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암석류의 파괴인성계수의 측정과 해석방법에 관한 연구

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Evaluation and Interpretation of the Fracture Toughness of Rocks

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ABSTRACT

암석의 파괴인성계수는 암석이 갖는 불균질성 및 비등방성에 의하여 시험조건에 따른 측정자료의 분산이 심하다. 즉, 시험편의 형태나 크기에 따른 변화가 심하여 기존의 선형 탄성 파괴역학 이론의 적용에 문제점이 있는 것으로 지적되고 있다. 이러한 자료의 분산을 최소화하기 위한 방법의 하나로 균열감응도를 적용한 해석을 제시하고 있다. 균열감응도란 파괴역학 실험 당시 시험편에 가해진 인공 균열의 감응도를 말하며 이는 3점 하중에 의한 파괴가 균열의 성장에 의한 파괴인지, 혹은 단순히 인장파괴에 의한 것인지를 판명함으로써 측정자료의 선택을 명확하게 하기 위한 방법의 하나로 적용될 수 있다.

1. Introduction

Fracture toughness of rock materials, which generally violates the fundamental assumptions of LEFM, often depends on the specimen size and test method employed. Hence, a standardized procedure for testing and data interpretation for determining fracture toughness of rock materials is required. Special attention has been given by the International Society for Rock Mechanics (ISRM) to the difficulties in obtaining the true fracture mechanics parameter for the

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wide variety of rock materials.

Rock specimens are usually cored from drill holes, and core-based specimens are more cost-effective for determining rock fracture toughness. Accordingly, single edge cracked round bar in bending (SECRBB), semi-circular bending (SCB), notched Brazilian discs, and the chevron bend (CB) specimens are most practical specimen types. Other core-based specimen types include the burst cylinder method, modified ring test, and the round compact tension. Among the various specimen geometries listed above, the SECRBB and CB specimens will be discussed here.

2. Specimen Geometries

2.1 The SECRBB Specimen

The geometry of the SECRBB specimen is shown in Fig. 1. Assuming a linear elastic behaviour, the compliance, C , of the specimen is defined as:

$$C = \frac{\delta_F}{F} \qquad \text{-------} (1)$$

where δ_F is the load point displacement (LPD) and F is the applied load. Within the limits of the slender beam theory, the compliance of a SECRBB specimen is given by:

$$\begin{aligned} C_{secrbb} &= \frac{S^3}{48\ E \cdot I_{secrbb}} \\ &= \frac{1}{E \cdot D} \cdot \frac{4\ (S/D)^3}{3\pi \cdot f\ (\alpha)} \end{aligned} \qquad \text{-------}(2)$$

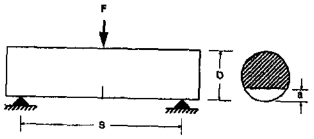

Fig. 1. Geometry of the SECRBB specimen.

where S is the support span, and $f\ (\alpha)$ is a dimensionless function derived from I_{secrbb} , the moment of inertia of the SECRBB cross-section. The mode I stress intensity factor for SECRBB specimen (S/D = 3.33) is then given by (Bush, 1976):

$$\begin{aligned} K_I &= \sqrt{\frac{F^2}{4\ (aD - a^2)^{0.5} \cdot D^2} \cdot \frac{\partial(cED)}{\partial(a/D)}} \\ &= \frac{F}{D^{1.5}} \cdot Y'_{secrbb} \end{aligned} \qquad \text{---}(3)$$

From a compliance calibration for the Ekeberg marble, Ouchterlony(1981) obtained Y'_{secrbb} as:

$$Y'_{secrbb} = 10.62\alpha^{0.5}(1 + 19.65\alpha^{4.5})^{0.5}/(1 - \alpha)^{0.25} \quad \text{-}(4)$$

which is valid for $0 \leq \alpha \leq 0.6$, and S/D = 3.33.

2.2 The Chevron Bend Specimen

The Chevron bend specimen is one of the two specimen geometries suggested by the ISRM. Chevron-notched specimen have several advantages over other specimen types, especially for materials exhibit brittle fracturing: the ligament shape enables a stable crack propagation from a self-produced sharp crack. A stable crack growth up to certain distance from the initial chevron tip produces a naturally sharp crack, so that the fatigue pre-cracking requirement of ASTM E399 can be omitted. Furthermore, assuming a flat R-curve for generally brittle rock materials, fracture toughness is calculated from the maximum load and initial specimen dimensions.

The geometry of the CB specimen is shown in Fig. 2. In level I testing, only the maximum load, $F_{\max}$, is recorded and the fracture toughness is calculated by (ISRM, 1988):

$$K_I = A_{\min} \cdot \frac{F_{\max}}{D^{1.5}} \quad \text{--------(5)}$$

With the notations given in Fig. 2, the dimensionless factor, $A_{\min}$ is given as:

$$A_{\min} = \left[1.84 + 7.15\left(\frac{a_0}{D}\right) + 9.85\left(\frac{a_0}{D}\right)^2\right]\left(\frac{S}{D}\right) \quad \text{-(6)}$$

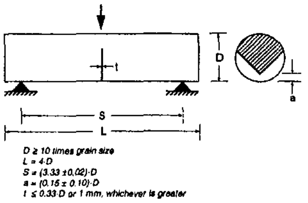

Fig. 2. Geometry of the CB specimen.

3. Notch Sensitivity

Two different types of failure mechanisms act within a notched specimen under loading. The first is a strength failure governed by the maximum tensile stress at the crack tip and the tensile strength of the rock material. The second is the failure due to crack extension, governed by the fracture toughness of the rock material. Therefore, it is necessary to determine which failure mechanism is critical at the instant of specimen failure.

Carpinteri (1982) employed the concept of notch sensitivity in analyzing fracture toughness test results of aggregative materials, like concrete or mortar, assuming homogeneous, isotropic and linear elastic material behaviour. He applied the dimensional analysis for physical similitude and scale modelling to define a non-dimensional brittleness number, s, given by;

$$s = \frac{K_{Ic}}{\sigma_t \cdot w^{0.5}} \quad \text{------(7)}$$

where w is the specimen width.

For example, the stress intensity factor for the three point bend specimen (that is, CB and SECRBB specimens);

$$K_I = \frac{F \cdot S}{b \cdot w^{1.5}} \cdot f\,(a/w) \quad \text{----(8)}$$

where S is the support span, and $f\,(a/w)$ is a dimensionless coefficient.

At the critical load F_c, the generalized force for crack extension F_{crack}, is;

$$F_{crack} = \frac{F_c \cdot S}{\sigma_t \cdot w^2 \cdot b} = \frac{s}{f\,(a/w)} \qquad ---(9)$$

Assuming that the linear elastic slender beam theory is still applicable for the notched three-point bend specimen, the generalized force for ultimate strength failure, $F_{strength}$, is;

$$F_{strength} = \frac{F_c \cdot S}{\sigma_t \cdot w^2 \cdot b} = \frac{2}{3}\left(1 - \frac{a}{w}\right)^2 \qquad ----(10)$$

It should be noted that the two generalized forces given in Eqs. (9) and (10) are both dimensionless, and are directly comparable. The notch sensitivity is then defined as a function of the brittleness number and crack length ratio, a/w : at a certain combination of these two parameters, if the generalized force for crack extension is smaller than that of ultimate strength failure, crack extension is more critical and the specimen is notch sensitive. A valid fracture toughness measurement for a given specimen geometry can be assured only if the specimen is notch sensitive.

Carpinteri (1982) concluded that some recurring experimental inconsistencies, such as the variance of K_{Ic} with the crack length, specimen size, and test geometry, can be explained by the notch sensitivity of the specimen.

4. Notch Sensitivity of the CB and SECRBB Specimen

The coordinates of the centroid, $\bar{x}$ and $\bar{y}$, and the moment of inertia with respect to the y direction, I_y , of a plane area in Fig. 3 are calculated from the theory of elasticity as:

$$\bar{x} = \frac{Q_y}{A} = \frac{\int x\,dA}{\int dA}$$
$$\bar{y} = \frac{Q_x}{A} = \frac{\int y\,dA}{\int dA} \qquad ---(11)$$

and,

$$I_y = \int x^2\,dA \qquad ---(12)$$

where Q_x and Q_y are the first moments of the area about the x and y axis, respectively, and A is the area of the plane.

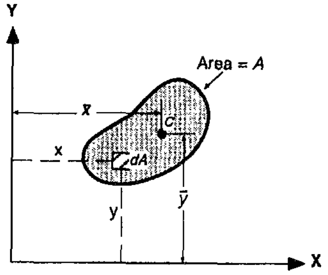

Fig. 3. Centroid of a Plane Area.

As shown in Fig. 4, the distance y' from the new centroid of the ligament plane of the notched specimens to the crack tip is calculated as:

$$y' = D \cdot h(\alpha) \qquad \text{------(13)}$$

where D is the specimen diameter and α is the crack length ratio, that is, a/D. The dimensionless function $h(\alpha)$ depends on the specimen geometry as:

$$\begin{aligned} h(\alpha)_{cb} &= 0.56 - 0.51\alpha - 0.02\alpha^2 - 0.06\alpha^3 \\ h(\alpha)_{secrbb} &= 0.50 - 0.85\alpha + 1.11\alpha^2 - 1.75\alpha^3 \end{aligned} \qquad \text{--(14)}$$

where the subscripts *cb* and *secrbb* represent the CB and SECRBB specimen, respectively. For the notched cross-section, the moment of inertia of the ligament plane is given by:

$$I = \pi D^4 \cdot i(\alpha) \qquad \text{----(15)}$$

where the dimensionless function $i(\alpha)$ also depends on the specimen geometry, such that:

$$\begin{aligned} i(\alpha)_{cb} &= 0.01 - 0.04\alpha + 0.05\alpha^2 - 0.02\alpha^3 \\ i(\alpha)_{secrbb} &= 0.02 - 0.02\alpha - 0.09\alpha^2 + 0.27\alpha^3 \end{aligned} \qquad \text{---(16)}$$

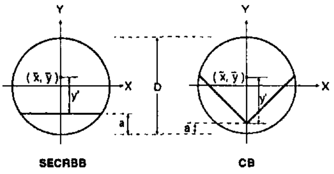

Fig. 4. Notched Cross-Section of the CB and SECRBB Specimens.

Assuming that the elastic slender beam theory is still applicable for the notched beam, the tensile stress developed at the notch tip in the three-point bend specimen is :

$$\sigma_t = \frac{F \cdot S \cdot \bar{y}}{4\, I} \qquad \text{------(17)}$$

From Eqs. (13), (15), and (17), the generalized force for the ultimate strength failure, $F_{strength}$ of the CB and SECRBB specimens (S/D = 3.33) is given by:

$$F_{strength} = \frac{F_{\max}}{\sigma_t \cdot D^2} = \frac{4\pi}{3.33} \cdot \frac{i(\alpha)}{h(\alpha)} \qquad \text{---(18)}$$

From Eqs. (3) and (5), the fracture toughness formulas for the CB and SECRBB specimens (S/D = 3.33) can be written as :

$$K_{Ic} = \frac{F_{\max}}{D^{1.5}} \cdot f(\alpha) \qquad \text{-----(19)}$$

The dimensionless stress intensity factor, $f(\alpha)$ is given in Eq. (4) for the SECRBB specimen, and in Eq. (6) for the CB specimen. The generalized force for crack extension, F_{crack} , is given as:

$$F_{crack} = \frac{F_{\max}}{\sigma_t \cdot D^2} = \frac{s}{f(\alpha)} \qquad \text{----(20)}$$

where the brittleness number, s , for the cylindrical specimen is;

$$s = \frac{K_{Ic}}{\sigma_t \cdot D^{0.5}} \qquad \text{--------(21)}$$

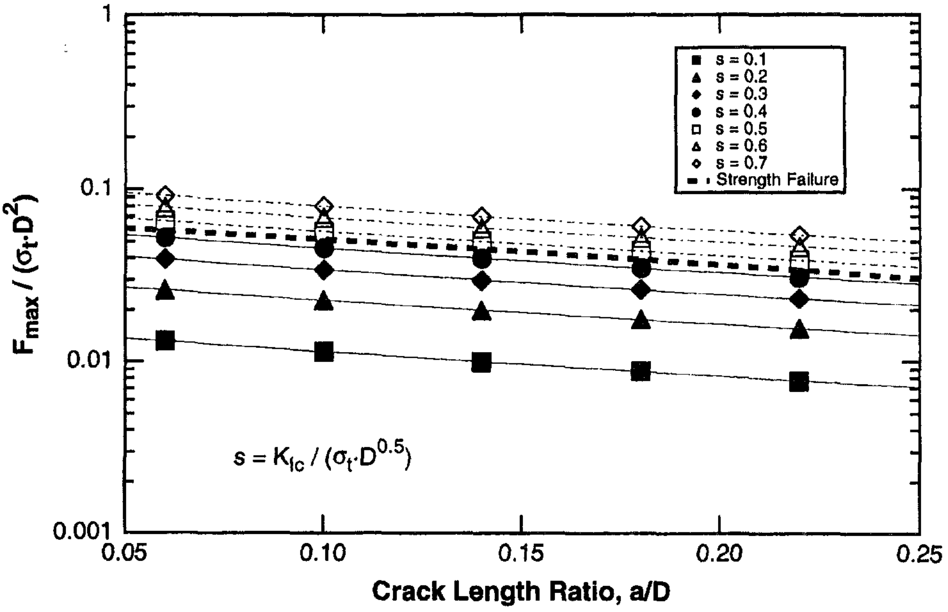

Fig. 5. Generalized Force Curve vs. α for the SB Specimen.

5. Concluding Remarks

Figure 5 shows the generalized crack extension force curves, for different brittleness numbers, and the generalized strength failure curve for the CB specimen. The range of the crack length ratio in the figure corresponds to the valid range of the dimensionless stress intensity factor for this specimen type.

The region where the crack propagation failure curves are below the ultimate strength failure curve defines the notch-sensitive range of the given specimen type. Within this region, the failure due to crack extension comes before the strength failure, and the test data is valid.

The brittleness number is calculated from the tensile strength and diameter of the specimen, and the measured fracture toughness which depends on the crack length ratio. As shown in this figure, the brittleness number calculated from the measured fracture toughness of the test specimen should be less than 0.45; otherwise, test result is regarded as invalid, according to the notch sensitivity.

The similar plot for the SECRBB specimen is given in Fig. 6. Consider a SECRBB specimen for which the brittleness number is calculated as 0.5.

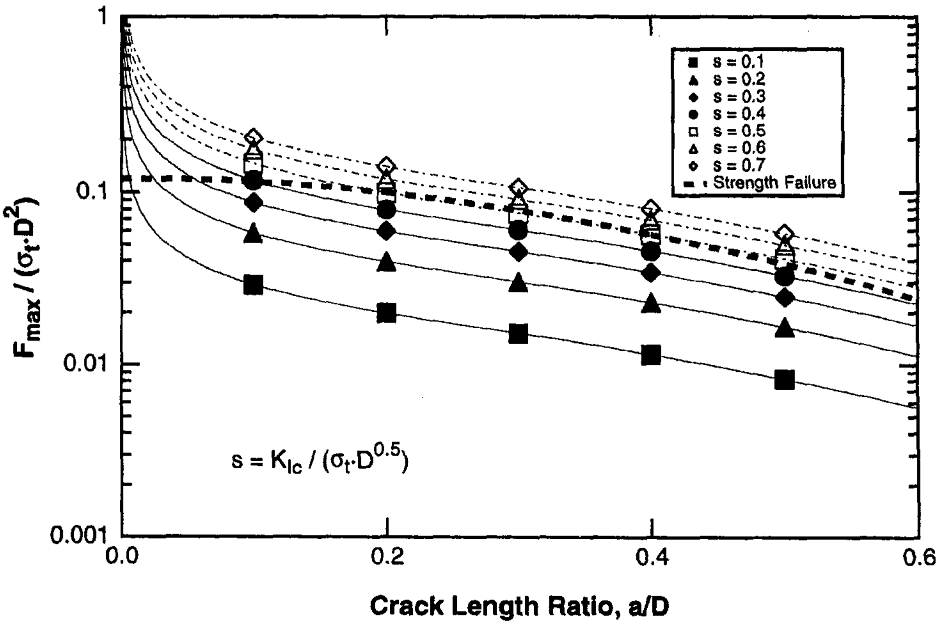

Fig. 6. Generalized Force Curves vs. α for the SECRBB Specimen.

If the crack length ratio is in the range of 0.2 to 0.3, fracture toughness testing on this specimen may be accepted, since the crack extension failure curve is tangent to the strength failure curve within this range. However, for other crack length ratios, for which the strength failure is more critical and the test result is rejected.

Thus, the notch sensitivity analysis with a fixed crack length ratio of the specimens can be applied to discriminate invalid fracture toughness values in screening stage. Similarly, with an estimated fracture toughness for a rock material, the valid range of the initial crack length ratio for a given type of specimen geometry can also be determined prior to the testing.

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