⊠ 연구논문

Periodic Inspection of a Random Shock Model*

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Abstract

A Markovian stochastic model for a system subject to random shocks is considered. Each shock arriving according to a Poisson process decreases the state of the system by a random amount. A repairman arrives at the system periodically for inspection and repairs the system only if the state is below a threshold. Costs are assigned to each inspection of the repairman, to each repair, and to the system being in bad states below the threshold. The expected long run average cost is obtained and compared with that of the random inspection introduced by Lee and Lee(1994).

1. Introduction

Lee and Lee(1993) introduced a random shock model for a system. It is assumed that the state of the system is initially $\beta > 0$ and thereafter deteriorates jumpwise due to the shocks which come to the system according to a Poisson process of rate $\nu > 0$. The shocks instantaneously decrease the state of the system by random amounts which are independently and identically distributed with distribution function H. It is further assumed that the system is repaired by a repairman who

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arrives at the system for inspections according to another Poisson process of rate $\lambda > 0$; if the state of the system when he inspects exceeds a threshold α $(0 < \alpha < \beta)$, he does nothing, otherwise he instantaneouly increases the state of the system up to β. They studied the distribution function of $X(t)$, the state of the system at time t. Lee and Lee(1994) also obtained the expected long-run average cost after assigning various costs to the system and then sought to minimize the average cost by varying the inspection rate λ of the repairman.

In this paper, we consider another inspection rule in which the time interval between two successive inspections is a fixed quantity τ instead of the exponential random variable of Lee and Lee(1993). Zuckerman(1980) and Abdel-Hameed(1987) also studied the periodic inspection and the replacement problem of similar models for the system and showed that the optimal replacement policy is a control limit policy i.e. replacing the system either upon the detection of a failure or at the first inspection time at which the accumulated damage exceeds a given fixed value, whichever occurs first. We, however, in this paper assume that the state of the system can be known only by an inspection of the repairman and the system is repaired perfectly or replaced by a new one only if the state of the system is less than the threshold α, when the repairman inspects the system. After assigning costs to each inspection, to each repair, and to the system being in bad states as Lee and Lee(1994) did in the earlier analysis, we obtain the expected long-run average cost $Cpe(\tau)$ for a given inspection interval τ. We compare this cost with the expected long-run average cost $C(\lambda)$ for a given inspection rate λ obtained in Lee and Lee(1994).

2. Expected long-run average cost

Let a denote the cost per inspection of the repairman, let b denote the expected cost of a repair and let $c\alpha$ denote the penalty cost per unit time of the system being in bad states below α.

The points where the actual repair occurs form an embedded renewal process because the system restarts probabilistically at these points. Let T be the generic random variable denoting the time interval between two successive renewals and

N be the number of inspections until the actural repair occurs during the cycle T Then

$$T \equiv \tau N, \tag{1}$$

where $\equiv$ denotes equality in distribution.

It can be easily seen that the expected total cost during a cycle for a given inspection interval τ is given by

$$C_T(\tau) = aE(N) + b + c_a E(T - T_a)$$

where T_a is the first passage time from state β to state α during a cycle. Lee and Lee(1993) showed that the distribution function $U_a(t)$ of T_a is given by

$$U_a(t) = 1 - \sum_{n=0}^{\infty} H^{(n)}(\beta - \alpha) \frac{\exp(-\nu t)(\nu t)^n}{n!}, \tag{2}$$

where $H^{(n)}$ is the n-fold recursive Stieltjes convolution of $H, H^{(0)}$ being the Heaviside function which is defined by

$$H^{(0)}(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

By applying the Renewal Reward theorem we obtain the expected long-run average cost $C_{pe}(\tau)$ for a given τ,

$$\begin{aligned} C_{pe}(\tau) &= \frac{aE(N) + b + c_a E(T - T_a)}{E(T)} \\ &= c_a + \frac{a}{\tau} - \frac{c_a E(T_a) - b}{\tau E(N)}, \end{aligned} \tag{3}$$

by equation (1). Here $E(T_a)$ is obtained from equation (2). To evaluate $E(N)$, notice that N is less than or equal to n if and only if the total damage up to $n\tau$ during a cycle is larger than or equal to $\beta - \alpha$, and hence

$$N = \inf\{n \mid \text{total damage up to } n\tau \text{ during the cycle } T \geq \beta - \alpha\}$$
$$\equiv \left\langle \frac{T_\alpha}{\tau} \right\rangle + 1,$$

where $\langle a \rangle$ is the largest integer less than or equal to a.

Therefore,

$$P\{N = n\} = \overline{U_\alpha}((n-1)\tau) - \overline{U_\alpha}(n\tau),$$

where $\overline{U_\alpha}(t) = 1 - U_\alpha(t)$ and

$$E(N) = \sum_{n=o}^{\infty} \overline{U_\alpha}(n\tau).$$

3. Comparison

Lee and Lee(1994) studied this random shock model with random inspection policy, that is, a repairman coming to the system according to a Poisson process of rate λ inspects the system and repairs (or replaces) the system if and only if the state is below the threshold α. They assigned to the system the same costs as those in this paper and obtained the expected long-run average cost $C(\lambda)$ given by

$$C(\lambda) = \frac{a\lambda^2 E(T_\alpha) + (a+b)\lambda + c_\alpha}{\lambda E(T_\alpha) + 1}. \quad (4)$$

They showed that there exists a unique arrival rate λ which minimizes $C(\lambda)$.

In this section, we compare $C_{pe}(\tau)$ obtained in section 2 with $C(\lambda)$ by putting $\lambda = 1/\tau$, that is, we equate the long-run inspection rate of the random inspection policy with that of the periodic inspection policy, which results in the same long-run inspection cost in both inspection policies. We obtain the following:

Theorem *Under the condition that* $c_\alpha E(T_\alpha) - b \geq 0$,

$$C_{pe}(\tau) \leq C(\frac{1}{\tau}),$$

and $C_{pe}(\tau) \geq C(\frac{1}{\tau})$, *otherwise.*

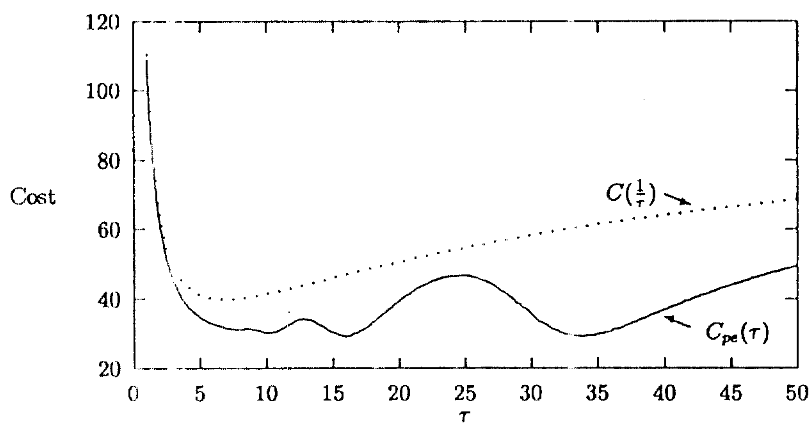
Proof Since $N \equiv \left\langle \frac{T_\alpha}{\tau} \right\rangle + 1$, we obtain that

$$E(N) \le \frac{E(T_\alpha)}{\tau} + 1.$$

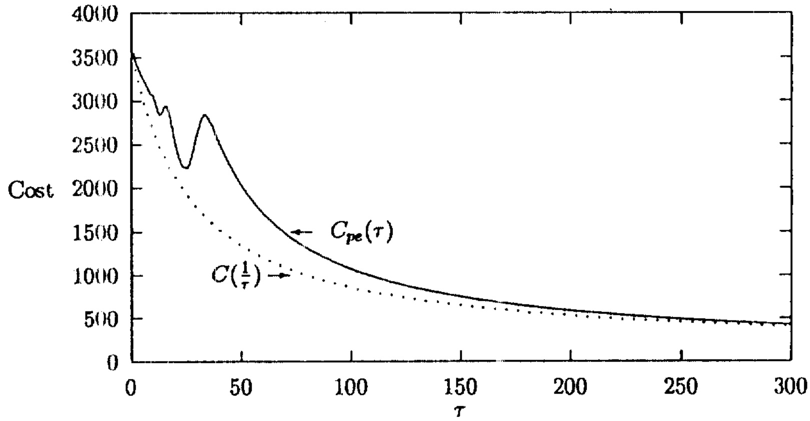
By substituting this relation into the equation (4), we obtain the desired result depending on whether $c_\alpha E(T_\alpha) - b \ge 0$ or not.

Remark

1. Since $C(\frac{1}{\tau})$ and $C_{pe}(\tau)$ include the same long-run inspection cost, the above condition does not depend upon the inspection cost a. The condition $c_\alpha E(T_\alpha) \ge b$ implies that the periodic inspection is better than the random inspection if the repair cost is relatively smaller than the penalty.
2. Specially, when H is an exponential distribution function with parameter $\xi > 0$, we illustrate two cost functions under two conditions $c_\alpha E(T_\alpha) \ge b$ and $c_\alpha E(T_\alpha) \le b$ in <Figure 1> and <Figure 2>, respectively.
3. As we can see in <Figure 1>, the uniqueness of τ which minimizes $C_{pe}(\tau)$ can not be guaranteed.



< Figure 1 > β=100, α=30, ν=5, ξ=2, a=100, b=200, c_α=100, c_0=0



< Figure 2 > β=100, α=30, ν=5, ξ=2, a=50, b=100000, c_{α}=100, c_0=0

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