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$\Gamma - BCK$–ALGEBRAS.

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ABSTRACT. In this paper we prove that if Y is a poset of the form $\underline{1} \oplus Y'$ for some subposet Y' then $BCK(Y)$ is a Γ-BCK-algebra. Moreover, if X is a BCI-algebra then $Hom(X, BCK(Y))$ is a positive implicative Γ-BCK-algebra.

The notion of BCK-algebras was introduced by K. Iséki, and many followers developed the algebra to several areas. Any BCK-algebra has the poset structure in it. A poset Z is called the *ordinal sum* of two posets X and Y, written $Z = X \oplus Y$, if $z_1 \leq z_2$ in Z then $z_1 \in X$, $z_2 \in Y$ or $z_1 \leq z_2$ in X or $z_1 \leq z_2$ in Y. We denote n-element chain by C_n, while $\underline{n}$ stands for any n-element antichain, i.e., $x \leq y$ if and only if $x = y$. S. Tanaka [8] constructed a BCK-algebra from the poset $X = \underline{1} \oplus X'$, where X' is any antichain. With this concept, J. Y. Kim, Y. B. Jun and H. S. Kim [6] applied to a poset $X = \underline{1} \oplus X'$, where X' is an arbitrary poset, and called it a *Tanaka-type algebra* ([6]). Y. Setō [7] also constructed a BCK-algebra from a poset $X = \underline{1} \oplus \underline{1} \oplus X'$ where X' is an arbitrary poset, called *Setō-algebra* ([6]). A. Grząślewicz [2] introduced the notion of Γ-BCK-algebra, and Y. B. Jun and J. Meng [4] studied $Hom(-,-)$ as BCK/BCI-algebras with this notion. In this paper we prove that if Y is a poset of the form $\underline{1} \oplus Y'$ for some subposet Y' then $BCK(Y)$ is a Γ-BCK-algebra. Moreover, if X is a BCI-algebra then $Hom(X, BCK(Y))$ is a positive implicative Γ-BCK-algebra.

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A *BCI-algebra* is a non-empty set X with a binary operation $*$ and a constant 0 satisfying the following axioms: for any $x, y, z \in X$,

(1) (1) $\{(x * y) * (x * z)\} * (z * y) = 0$,
(2) (2) $\{x * (x * y)\} * y = 0$,
(3) (3) $x * x = 0$,
(4) (4) $x * y = 0$ and $y * x = 0$ imply that $x = y$,
(5) (5) $x * 0 = 0$ implies $x = 0$.

If (5) is replaced by (6) $0 * x = 0$ for all $x \in X$, then the algebra is called a *BCK-algebra*. A partial ordering $\leq$ on X can be defined by $x \leq y$ if and only if $x * y = 0$. A BCK-algebra X satisfying $(x * y) * (y * z) = (x * y) * z$ for any $x, y, z \in X$ is said to be *positive implicative* ([3]).

MAIN ASSUMPTION. We discuss the posets with the least element 0.

THEOREM 1 [6]. *Let X be a poset with the least element 0 and a binary operation $*$ defined as follows:*

$$x * y = \begin{cases} 0 & \text{if } x \leq y, \\ x & \text{otherwise.} \end{cases}$$

Then $(X; *, 0)$ is a BCK-algebra .

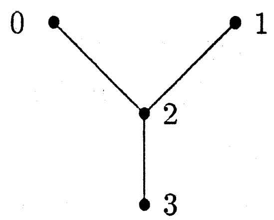
We denote the BCK-algebra in Theorem 1 by $BCK(X)$ and call it a *Tanaka-type algebra* ([6]). Y. Setō [7] introduced a binary operation $*$ on a special type of the poset $X = \underline{1} \oplus \underline{1} \oplus X'$ for some poset X', i.e., X has the least element 0 and a unique atom c, and $c < x$ for any $x \in X'$, and obtained a BCK-algebra . We call it *Setō-algebra* ([6]). A BCK-algebra X is called a *Γ-BCK-algebra* ([2]) if $x * y = y * x$ then $x = y$ for any $x, y \in X$.

THEOREM 2. *If X is a poset then the Tanaka-type algebra $BCK(X)$ is a Γ-BCK-algebra.*

PROOF. Let $x, y \in X$ with $x * y = y * x$. Assume that $x \neq y$. If $x < y$ then $y * x = x * y = 0$ and hence $x = y$, a contradiction. Similarly $y < x$ leads to a contradiction. If $x \circ y$ (i.e., $x \circ y$ means that x and y are incomparable) then $x = x * y = y * x = y$, a contradiction. This proves the theorem.

EXAMPLE 3 Consider the following Tanaka-type algebra with its Hasse diagram:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	2
3	3	3	3	0



We can easily see that it is a Γ-BCK-algebra.

REMARK Setō-algebra need not be Γ-BCK-algebra(see Example 4).

EXAMPLE 4. Consider the poset described in Example 3. We can easily construct Setō-algebra as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	1
3	3	1	1	0

We can see that $2 * 3 = 1 = 3 * 2$, but $2 \neq 3$.

E. Y. Deeba and S. K. Goel [1] proved that if X is a BCI-algebra and Y is a BCK-algebra then $Hom(X, Y) = \{f | f : X \to Y$: homomorphism $\}$ forms a BCK-algebra . Y. B. Jun and J. Meng [4] proved the following theorem.

THEOREM 5 [4]. *If X is a BCI-algebra and Y is a Γ-BCK-algebra then $Hom(X,Y)$is a Γ-BCK-algebra.*

By applying Theorem 2 and Theorem 5 we obtain the following corollary.

COROLLARY 6. *If X is a BCI-algebra and Y is a poset, then $Hom(X, BCK(Y))$ is a Γ-BCK-algebra.*

Y. B. Jun and J. Meng [4] studied positive implicativity on $Hom(X,Y)$.

PROPOSITION 7 [4]. *If X is a BCI-algebra and Y is a positive implicative BCK-algebra , then $Hom(X,Y)$ is a positive implicative BCK-algebra .*

J. Y. Kim, Y. B. Jun and H. S. Kim [6] proved that given a poset X of the form $\underline{1} \oplus X'$ for some (sub)poset X', the Tanaka-type algebra $BCK(X)$ is positive implicative. With this concept we construct the following proposition.

PROPOSITION 8. *If X is a BCI-algebra and Y is a poset, then $Hom(X, BCK(Y))$ is positive implicative.*

PROOF. If Y is a poset then $BCK(Y)$ is positive implicative ([5]). By applying Proposition 7 we can see that $Hom(X, BCK(Y))$ is positive implicative. This proves the proposition.

By applying Theorem 2 and Proposition 8 we obtaing the following theorem.

THEOREM 9. *If X is a BCI-algebra and Y is a poset, then $Hom(X, BCK(Y))$ is a positive implicative Γ-BCK-algebra.*

REFERENCES

1. E. Y. Deeba and S. K. Goel, *A note on BCI-algebras*, Math. Japonica 33 (1988), 517-522.

2. A. Grzaślewicz, *On the extensibility of commutative BCK-algebras*, Math. Seminar Notes **8** (1980), 389-394.
3. K. Iséki and S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japonica **23** (1978), 1-26.
4. Y. B. Jun and J. Meng, *On Hom*$(-,-)$ *as BCK/BCI-algebras*, Kyungpook Math. J. **35** (1995), 77-83.
5. Y. B. Jun, J. Y. Kim and H. S. Kim, *On Q-upper algebras*, (submitted).
6. J. Y. Kim, and Y. B. Jun and H. S. Kim, *BCK-algebras inherited from the posets*, Math. Japonica (to appear).
7. Y. Setō, *Some example of BCK-algebras*, Math. Seminar Notes **5** (1977), 397-400.
8. S. Tanaka, *Examples of BCK-algebras*, Math. Seminar Notes **3** (1975), 75-82.

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