Econ. Environ. Geol.
Vol. 29, No. 3, p. 325-333, 1996.

Development and Application of a Landfill Gas Migration Model

Yu-Chul Park*, Kang-Kun Lee*, Chul-Hwi Park and Yong-Woo Kim****

ABSTRACT : numerical model is developed to estimate gas flow in the landfill site. Darcy's law, the mass conservation law, and the ideal gas state equation are combined to compose the governing equation for the steady-state and transient-state gas flows. The finite element method (FEM) is used as the numerical solution scheme. Two-dimensional radial symmetric triangular ring element is used to discretize the simulation domain. The steady state model developed in this study is compared with AIRFLOW that is a commercial model developed by Hydrologic Inc. Mass balance test is performed on the transient gas flow simulation. The developed model is applied to analyze the gas extraction experiment performed by Daewoo Institute of Construction Technology at the Nanjido landfill in 1993. The developed model was registered at Korea Computer Program Protection Foundation.

INTRODUCTION

Sanitary landfilling is a major means of solid waste disposal in many countries for reasons of economy and simplicity. Current trends suggest the continuation of this method of disposal in the future. Anaerobic decomposition of the landfill refuses generates a large amount of CH_4 and CO_2 (Mohsen *et al.*, 1980). The problem of CH4 migration from landfills and subsequent explosions in underground structures has received increasing attention during recent years.

Many researchers have studied about the landfill gas. Moore (1979) studied about the generation, migration, and the control of the landfill gas. Mohsen *et al.* (1980) studied the gas migration and the vent design at landfill sites. Hartz and Ham (1982) analyzed the mechanism of gas generation and calculated the gas generation rates of a number of landfill waste samples. Hanashima *et al.* (1981) have analyzed the production of heat and gas in the semiaerobic landfill. Pacey (1981) used a variety of models to predict the methane gas production in a landfill and to assess the gas recovery potential at a landfill. Simulation models for gas-flow simulation can use the research results on gas gene- ration to compute gas pressure in the landfill. Com- pared with the study on landfill gas production ratio the study on landfill gas-pressure distribution is far more insufficient.

In order to control and contain migrating gases, reliable estimates of the extent of gas migration are necessary. These are difficult to acquire because only limited field data are available in most cases. The landfill site is the place where the soil is mixed with the wastes which has various physical and chemical properties. The gas and groundwater move through the pores which are the void spaces between wastes and soils. It is difficult to estimate gas migration by solving simple mathematical equations because the mixture of soil and waste are very heterogeneous and the gas flow is the density-dependent flow.

The gas flow is coupled with the moisture flow because the moisture content greatly influences on the gas permeability of a porous medium. Generally, simulation of gas flows is more complicated than the simulation of groundwater flow. A complete 3-D gas migration model is not present yet. The most advanced gas flow model developed up to date is the axisymmetric model where the axis indicates the gas extraction well and the gas flow is assumed to be axisymmetric. The study also is going to develop an axisymmetric model, but it tries to improve the method to implement a complicated boundary and well conditions and to apply it to both to steady- and transient-state gas flows.

The governing equations for one species gas is established in this study. The developed model is used to simulate gas flow scenarios in the Nanjido Landfill.

MATHEMATICAL MODEL

The governing equation is derived by combining the mass continuity equation and Darcy's equation

---

* Department of Geological Sciences. Seoul National University
** Daewoo Institute of Construction Technology

(Huyakorn and Pinder, 1983).

The continuity equation takes the form

$$-\partial \rho q_i/\partial x_i = \partial \phi \rho/\partial t \qquad (1)$$

where ρ is the density of gas, and ϕ is the porosity of a medium.

Combining (1) and Darcy's law, we obtain

$$\frac{\partial}{\partial x_j}\left[\frac{k_{ij}}{\mu}\rho\left(\frac{\partial p}{\partial x_j}+\rho g\frac{\partial z}{\partial x_j}\right)\right]=\frac{\partial}{\partial t}(\phi\rho) \qquad (2)$$

where k_{ij} is the component of the intrinsic permeability tensor.

For an ideal gas, the equation of state is given by

$$\rho = Mp/RT \qquad (3)$$

where M is the molecular weight, R is the universal gas constant, p is the density of gas, and T is the absolute temperature. If the flow takes place under isothermal conditions, then the relation between gas density and pressure is given by

$$\frac{\partial \rho}{\partial t}=\frac{M}{RT}\frac{\partial p}{\partial t} \qquad (4)$$

Substitution of (3) and (4) into (2) yields

$$\frac{\partial}{\partial x_i}\left(\frac{k_{ij}}{\mu}p\frac{\partial p}{\partial x_j}\right)+\frac{\partial}{\partial x_i}\left(\frac{k_{ij}}{\mu}p\rho g\frac{\partial z}{\partial x_j}\right)=\frac{\partial}{\partial t}(\phi p) \qquad (5)$$

which can be written as

$$\frac{\partial}{\partial x_i}\left(\frac{K_{ij}}{\mu}p\frac{\partial p}{\partial x_j}\right)+\left(\frac{K_{ij}}{\mu}p\rho g e_j\right)=(p\alpha+\phi)\frac{\partial p}{\partial t} \qquad (6)$$

where e_j is the upward positive vertical unit vector and α is the compressibility of the rock matrix. When the effects of gravity and rock compressibility are neglected, (6) reduces to

$$\frac{\partial}{\partial x_i}\left(\frac{k_{ij}}{\mu}p\frac{\partial p}{\partial x_j}\right)=\phi\frac{\partial p}{\partial t} \qquad (7)$$

which may be written in the form

$$\frac{\partial}{\partial x_i}\left(\frac{k_{ij}}{\mu}\frac{\partial u}{\partial x_j}\right)=\frac{\phi}{p}\frac{\partial u}{\partial t} \qquad (8)$$

where $u=p^2$.

The relationship between the pressure gradient and the flow velocity is given by the Darcy's law,

$$q_r(t)=-\frac{k_{rr}}{\mu}\frac{\partial p(t)}{\partial r},\quad q_z(t)=-\frac{k_{zz}}{\mu}\frac{\partial p(t)}{\partial z} \qquad (9)$$

where $q_r(t)$ and $q_z(t)$ are the components of the Darcy flux in the radial and vertical directions r and z at time t. The pressure difference changes as time passes, and the velocity changes with time.

The components of the average linear gas velocity are given by

$$v_r(t)=\frac{q_r(t)}{\phi},\quad v_z(t)=\frac{q_z(t)}{\phi} \qquad (10)$$

where ϕ is the porosity for gas flow.

The pathlines provide a clear visual description of the flow regime and they are useful in determining the radius of influence of an extraction system. The two-dimensional characteristic equation of a pathline is given by

$$s(r,\ z)=s(r_0,\ Z_0)+\int v(t)dt \qquad (11)$$

where s is a vector containing the r, z coordinates of the pathlines, $s(r_0,\ z_0)$ indicates the starting point of the pathline (initial condition), and v is the average gas velocity at time t.

NUMERICAL MODEL

The Galerkin finite element method is used to solve the equation of gas flow . Two-dimensional vertical section of a porous medium with the radial symmetry is represented by linear triangular elements (Figure 1). A computer program was coded for radial symmetric linear triangular elements (Huyakorn and Pinder, 1983). The radial symmetric linear triangular element is a body made by rotating of the two-dimensional linear triangular element.

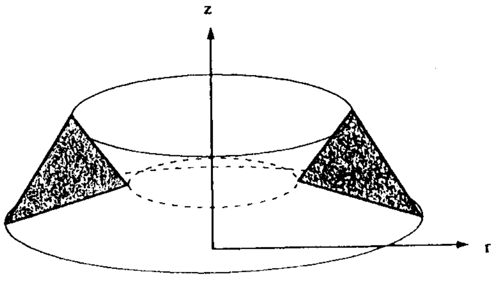


Fig. 1 A linear triangular ring element.

The boundary conditions allowed by the model is the constant pressure, the constant flux, and the time variation of flux. The Cauchy-type boundary condition seldom to be formed in landfill gas flows because flux boundaries are usually open to free air. All element sides located on the domain boundary are assumed to be impermeable unless they are specified.

The components of the Darcian flux are obtained from a further interpolation using the Galerkin method. From the Darcian fluxes the radial and vertical components of the pore velocities in the i-th element are calculated as

$$v_{r,i}(t)=\frac{q_{r,i}(t)}{\phi},\ v_{z,i}(t)\frac{q_{z,i}(t)}{\phi} \tag{12}$$

This formulation yields a discrete distribution of elemental velocities. The velocity vectors are defined at the element centroids.

The particle tracking method is used to trace the gas flow motion. Equation (11) is written in a discretized form as an explicit Euler-scheme:

$$r_{t+\Delta t}=r_t\pm v_{r_t}(t)\Delta t,\ z_{t+\Delta t}=z_t\pm v_{z_t}(t)\Delta t \tag{13}$$

where $\triangle t$ is the time increment, v_{r_t}, $v_{z_t}(t)$ are the (upstream) components of the average linear gas velocity at the current particle location at time t, and r and z are the pathline coordinates. The plus-sign in (13) corresponds to a forward tracking mode, the minus-sign corresponds to a reverse tracking mode. By using the forward tracking, the location of a particle at a future time can be predicted; by using the reverse tracking the location of a particle at a previous time can be traced.

The travel times are obtained from a single summation of all time increments:

$$t=t_0+\sum\Delta t \tag{14}$$

where t is the total travel time and t_0 is the initial time level.

The pathlines obtained with this upstream weighted Eulerian time integration scheme are piecewise linear. The quality of the Euler integration is a function of the length of the spatial step $\triangle s$:

$$\Delta s=v(t)\Delta t \tag{15}$$

The accuracy of the velocity v(t) at the current particle location strongly affects the quality of the pathline calculation. An interpolation scheme has to be used to process the discrete velocity field obtained from (12). In order to guarantee smooth and accurate pathlines, the spatial steps have to be small in areas where large changes in the direction and magnitude of the velocity field occur and the spatial steps can be larger in the areas of more uniform flow. These accuracy requirements are reflected in the automatic adjustment of the spatial and temporal increments.

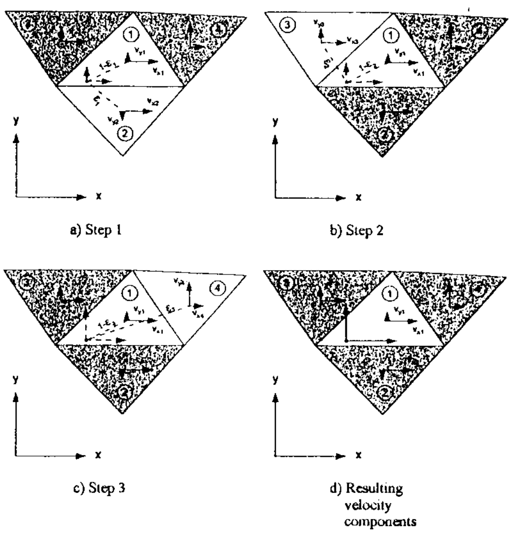

a) Step 1
b) Step 2
c) Step 3
d) Resulting velocity components

Fig. 2. Velocity interpolation.

The elemental velocities $v_{r,j}(t)$ and $v_{z,j}(t)$ are defined at the centroids of the grid elements. In order to calculate accurate pathlines and the velocity components, an interpolation scheme capable of calculating a velocity at any location in the domain must be employed. The velocity vector at an arbitrary particle location is calculated from a weighted average of the velocity components within the element that currently contains the particle and its three neighbor elements. The weights are inversely proportional to the distance between the current particle location and the centroids of these four elements (Figure 2). The velocity components at location (r, z) are obtained from a four step procedure:

Step 1: Interpolate between the velocities in the current element and the first adjacent elements.

$$\begin{aligned}V_{r,i}(t)&=\varepsilon_1 v_{r,1}(t)+(1-\varepsilon_1)v_{r,i}(t),\\ V_{z,i}(t)&=\varepsilon_1 v_{z,1}(t)+(1-\varepsilon_1)v_{z,i}(t)\end{aligned} \tag{16}$$

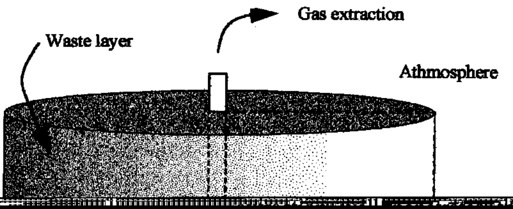
where $V_{r,1}(t)$ and $V_{z,1}(t)$ are the intermediate and interpolated velocity components, ε_1 is the weight, calculated as the ratio of $d_1/(d_1+d)$, where d is the distance between the particle and the element centroid of the element containing the particle, d_1 is the distance between the particle and the first adjacent element, $v_{r,i}(t)$ and $v_{z,i}(t)$ are the elemental velocity components defined at the centroid of the element containing the particle, and $v_{r,1}(t)$ and $v_{z,i}(t)$ are the elemental velocity components defined at the centroid of the first adjacent element.

Step 2: Repeat Step 1 for each adjacent element.

Step 3: Average the interpolated velocity components to obtain the current particle velocity.

$$v_r(r,z,t) = \frac{1}{3}\sum_{i=1}^{3} V_{r,i}(t), V_z(r,z,t) = \frac{1}{3}\sum_{i=1}^{3} V_{z,i}(t) \qquad (17)$$

where $v_1(r, z, t)$ and $v_z(r, z, t)$ are the final interpolated particle velocity components at location (r, z) and at time t. A particle is allowed to move a maximum distance of 10% of the average element size before a new velocity must be calculated. A new velocity is calculated whenever a particle intersects an element boundary. Particles are tracked until they are captured by a receptor (e.g. a sink) or leave the domain. The pathline calculation is terminated once the particle has intersected the boundary of an element that contains a well node. A new velocity is calculated whenever a particle intersects an element boundary. The terminal pathline coordinates are the well coordinates.



VERIFICATION OF MODEL

The developed numerical model is verified in two ways. For the steady state model, the simulation result was compared with that of AIRFLOW model. The AIRFLOW model is developed by Franz and Guiguer (1992) of Waterloo Hydrogeologic Software. The gas flow simulation for a simple model is performed by AIRFLOW and the model developed in this study. The simulation problem is described in Figure 3. The

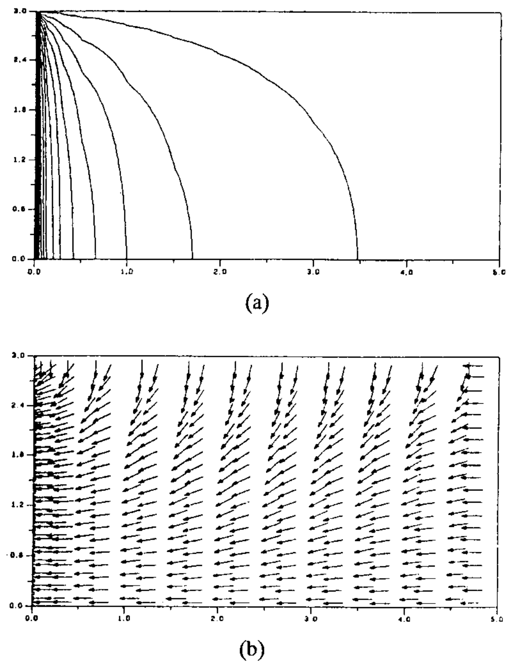


Fig. 5. Simulation results by using the developed model: pressure distribution (minimum=200 Torr, maximum=760 Torr, contour interval=290 Torr) (a) and velocity directions (b).

atmospheric pressure is 760 Torr and wellbore pressure is 200 Torr. Figure 4 is the simulation result of AIRFLOW. The pressure distribution and velocity profile are obtained by using AIRFLOW. Figure 5 is the simulation result of the developed model. The results are almost exactly same.

In order to verify the developed model for transient simulations, the mass balance is computed for a transient gas flow problem. If the mass balance is kept through all the model domain, the developed model has a certain degree of validity. If the mass balance is zero or near zero at every node at any time and the gas pressure converges to the steady-state gas pressure, then it is possible to say that there is no loss or gain of mass in the model and the gas pressure profile has the right form. It means that the mass conservation is kept in the model and the model has the validity in that point of view though it does not guarantee a perfect verification of the model. The mass balance error is less than 1% for the simulation from t=0 to t= 12 hours and 1,000 time steps to reach the ending time. Pressure distribution after long time is almost the same as that of the steady state model. The model parameters are equal to the previous steady state simulation shown in Figure 4.

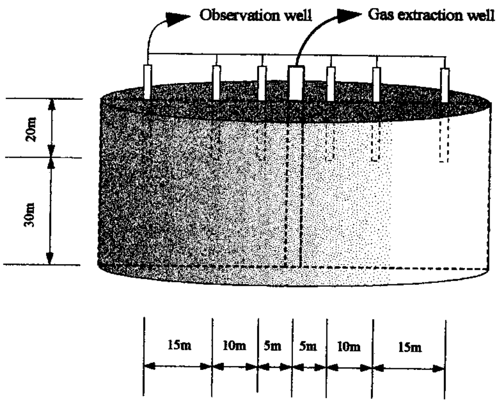


Fig. 6. Schematic diagram for the extraction experiment. The experiment was performed at the Nanjido Landfill in 1993 by Daewoo Institute of Construction Technology. One extraction well and six observation wells are established.

MODEL APPLICATION

Gas Extraction Experiment

The numerical model developed in this study can be used for the simulation of gas flow in a landfill. The model is very useful to design the gas extraction system in landfills. The developed model is applied to the experimental data at the Nanjido landfill. Nanjido landfill is used as the solid waste landfill from 1978 to 1992. The dump site has about 80 m height and 2,700,000 m^2 area. The wastes are buried without any sanitary design. The gas extraction experiment was performed by Daewoo Institute of Construction Technology in 1993. An extraction well with 50 m depth and six observation wells located at 5, 15, 30 m apart from the extraction well were installed (Figure 6). The gas pressure is recorded at each observation well. The extraction experiments were performed with three different rates, 6, 10, and 15 m^3/min.

Unfortunately, there are several problems which might degrade the validity of the test results. The gas pressure changes according to the atmospheric pressure and surface temperature. For the comparison of the gas pressure data with the simulation result, the gas pressure data should be corrected for the atmospheric pressure and temperature changes.

The complexity of physical properties in landfill adds the difficulty to the comparison. It is not clear that the Nanjido landfill has unsaturated zone up to 50 m depth which is the depth of the gas extraction well. In addition to that, there is a strong evidence

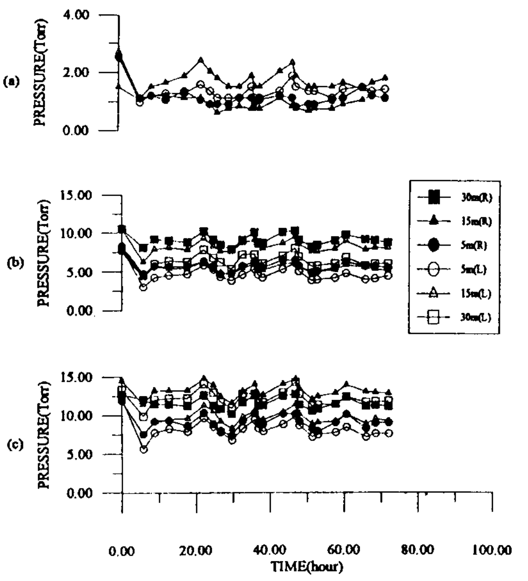


Fig. 7. The gas pressure drops with constant extraction rate Q=15 m^3/min according to time: at 5 m depth (a), at 10 m depth (b), and 15 m depth (c).

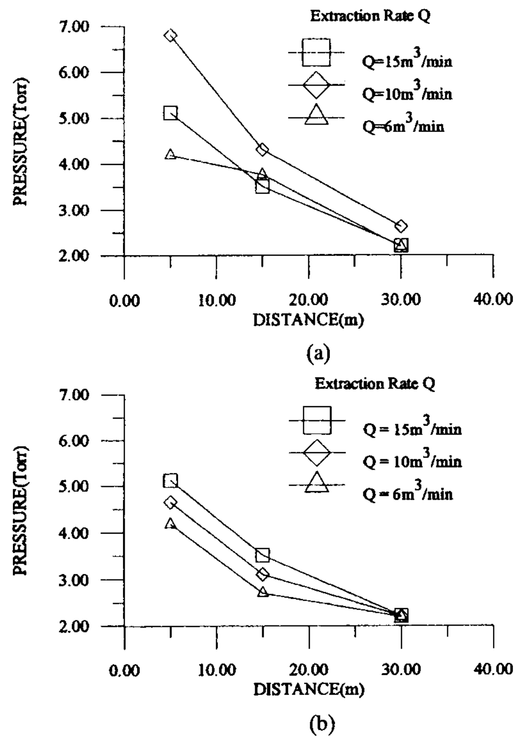


Fig. 8. The comparison of the filed data and the simulation results: the field data of the gas extraction experiment (a) and the simulation result by the numberical model (b).

that perched floating water layer is formed around the depth of 10~15 m below the landfill surface. Vapor condensation due to the temperature decrease along the upward direction, the gas-pressure increase due to the decreasing gas permeability, and hydraulic-conductivity changes are considered to be the important factors in the formation of the floating water layers. The field data, however, do not have the quality as to be used in the comparison with the numerical results.

Figure 7 shows the measured gas pressure with time for a constant extraction rate. The measured gas pressure undulates with high amplitude which is not theoretically understandable if it is not measurement error or experimental error. However, with some inferiority in quality, the outline of the field data can be compared with that of the simulation result. Figure 8 represents the experimental data and the simulation results. For the case of extraction rate Q=15 m^3/min and 6 m^3/min, the measured gas pressure has the magnitudes and changing trends very similar to the simulation result. But for the case of Q=10 m^3/min, the field data seem to be very strange because the gas pressure drop for the extraction rate of Q=10 m^3/min is greater than that of Q=15 m^3/min.

The conceptual model for the gas flow simulation at the Nanjido landfill is described in Figure 9. The simulation parameters are tabulated in Table 2. The simulation domain is divided into three different

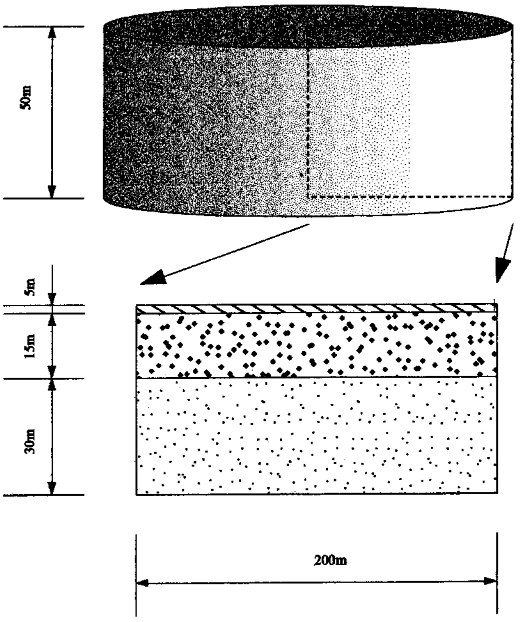


Fig. 9. Conceptual model for gas flow simulation in the Nanjido Landfill.

layers. The top layer is the surface soil layer, and the middle layer is the waste layer which is buried recently, and the bottom layer is the waste layer which is buried long time ago. The horizontal permeability of the waste layer is assumed 20 darcy which is general value for the solid waste landfill. The vertical permeability is assumed as 1/10~1/100 of the horizontal permeability because of compacting of the waste layer. The rate of gas generation for a m^3 waste volume is assumed to be 2.4 m3/year for the middle waste layer based on the report of the City of Seoul (1994). The gas production rate changes with the depth from landfill surface because the gas production rate decreases as time passes. The permeable surface is covered with top soils so that there is no gas generation in the surface soil layer.

Virtual Experiments

Gas flows for several situations are simulated by using the developed model. The simulation parameters are described in Table 2. The gas flow with the constant extraction pressure is simulated at first. And then, gas flows with three different extraction rates are simulated. Pressure accumulation with the embedded vertical fractures are simulated at last.

Table 2. Parameters for gas extraction simulation.

	Parameter	Value
μ	dynamic viscosity	4.3×10^{-2} cp
φ	porosity	10%
M	air molar mass	2.9×10^{-2} kg
R	universal gas constant	8.31 J/mol/°K
T	temperature	40°C(=313°K)
k	intrinsic permeability	20 darcy

1) Extraction with Constant Pressure

An extraction well is located at the center of the conceptual model in Figure 9. Gases are extracted at the depth of 15~20 m from surface. The extraction pressure is given to be 717 mmHg. The gas pressure profile and flow vectors are shown in Figure 10. The gas flow disturbed near the extraction well. The equi-pressure line is the shape of ellipse. The gas inside the capture zone is extracted into the well from a distance. The range of extraction could be defined as the distance from the well to furthest extracted gas.

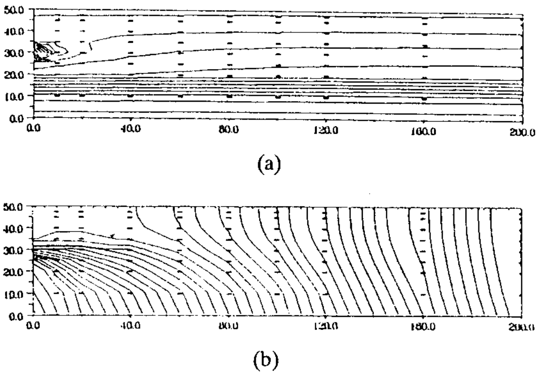


Fig. 10 Simulation results of gas extraction with a constant pressure, 717 Torr: gas pressure distribution (a) and gas flow pathlines (b).

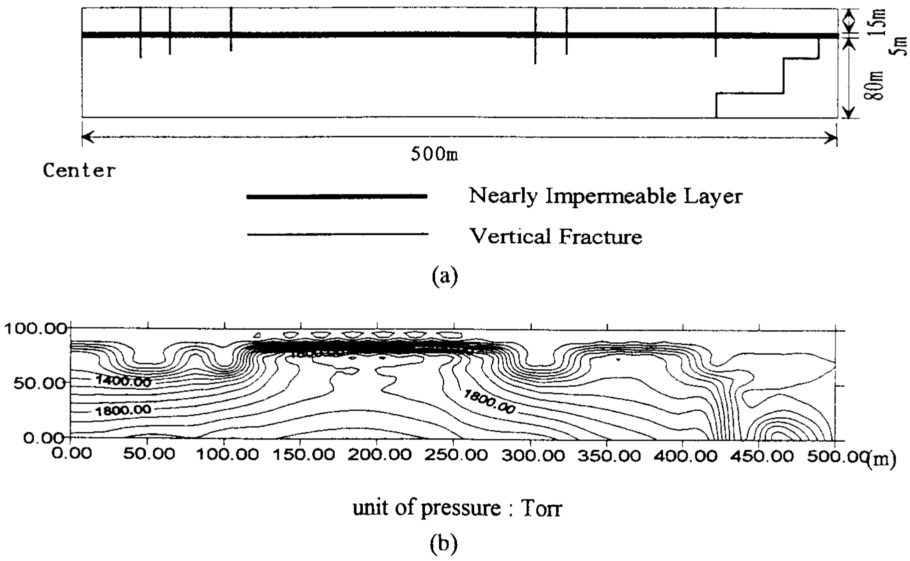


Fig. 11. Landfill model with vertical fractures (a) and gas pressure accumulation at relatively low permeable regions (b).

2) Extraction with Constant Rate

The gas pressure distribution when the gas is extracted with a constant gas extraction rate 15 m^3/min. The pattern of pressure distribution is similar to that of constant pressure extraction. The way the gas pressure changes with time is interesting because the change of gas pressure is greater in the early stage than in the later stage. If the permeability of a geologic medium is very low, than the change is greater around the extraction well. The gas pressure after 12 hours from the beginning of the extraction is estimated.

3) Embedded Vertical Fractures

Figure 11(a) is a model for simulation of the gas pressure distribution due to distribution of low permeability regions and high permeable fractures. Gas pressure accumulates along the low permeable region (Figure 11(b)). This phenomena may indicate that downward moisture flux terminates or changes the direction due to the upward gas pressure.

CONCLUSION AND DISCUSSION

The model for gas flow simulation in a landfill site is developed using the finite element method (FEM). Gas flow is a density-dependent flow and the landfill site is extremely heterogeneous. So it is impossible to estimate gas flow by solving simple mathematical equation. The governing equation for the gas flow is developed by combination of the Darcy's law and the mass conservation law. The landfill gas is assumed as the ideal gas, and the ideal state equation is introduced for simplicity of equation. The developed governing equation could represent the gas flow both of the steady state and of the transient state. Galerkin's technique is used to develop the finite element model. Two-dimensional radial symmetric triangular ring element is mainly used.

The simplified conceptual model for the Nanjido landfill is developed. It has three different layers. Top layer is the covering soil layer, with no gas generation. Middle and bottom layers are assumed as waste layers. The gas generation rate is assumed to be higher in the middle layer than in the bottom layer. The horizontal permeability of waste layer is set 20 darcy which is average value in the solid waste landfill.

The simulations of the steady state gas flow are performed in two ways. One is for the extraction with a constant pressure, and the other is the extraction with a constant rate. To find out the initial gas pressure condition, the simulation is performed for the case of no extraction. The gas flows upward and the equi-pressure line is horizontal. The zone of captured gas has the ellipsoid shape. The simulations of the gas extraction with constant rates are performed.

The simulations of transient-state gas flows are performed for the case of the gas extraction with constant rates. The influence of gas extraction on the transient-state gas flow shows the similarity to that of the steady-state gas flow. The differences are found in the scale of influence and length of pathline. The developed model code was registered in 1994 as a software according to the registration procedure of Korea Computer Program Protection Foundation.

This model was developed to analyze the landfill gas extraction tests. Axisymmetric approach is not applicable to the simulations of long-time extraction in an irregularly shaped small landfill or a landfill where the extraction well is located near the landfill boundary. The axisymmetric approach cannot be used for landfill with heterogeneous materials. The axisymmetric approach, however, has advantages in simplifying the computation domain, decreases the computation time and effort, and effectively simulates the gas extraction test.

The axisymmetric approach cannot represent a genuine 3-D gas pressure distribution. A genuine 3-D model is necessary to simulate actual 3-D gas flows. However, genuine 3-D model, which has not developed by anybody yet, cannot be effectively used in at this point because it requires information on 3-D domain geometry and 3-D gas permeability distribution. The 3-D gas permeability is coupled with the moisture content distribution in the vadose zone. The 3-D moisture content itself is very difficult to simulate and the model for the simulation of 3-D moisture has not been developed yet. Thus, the general 3-D gas flow model cannot be effectively used at this moment, but it probably increases the uncertainty in its simulation results.

The axisymmetric model also requires the gas permeability distribution in a rectangular profile. Layered structure are usually used for the hydraulic properties input and the values for the layered structures can be estimated within a range from theoretical and empirical data. With this environment, the axisymmetric model might be more practical than a genuine 3-D model in many cases.

The model developed in this study can be used in designing the gas extraction system in landfill site. For more complex system, the model can be extended to the three-dimensional model.

ACKNOWLEDGMENT

This study was supported by Daewoo Institute of

Construction Technology, and partly by the G7 project (Title : Decelopment of technology for soil/groundwater remediation).

We thank Mr. Tae-hee Kim for his efforts in making figures and correcting on early manuscript.

REFERENCES

Anderson, M.P. and Woessener, W.W. (1992) Applied Groundwater modeling, Simulation of flow and advective transport. Academic Press. Inc., San Diego, 381p.

Franz, T. and Guiguer, N. (1992) AIRFLOW, version 2, Steady-state radial-symmetric vapor flow simulation model. Water Hydrogeologic Software, Waterloo, 70p.

Hanashima, M., Yamasaki, K., Kuroki, T, and Onishi, K. (1981) Heat and gas analysis in semiaerobic landfill. J. Env. Eng. Div., 107, EE1.

Hartz, K.E. and Ham, R.K. (1982) Gas generation rates of landfill samples. Conservation & Recycling, v. 5, p. 133-147.

Huyakorn, P.S. and Pinder, G.F. (1983) Computational methods in subsurface flow. Academic Press, Inc., San Diego, 473p.

Mohsen, M.F.N., Farquhar, G.J. and Kouwen, N. (1980) Gas migration and vent design at landfill sites. Water, Air, and Soil Pollution, v. 13, p. 79-97.

Moore, C.A. (1979) Landfill gas generation, migration, and controls. Critical Reviews in Env. Control., Nov., p. 157-183.

Pacey, J.G. (1981) Prediction of landfill gas production and recovery. Int. Gas Res. Conf., p. 896-915.

City of Seoul (1994) Basic design for the stabilization of the Nanjido Landfill, 919p.

Manuscript received 26 March 1996

폐기물 매립지에서의 가스 거동에 관한 모델 개발과 적용

박유철* · 이강근* · 박철휘** · 김용우**

요 약 : 폐기물 매립지 내에서 가스 흐름을 추정하기 위하여 수치 모델을 개발하였다. 가스 흐름은 밀도에 영향 받는 밀도류이고 매립지 내의 매체는 매우 불균질하므로 단순한 수학적 해법으로 가스 흐름을 추정하기 어렵다. Darcy 방정식과 질량 보존의 법칙을 결합하고 이상 기체 방정식을 도입하여 지배방정식을 만들어지고, 이 지배방정식을 유한요소법으로 풀이하였다. 정류상태 모의의 결과는 AIRFLOW 모델에 의한 모의 결과와 일치한다. 부정류상태 모의 결과는 질량 보존 평가와 일정 시간 후의 결과를 정류상태의 결과와 비교하여 간접적으로 결과의 신뢰성을 평가하였다. 대우건설에서 1993년에 실시한 난지도 매립지 가스 추출 시험 결과를 모델을 사용하여 분석하였다. 개발된 모델은 1994년 재단법인 한국 컴퓨터 프로그램 보호회에 등록되었다.