1.91	3.54	6.41	11.27	19.37	32.53	53.49
155					1.85	3.35	5.89	10.13	17.02	27.98
165						1.81	3.18	5.46	9.17	15.07
175							1.76	3.02	5.08	8.35
185								1.72	2.88	4.74
195									1.68	2.76
205										1.64

In <Table 3>, the first column is the designed normal stress temperature (corresponds to T_1) from 25℃ to 205℃. The first row is the accelerated stress temperature (corresponds to T_2) from 125℃ to 215℃. For example, if the usual operation high temperature is 75℃(T_1 = (75+273.16) ^oK) and we wish to run our burn-in under stress temperature at 145℃(T_2 = (145+273.16) ^oK), the acceleration factor is 265 from <Table 3>. Thus, burn-in for 3 hours at 145℃ coresponds to normal high temperature operation for almost 800 hours.

6. Illustrative Examples

Example 1: Here, we use the example of Jensen & Petersen (1982). A manufacturer of electronic circuits receives a batch of 1000 CMOS transistors From previous experience, engineering expects to find approximately $p = 0.05$ weak transistors in the batch. Assume that the time-to-failure distribution of the weak population is exponential with an mean time to failure equals to 200 hours at a test temperature of 125℃. At the same temperature, the mean time to failure of main population is known to be around 100,000 hours. Past investigation suggest the time-to-failure distribution of the main population is also exponential In this case, the failure rate of the weak components and strong subpopulation are constants. However, the failure rate of the mixed population decreases throughout the early failure period. <Figure 3> shows the failure rate curve of this mixed population. From the shape of the curve in Figure 3, an initial estimate for the burn-in time is between 1000 hours and 2000 hours to reach the useful failure region.



< Figure 3 > Bathtub Curve of Mixed Population in Example 1

Now, let us investigate this example further. What should be the expected burn-in time for this case? Assuming that $\widehat{\beta}_1 = \widehat{\beta}_2 = 1$, $\widehat{\eta}_1 = 200$ hours, $\widehat{\eta}_2 = 100,000$ hours and $\widehat{p} = 0.05$ are reasonable estimates, then by equation (12), the optimal burn-in time

$$t = \frac{20000000}{100000-200}\left[\ln\left(\frac{C_{BP}-C_{BF}}{C_{GF}}\right)+\ln\left(\frac{5000}{190}\right)\right]$$

$$=200.4008\left[\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)+3.2701691\right]$$

Suppose that $\left(\frac{C_{BP}}{C_{BF}}\right)=200$, then $t=1577$ hours. Thus, the optimum burn-in time t for the two-mixed exponential distribution is 1577 hours if the ratio of a field failure cost or repair cost possibly further downstream assembly and the replacing cost when a bad component fails the burn-in test is 200. The probability that an initially weak transistor passes burn-in (p_B) is 0.000376 and the probability that an initially main transistor passes the burn-in $(1-p_G)$ is 0.98435 by equation (5). Thus, more than 99% of weak transistors are eliminated while about 98% of main transistors are survived at burn-in time 1577 hours with minimum total cost.

Now, we can estimate the probability model for Example 1 using the values $\hat{p}=0.05, p_B=0.000376$ and $(1-p_G)=0.98435$. The estimated probabilities are shown in <Table 4>.

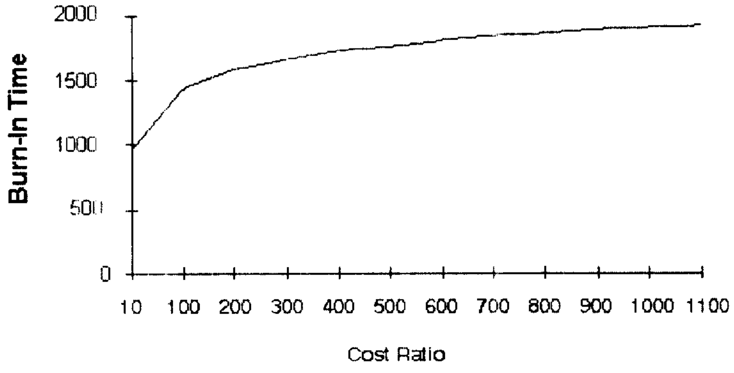
< Table 4 > Probability Model of Example 1 for Burn-In Time 1577 hours

Results of Burn-In	Main population	Weak population
Pass	0.93513395	0.000018797
Fail	0.01486605	0.0499812

In <Table 4>, the probability that a component survives the burn-in and was from the main portion of the 1000 CMOS transistors is 0.93513395 ($(1-p)(1-p_G)$) and the probability that a component survives the burn-in and was from the weak portion of the 1000 CMOS transistors is 0.000018797 (pp_B) These imply that 935 transistors out of 950 main CMOS transistors will pass the burn-in while 1 transistor out of 50 weak CMOS transistors may pass the burn-in.

<Figure 4> represents the relationship between the optimal burn-in time t and the cost ratio for this problem. The optimal burn-in time is increasing with the cost ratio. An approximate burn-in time of 1000-2000 hours might be considered

depending on the cost ratio of C_{BP} and C_{BF}. The average burn-in cost can be calculated from the equation (4) if all the cost functions and the probability of the weak population are assigned.



< Figure 4 > Optimal Burn-In Time t versus Cost Ratio for Example 1

Suppose that E_a is 1.0 eV and increase the stress temperature to 215℃ from the normal temperature 125℃. Then using equation (14) (or reading directly from <Table 3>), the acceleration factor is

$$A = \exp\{(1.0/8.6171\times 10^{-5})[(1/398.16)-(1/488.16)]\} = 216.$$

Under normal stress temperature at 125℃, the estimated optimum burn-in time is 1577 hours if $\left(\frac{C_{BP}}{C_{BF}}\right) = 200$. At the accelerated temperature 215℃, $t = \frac{1,577}{216} = 7.3$ hours. Therefore, burn-in under stress temperature of 215℃ for about 7 hours corresponds to 1577 hours under normal stress of 125℃. This represents a significant reduction in burn-in.

Example 2: This example is adapted from Kececioglu and Sun (1994). For the failure data of 19 components from $n = 150$ CMOS components tested at 125℃ and $5V$, the estimates of parameters were found using the Bayesian approach (Kamath, 1978) as follows:

$$\widehat{p} = 0.067226 \quad \widehat{\beta}_1 = 1.62, \quad \widehat{\eta}_1 = 535 \text{ hours}, \quad \widehat{\beta}_2 = 4.3, \quad \widehat{\eta}_2 = 8,250 \text{ hours}$$

Our goal is to estimate an optimal burn-in time with minimum total average burn-in cost from the above estimated parameter values. Assuming that the Bayesian estimates are reasonable estimates, then from equation (11), we have

$$(2.68)\ln t+\left(\frac{t}{535}\right)^{1.62}-\left(\frac{t}{8250}\right)^{4.3}=\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)$$

$$+\ln\left(\frac{0.067226\times 1.62\times(8250)^{4.3}}{0.932774\times 4.3\times(535)^{1.62}}\right)$$

Now, suppose that $\left(\frac{C_{BP}}{C_{BF}}\right)=200$, that is cost of field repair is 200 times the cost for replacing bad component. Then optimal burn-in time is approximately $t=2092$ hours.

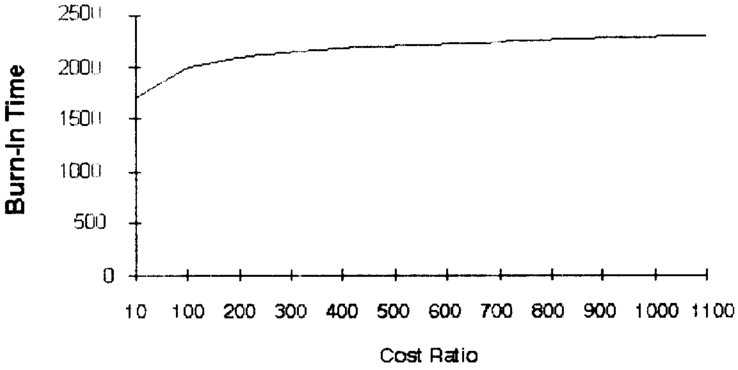
Let us now consider the probabilities of errors during burn-in. The probability that an initially weak component passes burn-in (p_B) is almost zero and the probability that an initially main component passes burn-in (($1-p_G$)) is 0.9973 by equation (5). The results imply that more than 99% of weak components are eliminated while about 99% of main components are survived through burn-in with minimized total cost. Now, the probability model for this example is shown in <Table 5>.

< Table 5 > Probability Model of Example 2 for Burn-In Time 2092 hours

Results of Burn-In	Main population	Weak population
Pass	0.9302231	0.00000667
Fail	0.0025509	0.06721932

From <Table 5>, we see that the probability that a component survives the burn-in and was from the main portion of the population is 0.9302231 ($(1-p)(1-p_G)$) and probability that a component survives the burn-in and was from the weak portion of the population is 0.00000667 (pp_B). Since $\widehat{p}$ = 0.067226, the weak components are 10 components out of 150 tested components (0.067226150 × 150=10 and the main components are 140 components out of 150 tested components ((1 - 0.067226) × 150 = 140). Thus, almost all weak CMOS components (10 components are eliminated and about 10 components from 140 main components are damaged after 2092 hours of burn-in.

<Figure 5> represents the relationship between the optimal burn-in time t and the cost ratio for this problem. The optimal burn-in time is increasing while the cost ratio is increasing and an approximate burn-in time of 1700-2300 hours might be estimated depending on the cost ratio of C_{BP} and C_{BF}.



< Figure 5 > Optimal Burn-In Time t versus Cost Ratio for Example 2

Suppose that E_a is 1.0 eV and increase the stress temperature to 215℃. Then using equation (14), acceleration factor is

$$A = \exp\{(1.0/8.6171\times10^{-5})[(1/398.16)-(1/488.16)]\} = 216.$$

Under the stress temperature at 125℃, the estimated optimum burn-in time is 2092 hours if $\left(\frac{C_{BP}}{C_{BF}}\right) = 200$. At the accelerated temperature 215℃, $t = \frac{2,092}{216} =$ 9.68 hours. Therefore, burn-in about 10 hours under the accelerated temperature 215℃ corresponds to burn-in 2,092 hours at temperature of 125℃. This represents a significant reduction in burn-in.

Example 3: A life test carried out on a sample of 30 electronic systems (Jensen & Petersen, 1982). Using Jensen & Petersen method and Baysian approach, the estimated values were obtained as follows:

$$\hat{p} = 0.18,\quad \hat{\beta}_1 = 0.75,\quad \hat{\eta}_1 = 20 \text{ hours},\quad \hat{\beta}_2 = 1.0,\quad \hat{\eta}_2 = 2200 \text{ hours}$$

From the analyses, they estimated approximate burn-in time 48-72 hours for eliminating about 90% of the weak population. Let us assume that 48-72 hours are reasonable estimates. Then, the estimation of the probabilities that an initially weak population is eliminated through burn-in ($1-p_B$) and the probabilities that an initially main population passes burn-in (($1-p_G$)) are shown in <Table 6>. In <Table 6>, the first column shows the burn-in time from 48 hours to 72 hours in a fixed interval.

< Table 6 > Probabilities of ($1-p_B$) and ($1-p_G$) for 48 hours to 72 hours of burn-in

Burn-In Time	$1-p_B$	$1-p_G$
48	0.85459440	0.97841811
52	0.87094657	0.97664079
56	0.88519893	0.97486669
60	0.89766536	0.97309582
64	0.90860441	0.97132816
68	0.91823084	0.96956371
72	0.92672430	0.96780247

From <Table 6> we see that the probabilities of main systems passes burn-in decreases and the probabilites of eliminated weak systems increases when the burn-in time increases. The results imply that about 85-93% of weak systems are eliminated while about 97-98% of main systems are survived through 48-72 hours of burn-in without considering burn-in cost factors. Therefore, the estimated burn-in times by Jensen & Petersen are reasonable for eliminating about 90% of weak systems. For this example, the optimal burn-in time with minimum cost is calculated from the following:

$$(0.25)\ln t+\left(\frac{t}{20}\right)^{0.75}-\left(\frac{t}{2200}\right)=\ln\left(\frac{1}{2}\left(\frac{C_{BP}}{C_{BF}}-1\right)\right)+\ln\left(\frac{0.18\times 0.75\times 2200}{0.82\times 20^{0.75}}\right)$$

Now, suppose that $\left(\frac{C_{BP}}{C_{BF}}\right)=10$, then optimal burn-in time is approximated, $t=127$ hours. Then, the probability that an initially weak system is eliminated through burn-in is 0.982 and the probability that an initially main system passes the test is 0.944 by equation (5). This result implies that more than 98% of weak systems are eliminated while about 94% of main systems are survived through burn-in. Using Jensen & Petersen's solution, about 85-93% of weak systems were eliminated and about 97-98% of main systems were survived through 48-72 hours of burn-in without considering burn-in cost factors. Even though the optimal solution has longer burn-in time and less surviving main systems, the optimal solution is better choice than Jensen & Petersen's solution since the total cost is the minimum and eliminating weak systems' probability is very high.

For the whole tested systems, we construct probability model as in <Table 7>. In <Table 7>, the probabilities are for the whole systems.

< Table 7 > Probability Model of Example 3 for 48 hours to 72 hours of Burn-In

Burn-In Time	$(1-p)(1-p_G)$	pp_B	$(1-p)p_G$	$p(1-p_B)$
48	0.80230285	0.02617301	0.00388474	0.15382699
52	0.80084545	0.02322962	0.00420466	0.15677038
56	0.79939069	0.02066419	0.00452400	0.15933581
60	0.79793857	0.01842024	0.00484275	0.16157976
64	0.79648909	0.01645121	0.00516093	0.16354879
68	0.79504224	0.01471845	0.00547853	0.16528155
72	0.79359803	0.01318963	0.00579556	0.16681037

<Table 7> shows that main electronic systems pass the burn-in are about 80% of tested electronic systems while weak electronic systems pass the burn-in are about 2-3% of tested electronic systems. Thus, about 82% of tested electronic systems pass the 48-72 hours of burn-in.

The estimated probability model of the optimal solution is defined as in <Table 8>. <Table 8> shows the estimated probabilities for each possible event during 127 hours of burn-in.

< Table 8 > Probability Model of Example 3 for 127 hours of Burn-In

Results of Burn-In	Main population	Weak population
Pass	0.77400402	0.0032962
Fail	0.01009668	0.1767038

<Table 8> shows that main electronic systems pass the burn-in are about 77% of tested electronic systems while weak electronic systems pass the burn-in are

about 0.3% of tested electronic systems. Thus, about 77% of tested electronic systems pass the 127 hours of burn-in.

Now, let us compare the <Table 7> and <Table 8>. Weak electronic systems pass the burn-in are about 0.3% of tested electronic systems for the optimal solution <Table 8> and about 2-3% of tested electronic systems for Jensen & Petersen' solution <Table 7>. Damaged main electronic systems from burn-in are about 1% of tested electronic systems for the optimal solution <Table 8> and about 0.3-0.5% of tested systems for Jensen & Petersen' solution <Table 7>. In practice, the cost of passed weak population from burn-in is much higher than the cost of damaged main population from burn-in. Therefore, the optimal solution is prefered for this example for the purpose of minimizing the cost and eliminating maximum weak systems.

7. Conclusion

We open encounter the reliability situations associated with weak (defective) and main (strong) subpopulations for many components. In these situations, eliminating the weak population should be done to improve reliability of components. In this paper, burn-in probability model and burn-in cost model were formulated to find the optimal burn-in time for a process with weak population. The new model includes the errors of burn-in and corresponding probability. The optimal burn-in time is determined to minimize the total burn-in cost.

When a process has mixed population, weak population fails faster than main population for stress. Thus, the characteristic life of weak population is much shorter than the characteristic life of main population. Under this assumption, three formulas were proposed to find the optimal burn-in times for three possible different situations. The proposed formulas will perform well as long as moderately accurate estimated parameter values are used. As shown in the examples, the optimal burn-in time for a product depends on the ratio of a field failure cost (or repair cost possibly further downstream assembly) and the replacing cost. Therefore, defining these costs should be done before planning a burn-in procedure.

The findings of this study can be used by a reliability engineer or analyst as a guide for planning an optimal burn-in procedure for a process with weak population.

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