

## 침투력을 고려한 사면안정의 이론적 해석

### The Theoretical Analysis of the Slope Stability subjected to Seepage Force

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(1996년 7월 12일 접수, 1996년 12월 14일 채택)

#### ABSTRACT

The main purpose of this study was to develop a useful method for analysing slope stability by seepage force. The stability of an embankment impounding a water reservoir is highly depend upon the location of seepage line with the embankment, it is important to illustrate the seepage phenomenon.

Of particular interest is the stability following a rapid rise change of reservoir level. Seepage forces in embankments are easily determined if frictional forces are expressed in relation to hydraulic gradient  $i$ .

Seepage forces can combine with soil weights to improve stability or worsen it, depending on the direction in which the forces act in relation to the geometric cross section.

#### 국문요약

사면안정 해석에 있어서 기존의 연구는 대부분 침투력의 값을 고려하지 않고 안전율을 계산하여 왔다. 그러나 비정상 침투시 침투력은 안전율에 많은 영향을 미친다.

따라서 본 연구에서는 사면안정해석에 있어서 침투력의 영역이 사면의 안전율에 미치는 영향이 큰 것을 확인하기 위해서 Bishop's Simplified Method를 이용하여 이론적으로 침투력을 고려한 안전율계산 수식을 유도하였다.

사면안정 이론식의 전개방법은 체체에 침윤선이 형성될 경우 침윤선을 기준으로 각 절편을 수중상태와 습윤상태로 구분하고, 이 습윤상태의 절편토체에 작용하는 침투력을 고려하여 사면안정수식을 해석했다.

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# 1. The Theoretical Analysis of the Slope Stability

## 1.1 The Use of Limit Design Methods

In this study may be introduced the estimate of stability not only by the use of approximate methods of stability analysis, but also by seepage force D. Unless equal attention is paid to each factor, an elaborate mathematical treatment may lead to a fictitious impression of accuracy.

However, in a number of cases the uniformity of the soil conditions or the importance of the problem will justify a more accurate analysis.

As the majority of stability problems occur in slopes and embankments having lower factors of safety than this, a state of plastic equilibrium must be considered to exist throughout at least part of the slope.

Under these conditions a quantitative estimate of the factor of safety can be obtained by examining the conditions of equilibrium when incipient failure is postulated, and comparing the strength necessary to maintain limiting equilibrium with the available strength of the soil. The factor of safety (F) is thus defined as the ratio of the available shear strength of the soil to that required to maintain equilibrium. The strength mobilized is, therefore, equal to  $\bar{s}$ , where :

$$\bar{s} = \frac{1}{F} \{ C + (\sigma_n - U) \tan \phi' + D \} \dots\dots\dots (1)$$

$\phi'$  = denotes angle of shearing resistance, (in terms of effective stress)

C = denotes cohesion, (in terms of effective stress)

U = denotes pore pressure,

F = denotes the factor of safety,

$\sigma_n$  = denotes total normal stress,

D = denotes seepage force,

Failure along a continuous rupture surface is usually assumed, but, as the shape and position of this surface is influenced by the distribution of seepage force and the variation of the shear parameters within the slope, a generalized analytical

solution is not possible and a numerical solution is required in each individual case. The rigorous determination of the shape of the most critical surface presents some difficulty, and in practice a simplified shape, usually a circular arc, is adopted, and the problem is assumed to be one of plane strain.

## 1.2 Mechanics of the Circular Arc Analysis

In order to examine the equilibrium of the mass of soil above the slip surface it follows from equation (1) that it is necessary to know the value of the normal stress at each point on this surface, as well as the magnitude of the seepage force. It is possible to estimate the value of the normal stress by following the method developed by Fellenius (1927, 1936), in which the conditions for the static equilibrium of the slice of soil lying vertically above each element of the sliding surface are fully satisfied.

The significance of this assumption may be examined by considering the equilibrium of the mass of soil (of unit thickness) bounded by the circular arc ABCD, of radius R and centre at O (Fig. 1). In the case where no external forces act on the surface of the slope, equilibrium must exist between the weight of the soil above ABCD and the resultant of the total forces acting on ABCD.

Let  $E_n, E_{n+1}$  denote the resultants of the total horizontal forces on the sections n and n+1 respectively,

and  $X_n - X_{n+1}$  = the vertical shear forces,

W = the total weight of the slice of soil,

$W_1$  = the weight of the slice of wet soil,

$W_2$  = the weight of the slice of submerged soil,

P = the total normal force acting on its base,

S = the shear force acting on its base,

H = the height of the water level,

$h_1$  = the height of the wet soil mass,

$h_2$  = the height of the submerged soil mass,

b = the breadth of the element,

$\ell$  = the length BC,

$\alpha$  = the angle between BC and the horizontal,

x = the horizontal distance of the slice from the centre of rotation

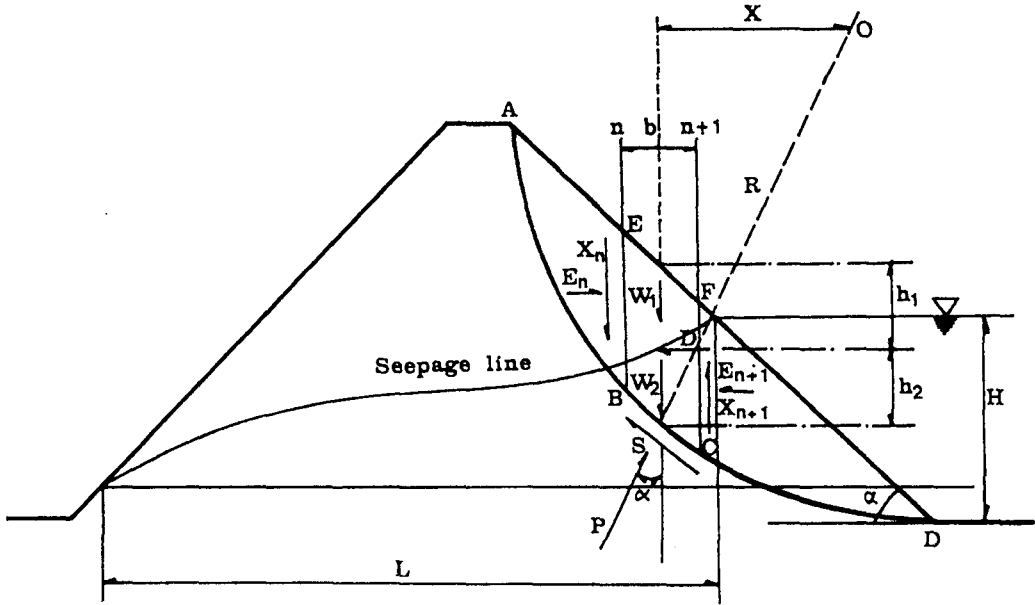


Fig. 1 Forces in the slices method

The total normal stress is  $\sigma_n$ , where  
 $\sigma_n = P/l$  ..... (2)

Hence, from equation (1), the magnitude of the shear strength mobilized to satisfy the conditions of limiting equilibrium is  $\bar{s}$  where :

$$\bar{s} = \frac{1}{F} \left\{ C_i' + \left( \frac{P}{l} - U \right) \tan \phi_i' + D \right\} \dots\dots (3-a)$$

$$S = \frac{1}{F} \sum \left\{ C_i' l + (P - U l) \tan \phi_i' + D l \right\} \dots\dots (3-b)$$

The shear force  $S$  acting on the base of the slice is equal to  $\bar{s} l$ , and thus, equating the moment about  $O$  of the weight of soil within  $ABCD$  with the moment of the external forces acting on the sliding surface, we obtain :

$$\sum Wx = \sum (W_1 + W_2)x = \sum SR = \sum \bar{s} l R \dots\dots (4)$$

It follows, therefore, from equation (3-a) that :

$$\sum (W_1 + W_2)x = \sum \frac{R}{F} [C_i' l + (P - U l) \tan \phi_i' + D l] \dots\dots (5)$$

$$F = \frac{R}{\sum (W_1 + W_2)x} \sum [C_i' l + (P - U l) \tan \phi_i' + D l] \dots\dots (6)$$

From the equilibrium of the soil in the slice above  $BC$ , we obtain  $P$ , by resolving in a direction normal to the slip surface :

$$P = (W_1 + W_2 + X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha \dots\dots (7)$$

The expression for  $F$  thus becomes :

$$F = \frac{R}{\sum (W_1 + W_2)x} \sum [C_i' l + \tan \phi_i' \{ (W_1 + W_2) \cos \alpha - U l \} + \tan \phi_i' \{ (X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha \} + D l] \dots\dots (8)$$

Since there are no external forces on the face of the slope, it follows that :

$$\sum (X_n - X_{n+1}) = 0 \dots\dots (9-a)$$

$$\sum (E_n - E_{n+1}) = 0 \dots\dots (9-b)$$

However, except in the case where  $\phi'$  is constant along the slip surface and  $\alpha$  is also constant, the terms in equation (8) containing  $X_n$  and  $E_n$  do not disappear.

A simplified form of analysis implies that the sum of these terms

$$\sum \tan \phi_i' \{ (X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha \}$$

may be neglected without serious loss in accuracy.

This is the method at present used, for example, by the U.S. Bureau of Reclamation(Daehn and Hilf, 1951)

Putting  $x=R \sin \alpha$ , the simplified form may be written :

$$F = \frac{1}{\Sigma(W_1+W_2)\sin \alpha} \Sigma [C_i' \ell + \tan \phi_i' \{ (W_1+W_2)\cos \alpha - U \ell \} + D \ell] \dots\dots\dots (10)$$

To derive a method of analysis which largely avoids this error it is convenient to return to equation(6). If we denote the effective normal force (P-U l) by P, and resolve the forces on the slice vertically, then we obtain, on re-arranging.

$$W_1+W_2 = P \cos \alpha + S \sin \alpha \dots\dots\dots (11-a)$$

$$P \cos \alpha = W_1+W_2 - S \sin \alpha \dots\dots\dots (11-b)$$

The expression used in Eq. (3-b)

$$P \cos \alpha = W_1+W_2 - \frac{\sin \alpha}{F} \Sigma [C_i' \ell + (P-U \ell) \tan \phi_i' + D \ell] \\ = W_1+W_2 - \Sigma \left[ \frac{C_i' \ell \sin \alpha}{F} + \frac{P \sin \alpha}{F} \tan \phi_i' - \frac{U \ell \sin \alpha}{F} \tan \phi_i' + \frac{\sin \alpha}{F} D \ell \right] \dots\dots\dots (12)$$

$$\Sigma P \left( \cos \alpha + \frac{\sin \alpha \tan \phi_i'}{F} \right) = W_1+W_2 - \Sigma \left[ \frac{C_i' \ell \sin \alpha}{F} - \frac{U \ell \sin \alpha \tan \phi_i'}{F} + \frac{\sin \alpha}{F} D \ell \right] \dots\dots\dots (13)$$

$$P = \Sigma \left[ \left( W_1+W_2 - \frac{C_i' \ell \sin \alpha}{F} + \frac{U \ell \sin \alpha \tan \phi_i'}{F} - \frac{\sin \alpha}{F} D \ell \right) / \left( \cos \alpha + \frac{\sin \alpha \tan \phi_i'}{F} \right) \right] \dots\dots (14)$$

where, Eq. (14)-U l

$$P-U \ell = \Sigma \left[ \frac{W_1+W_2 - \Sigma \left\{ \frac{C_i' \ell \sin \alpha}{F} - \frac{U \ell \sin \alpha \tan \phi_i'}{F} + \frac{\sin \alpha}{F} D \ell \right\}}{\cos \alpha + \frac{\sin \alpha \tan \phi_i'}{F}} \right] - U \ell \dots\dots\dots (15)$$

$$P-U \ell = \Sigma \frac{1}{\cos \alpha + (\sin \alpha \tan \phi_i'/F)} \left[ W_1+W_2 - \frac{C_i' \ell \sin \alpha}{F} + \frac{U \ell \sin \alpha \tan \phi_i'}{F} - U \ell \cos \alpha \right]$$

$$- \frac{U \ell \sin \alpha \tan \phi_i'}{F} - \frac{\sin \alpha}{F} D \ell ] \dots\dots\dots (16)$$

$$P-U \ell = \Sigma \frac{1}{\cos \alpha [1 + (\tan \alpha \tan \phi_i'/F)]} \left[ W_1 + W_2 - \frac{C_i' \ell \sin \alpha}{F} - U \ell \cos \alpha - \frac{\sin \alpha}{F} D \ell \right] \dots\dots\dots (17)$$

Putting,  $\sec \alpha = \frac{1}{\cos \alpha}$

$$P-U \ell = \Sigma \frac{\sec \alpha}{1 + (\tan \alpha \tan \phi_i'/F)} \left[ (W_1+W_2) - \frac{C_i' \ell \sin \alpha}{F} - U \ell \cos \alpha - \frac{\sin \alpha}{F} D \ell \right] \dots\dots (18)$$

Moment Eq. (6), Put(18) ( $x=R \sin \alpha$ )

$$F = \frac{\Sigma \left[ C_i' \ell + \left\{ W_1+W_2 - \frac{C_i' \ell \sin \alpha}{F} - U \ell \cos \alpha - \frac{\sin \alpha}{F} D \ell \right\} \frac{\sec \alpha \tan \phi_i'}{1 + (\tan \alpha \tan \phi_i'/F)} \right] R}{R \Sigma (W_1+W_2) \sin \alpha} \dots\dots\dots (19)$$

$$F = \frac{1}{\Sigma (W_1+W_2) \sin \alpha} \Sigma \left[ C_i' \ell + \frac{C_i' \ell \tan \alpha \tan \phi_i'}{F} + (W_1+W_2) \sec \alpha \tan \phi_i' - \frac{C_i' \ell \tan \alpha \tan \phi_i'}{F} - U \ell \tan \phi_i' - \frac{D \ell \tan \alpha \tan \phi_i'}{F} \right] \frac{1}{1 + (\tan \alpha \tan \phi_i'/F)} \dots\dots (20)$$

where,  $b = \ell \cos \alpha$  replace put,  $\ell = b \sec \alpha$

$$F = \frac{1}{\Sigma (W_1+W_2) \sin \alpha} \Sigma \left[ \frac{C_i' b \sec \alpha + (W_1+W_2) \sec \alpha \tan \phi_i' - U b \sec \alpha \tan \phi_i' - D b \sec \alpha \tan \alpha \tan \phi_i'/F}{1 + (\tan \alpha \tan \phi_i'/F)} \right] \dots\dots\dots (21)$$

$$F = \frac{1}{\Sigma (W_1+W_2) \sin \alpha} \Sigma \frac{\sec \alpha}{1 + (\tan \alpha \tan \phi_i'/F)} \left[ C_i' b + \tan \phi_i' \{ (W_1+W_2) - U b - D b \tan \alpha / F \} \right] \dots\dots\dots (22)$$

where,

$$m_a = \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi_i'}{F} \right) \\ F = \frac{1}{\Sigma (W_1+W_2) \sin \alpha} \Sigma [C_i' b + \tan \phi_i' \{ (W_1+W_2) - U b - D b \tan \alpha / F \}] \frac{1}{m_a} \dots\dots (23)$$

It should also be noted that, since the seepage force in the simplified method given in equations

(23) varies with the central angle of the arc, it will lead to a different location for the most critical circle than that given by the more rigorous method. It will, therefore, generally be necessary to examine a number of trial circles by this method.

It is can be seen that for partially submerged slopes an analogous method is obtained by using submerged densities for those parts of the slices which lie below the level of the external free water surface, and by expressing the seepage forces there as an excess above the hydrostatic pressure corresponding to this water level.

**2. Bishop's Simplified Method**

It has been shown elsewhere by a relaxation analysis of a typical earth dam (Bishop, 1955) that, even assuming idealized elastic properties for the soil, local overstress will occur when the factor of safety (by a slip circle method) lies below a value.

$$F = \frac{1}{\sum W \sin \alpha} \sum [C_i' b + \tan \phi_i' (W - Ub)] \frac{1}{m_a} \dots\dots\dots (24)$$

$$m_a = \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi_i'}{F} \right) \dots\dots\dots (25)$$

**3. Conclusion**

In this study, it was reported about the introduction of the analysis of the slope stability under the

seepage force by Bishop's Simplified Method.

Embankment under the seepage force was divided into two groups, submerged condition and wet condition. The analysis of the slope stability was performed by considering seepage force term that acts on slice mass in the submerged condition.

For the more correct computation of the factor of safety, it is require the considering seepage force term, but not yet, the analysis have been performed without considering of seepage force in the formation of seepage line.

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