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COCOMPACT F-BASES AND RELATION BETWEEN COVER AND COMPACTIFICATION

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ABSTRACT. Observing that a locally weakly Lindelöf space is a quasi-F space if and only if it has an F-base, we show that every dense weakly Lindelöf subspace of an almost-p-space is C-embedded, every locally weakly Lindelöf space with a cocompact F-base is a locally compact and quasi-F space and that if Y is a dense weakly Lindelöf subspace of X which has a cocompact F-base, then βY and X are homeomorphic. We also show that for any a separating nest generated intersection ring $\mathcal{F}$ on a space X, there is a separating nest generated intersection ring $\mathcal{G}$ on Φ_Y^{-1}(X) such that QF(w(X, $\mathcal{F}$)) and $w(\Phi_Y^{-1}$(X), $\mathcal{G}$) are homeomorphic and $\Phi_{Y_X}(\mathcal{G}^{\#}) = \mathcal{F}^{\#}$.

1. Introduction

Henriksen introduced pretty bases and showed that every locally weakly Lindelöf space with a cocompact pretty base is locally compact and basically disconnected ([6]). Henriksen, Vermeer and Woods showed that for a compact space X, $\Phi_X(Z(QF(X))^{\#}) = Z(X)^{\#}$, where (QF(X), Φ_X) is the minimal quasi-cover of X ([7]). In [8], the concept of F-bases which are generalized pretty bases was introduced and it was shown that every locally weakly Lindelöf space with an F-base is a quasi-F space.

Each normal base for a space X leads to a Wallman compactification ([1]) and a separating nest generated intersection ring was studied in [3], [4] and [10].

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In this paper, we will show that every dense weakly Lindelöf subspace of an almost-p-space is C-embedded, every locally weakly Lindelöf space with a cocompact F-base is a locally compact and quasi-F space and that if X has a cocompact F-base and Y is a dense weakly Lindelöf subspace of X, then βY and X are homeomorphic. Moreover, we will show that for any a separating nest generated intersection ring $\mathcal{F}$ on a space X, there is a separating nest generated intersection ring $\mathcal{G}$ on $\Phi_Y^{-1}(X)$ such that QF(Y) and $w(\Phi_Y^{-1}(X), \mathcal{G})$ are homeomorphic and $\Phi_{Y_X}(\mathcal{G}^{\#}) = \mathcal{F}^{\#}$, where Y = $w(X, \mathcal{F})$ is the Wallman compactification of X associated with $\mathcal{F}$.

Except for the parts of this paper dealing with cotopologies, all of the spaces considered will be Tychonoff spaces and for the terminology, we refer to [5] and [9].

2. Cocompact F-bases

Recall that a subspace Y of a space X is C* (C, resp.)-embedded in X if for any (bounded, resp.) continuous map $f : Y \longrightarrow R$, there is a (bounded, resp.) continuous map $g : X \longrightarrow R$ with $g|_Y = f$, where R is the space of real numbers endowed with the usual topology.

Definition 2.1 A space X is called a *quasi-F space* if for any zero-sets A, B in X, $cl_X(int_X(A \cap B)) = cl_X(int_X(A)) \cap cl_X(int_X(B))$, equivalently, every dense cozero-set in X is C*-embedded in X.

Lemma 2.2 Let Y be a dense subspace of a space X. Then for any closed set A in X, $int_X(A) \cap Y = int_Y(A \cap Y)$.

Proof. Clearly, $int_X(A) \cap Y \subseteq int_Y(A \cap Y)$. Let $x \in int_Y(A \cap Y)$, then there is an open neighborhood U of x in X with $(U \cap Y) \subseteq (A \cap Y)$. Since Y is dense in X and U is open in X, $cl_X(U) = cl_X(U \cap Y) \subseteq A$ and hence $x \in int_X(A) \cap Y$.

A subspace Y of a space X is said to be *z-embedded* in X if for any zero-set A in Y, there is a zero-set Z in X with $A = Z \cap Y$. For any space X, βX denotes the

Stone-Čech compactification of X. Using the above lemma, we will prove that the following:

Proposition 2.3 A space X is a quasi-F space if and only if every dense z-embedded subspace of X is C*-embedded in X.

Proof. Suppose that X is a quasi-F space. Take any dense z-embedded subspace Y of X and disjoint zero-sets A, B in Y. Then there are disjoint zero-sets C, D in Y such that $A \subseteq int_Y(C)$ and $B \subseteq int_Y(D)$. Since Y is z-embedded in X, there are zero-sets E, F in βX such that $C = E \cap Y$ and $D = F \cap Y$. Since Y is dense in βX and $F \cap E \cap Y = \emptyset$, $int_{\beta X}(F) \cap int_{\beta X}(E) = \emptyset$. Since X is a quasi-F space, βX is also quasi-F ([7]) and hence $cl_{\beta X}(int_{\beta X}(F)) \cap cl_{\beta X}(int_{\beta X}(E)) = \emptyset$. By the above lemma, $cl_{\beta X}(int_Y(C)) \cap cl_{\beta X}(int_Y(D)) = cl_{\beta X}(int_Y(E \cap Y)) \cap cl_{\beta X}(int_Y(F \cap Y))$ $= cl_{\beta X}(int_{\beta X}(E) \cap Y) \cap cl_{\beta X}(int_{\beta X}(F) \cap Y) = cl_{\beta X}(int_{\beta X}(E)) \cap cl_{\beta X}(int_{\beta X}(F)) =$ $\emptyset$. Thus $cl_{\beta X}(A) \cap cl_{\beta X}(B) = \emptyset$ and therefore Y is C^*-embedded in X ([5]). Since every cozero-set in X is z-embedded in X ([2]), the converse is trivial.

For any space X, Coz(X) denotes the set of cozero-sets in X.

Definition 2.4 Let X be a space and $\mathcal{B} \subseteq Coz(X)$. Then a base $\mathcal{B}$ for X is said to be an *F-base* for X if $\mathcal{B}$ is closed under countable unions and for any $A, B \in \mathcal{B}$, $int_X(cl_X(A \cup B)) = int_X(cl_X(A)) \cup int_X(cl_X(B))$.

For any quasi-F space X, Coz(X) is an F-base for X. A base $\mathcal{B}$ for a space X is called a *pretty base* if each B in $\mathcal{B}$ is clopen in X and for any sequence (B_n) in $\mathcal{B}$, $cl_X(\cup\{B_n : n \in N\}) \in \mathcal{B}$ ([6]). Clearly, for any pretty base $\mathcal{B}$ for a space X, $\{\cup\mathcal{B}' : \mathcal{B}'$ is a countable subfamily of $\mathcal{B}\}$ is an F-base for X.

Recall that a space X is called *weakly Lindelöf* if for any open cover $\mathcal{U}$ of X, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\mathcal{V}$ is dense in X and that a space X is called *locally weakly Lindelöf* if every element of X has a weakly Lindelöf neighborhood.

Proposition 2.5 ([8]) A locally weakly Lindelöf space X is quasi-F if and only if X has an F-base.

Lemma 2.6 Let X be a space. If X has a dense weakly Lindelöf subspace, then X is also weakly Lindelöf.

Proof. Let Y be a dense weakly Lindelöf subspace of X and $\mathcal{U}$ an open cover of X. Since Y is a weakly Lindelöf space, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $(\cup \mathcal{V}) \cap$ Y is dense in Y. Since Y is dense in X, $\cup \mathcal{V}$ is dense in X.

Proposition 2.7 Let X be a space with an F-base $\mathcal{B}$ and Y a dense weakly Lindelöf subspace of X. Then Y is C*-embedded in X.

Proof. By Lemma 2.6, X is a weakly Lindelöf space and by Proposition 2.5, X is a quasi-F space. Take any disjoint zero-sets A, B in Y. Then there are zero-sets C, D in Y such that $A \subseteq int_Y(C)$, $B \subseteq int_Y(D)$ and $C \cap D = \emptyset$. Since Y is weakly Lindelöf, Y - C and Y - D are weakly Lindelöf ([2]). Since $\mathcal{B}$ is a base for X and $\mathcal{B} \subseteq Coz(X)$, there are zero-sets E, F in X such that $cl_Y(int_Y(C)) = cl_X(int_X(E)) \cap Y$ and $cl_Y(int_Y(D)) = cl_X(int_X(F)) \cap Y$. Since X is a quasi-F space and Y is dense in X, $\emptyset = cl_X(int_X(E)) \cap cl_X(int_X(F)) = cl_X(int_Y(C)) \cap cl_X(int_Y(D))$ and hence $cl_X(A) \cap cl_X(B) = \emptyset$. Since βX is quasi-F, Y is C*-embedded in X.

Definition 2.8 A space such that every zero-set in it is a regular closed set is called *almost-p*.

It is well-known that if a space X is locally compact and realcompact, then βX - X is a compact almost-p-space ([9]). For a space X, υX denotes the Hewitt realcompactification of X.

Corollary 2.9 Suppose that X is an almost-p-space. Then we have the following:

(a) every dense weakly Lindelöf subspace Y of X is C-embedded in X,
(b) if Y is also realcompact, then Y = X.

Proof. (a) Since X is an almost-p-space, X is a quasi-F space and hence Coz(X) is an F-base. By Proposition 2.7, Y is C*-embedded in X. Take any zero-set Z in X with $Y \cap Z = \emptyset$. Since Y is dense in X, $int_X(Z) = \emptyset$ and hence $Z = \emptyset$, because

X is an almost-p-space. Thus Y and Z are completely separated in X and so Y is C-embedded in X ([5]).

(b) By (a), Y is C-embedded in X and hence υX = υY. Since Y $\subseteq$ X and Y is realcompact, X = Y ([5]).

Definition 2.10 Let $\mathcal{B}$ be a base for a space (X, τ) and τ^* the topology on X generated by {X - cl_τ(B) : B $\in \mathcal{B}$}, where cl_τ(B) is the closure of B in (X, τ). Then τ^* is called *the cotopology on X generated by $\mathcal{B}$*. If $\mathcal{B}$ is an F-base and (X, τ^*) is quasi-compact, then $\mathcal{B}$ is called a *cocompact F-base.*

Lemma 2.11 Let (X, τ) be a space which has an F-base $\mathcal{B}$ and τ^* the cotopology on X generated by $\mathcal{B}$. Then for any weakly Lindelöf subspace Y of (X, τ), cl_τ(Y) is closed in (X, τ^*).

Proof. Let T = cl_τ(Y) and $x \in$ X - T. Since (X, τ) is a regular space and $\mathcal{B}$ is a base for (X, τ), {cl_τ(B) : $x \in$ B $\in \mathcal{B}$} is a local base at x in (X, τ). Hence there is $B_x \in \mathcal{B}$ with $x \in B_x \subseteq cl_\tau(B_x) \subseteq$ X - T. For any $t \in$ T, there is $B_t \in \mathcal{B}$ such that $t \in B_t$ and $B_t \cap B_x = \emptyset$. Since T is weakly Lindelöf, there is a sequence (x_n) in T such that T $\subseteq cl_\tau(\cup\{B_{x_n} : n \in N\})$. Let B = $\cup\{B_{x_n} : n \in N\}$, then B $\in \mathcal{B}$ and B $\cap B_x = \emptyset$. Hence $x \in$ X - cl_τ(B) and T $\cap$ (X - cl_τ(B)) = $\emptyset$. So $x \notin cl_{(X,\tau^*)}$(T). Thus T is closed in (X, τ^*).

Theorem 2.12 Suppose that (X, τ) has a cocompact F-base $\mathcal{B}$. Then we have the following:

(a) if (X, τ) is a quasi-F space and Z is a zero-set in X such that $cl_\tau(int_\tau(Z))$ is weakly Lindelöf, then $cl_\tau(int_\tau(Z))$ is a compact subset in (X, τ), where int_τ(Z) is the interior of Z in (X, τ),

(b) if (X, τ) is a locally weakly Lindelöf space, then (X, τ) is quasi-F and locally compact, and

(c) if Y is a dense weakly Lindelöf subspace of (X, τ), then βY and X are homeomorphic.

Proof. (a) Let Z be a zero-set in (X, τ) such that $cl_\tau(int_\tau(Z))$ is weakly Lindelöf. Let A = $cl_\tau(int_\tau(Z))$ and $\mathcal{U}$ a family of open sets in X with A $\subseteq \cup\mathcal{U}$. For any $a \in$

A, there is $U_a \in \mathcal{U}$ with $a \in U_a$ and there is $B_a \in \mathcal{B}$ with $a \in B_a \subseteq cl_\tau(B_a) \subseteq U_a$. Let $\mathcal{V} = \{int_\tau(cl_\tau(B_a)) : a \in A\}$, then $\mathcal{V}$ is a refinement of $\mathcal{U}$. Let $a \in A$ and $F = cl_A((X - cl_\tau(B_a)) \cap A)$. Then F is a regular closed set in A and so weakly Lindelöf ([2]). By Lemma 2.11, $cl_\tau(F)$ is closed in (X, τ^*). Note that $cl_\tau((X - cl_\tau(B_a)) \cap A) = cl_\tau(int_\tau(X - B_a) \cap A)$. Since (X, τ) is a quasi-F space and X - B_a is a zero-set in (X, τ), $cl_\tau(int_\tau(X - B_a) \cap A) = cl_\tau(int_\tau(X - B_a)) \cap A$. Hence $cl_\tau(int_\tau(X - B_a)) \cap A$ is closed in (A, τ_A^*) and so $A - (cl_\tau(int_\tau(X - B_a)) \cap A) = int_\tau(cl_\tau(B_a)) \cap A$ is open in (A, τ_A^*). Since $\{int_\tau(cl_\tau(B_a)) \cap A : a \in A\}$ is an open cover of (A, τ_A^*) and (A, τ_A^*) is compact, there are $a_1, a_2, ..., a_n$ in A with $\cup_{i=1}^n(int_\tau(cl_\tau(B_{a_i})) \cap A) = A$. Hence $A \subseteq \cup_{i=1}^n(int_\tau(cl_\tau(B_{a_i})) \subseteq \cup_{i=1}^n U_{a_i}$. Thus A is a compact subset in (X, τ).

(b) By Proposition 2.5, X is a quasi-F space. Take any $x \in X$, then there is a zero-set neighborhood Z of x in (X, τ) such that $cl_\tau(int_\tau(Z))$ is weakly Lindelöf. By (a), $cl_\tau(int_\tau(Z))$ is a compact subset in (X, τ). Hence (X, τ) is a locally compact space.

(c) Let Y be a dense weakly Lindelöf subspace of X. By Lemma 2.6 and Proposition 2.7, Y is C*-embedded in X and by (a), (X, τ) is compact. Hence βY and X are homeomorphic.

3. Relation between cover and compactification

For any space X, let R(X) denote the regular closed sets in X, Z(X) the set of zero-sets in X and $Z(X)^\# = \{int_X(cl_X(A)) : A \in Z(X)\}$. It is well-known that R(X) is a Boolean algebra under the inclusion relation and $Z(X)^\#$ is a sublattice of R(X). Recall that a subset F of a lattice $(L, \leq)$ with the top element 1 and the bottom element 0 is called an *L-filter* if (i) $F \neq \emptyset$, $0 \notin F$, (ii) $a \in F$ and $a \leq b \in L$ implies $b \in F$, and (iii) F is closed under finite meets and that a maximal L-filter F is called an L-ultrafilter.

Let X be a compact space and $QF(X) = \{\alpha : \alpha \text{ is a } Z(X)^\#\text{-ultrafilter}\}$. For any $A \in Z(X)^\#$, let $A^* = \{\alpha : A \in \alpha\}$. Then $\{A^* : A \in Z(X)^\#\}$ is a base for closed sets of some compact topology τ on QF(X). Let QF(X) be the topological space with the topology τ. Define a map Φ_X : QF(X) $\longrightarrow$ X by $\Phi_X(\alpha) = \cap\alpha$. Then (QF(X), Φ_X) is the minimal quasi-F cover of X, that is, QF(X) is a quasi-F space and Φ_X is a covering map (= perfect irreducible) and for any quasi-F space Y and any covering map f : Y $\longrightarrow$ X, there is a covering map g : Y $\longrightarrow$ QF(X) with $\Phi_X \circ g = f$ ([7]).

Definition 3.1 Let X be a space and $\mathcal{F}$ a family of closed sets in X. Then $\mathcal{F}$ is called *a separating nest generated intersection ring* on X if (i) for each closed set H in X and $x \notin$ H, there are disjoint sets in $\mathcal{F}$, one containing H and the other containing x, (ii) it is closed under finite unions and countable intersections, and (iii) for any F $\in \mathcal{F}$, there are sequences (F_n) and (H_n) in $\mathcal{F}$ such that for any $n \in$ N, X - $H_{n+1} \subseteq F_{n+1} \subseteq$ X - $H_n \subseteq F_n$ and F $= \cap\{F_n : n \in N\}$.

For a subspace X of a space Y and a separating nest generated intersection ring $\mathcal{F}$ on Y, $\mathcal{F}_X = \{A \cap X : A \in \mathcal{F}\}$ is a separating nest generated intersection ring on X ([10]).

Let X be a space and $\mathcal{F}$ a separating nest generated intersection ring on X. Then $\mathcal{F}$ is a normal base for X ([10]). Let w(X, $\mathcal{F}$) be the Wallman compactification of X associated with $\mathcal{F}$ ([1]). Then $\mathcal{F} = Z(w(X, \mathcal{F}))_X$ and if Y is a compactification of X such that $\mathcal{F} = Z(Y)_X$, then there is a continuous map $f : w(X, \mathcal{F}) \longrightarrow$ Y with $f \circ w = j$, where j : X $\longrightarrow$ Y and w : X $\longrightarrow w(X, \mathcal{F})$ are dense embeddings ([10]).

Lemma 3.2 ([9]) Let K and Y be compactifications of a space X. Then for any disjoint closed sets A, B in K, $cl_Y(A \cap X) \cap cl_Y(B \cap X) = \emptyset$ if and only if there is a continuous map f : Y $\longrightarrow$ K with $f \circ j_1 = j_2$, where j_1 : X $\longrightarrow$ Y and j_2 : X $\longrightarrow$ K are dense embeddings.

Theorem 3.3 Let X be a space and Y $= w(X, \mathcal{F})$ for some a separating nest generated intersection ring $\mathcal{F}$ on X. Then there is a separating nest generated intersection ring $\mathcal{G}$ on $\Phi_Y^{-1}(X)$ such that QF(Y) and $w(\Phi_Y^{-1}(X), \mathcal{G})$ are homeomorphic.

Proof. Let T $= \Phi_Y^{-1}(X)$ and $\mathcal{G} = Z(QF(Y))_T$. Then $\mathcal{G}$ is a separating nest generated intersection ring on T. Let K $= w(\Phi_Y^{-1}(X), \mathcal{G})$. Since $\mathcal{G} = Z(QF(Y))_T$, there is a continuous map $h : \mathrm{K} \longrightarrow \mathrm{QF(Y)}$ with $h \circ w = j$, where $j : \mathrm{T} \longrightarrow \mathrm{QF(Y)}$ and $w : \mathrm{T} \longrightarrow \mathrm{K}$ are dense embeddings . Take any disjoint closed sets A, B in K. Since K is compact, there are disjoint zero-set neighborhoods C and D of A and B in K, respectively and since $\mathcal{G} = \mathrm{Z(K)}_T$, C $\cap$ T, D $\cap$ T $\in \mathcal{G}$. So there are zero-sets E, F in QF(Y) such that C $\cap$ T = E $\cap$ T and D $\cap$ T = F $\cap$ T. Since K and QF(Y) are compactifications of T, by Lemma 2.2, $\mathrm{int}_{QF(Y)}(\mathrm{E}) \cap \mathrm{T} = \mathrm{int}_K(\mathrm{C}) \cap \mathrm{T}$ and $\mathrm{int}_{QF(Y)}(\mathrm{F}) \cap \mathrm{T} = \mathrm{int}_K(\mathrm{D}) \cap \mathrm{T}$. Hence $\mathrm{int}_{QF(Y)}(\mathrm{E} \cap \mathrm{F}) = \emptyset$. Since QF(Y) is a quasi-F space, $\mathrm{cl}_{QF(Y)}(\mathrm{int}_{QF(Y)}(\mathrm{E})) \cap \mathrm{cl}_{QF(Y)}(\mathrm{int}_{QF(Y)}(\mathrm{F})) = \emptyset$. Since $\mathrm{cl}_{QF(Y)}(\mathrm{int}_{QF(Y)}(\mathrm{E})) \cap \mathrm{cl}_{QF(Y)}(\mathrm{int}_{QF(Y)}(\mathrm{F})) = \mathrm{cl}_{QF(Y)}(\mathrm{int}_K(\mathrm{C}) \cap \mathrm{T}) \cap \mathrm{cl}_{QF(Y)}(\mathrm{int}_K(\mathrm{D}) \cap \mathrm{T})$, $\mathrm{cl}_{QF(Y)}(\mathrm{A} \cap \mathrm{T}) \cap \mathrm{cl}_{QF(Y)}(\mathrm{B} \cap \mathrm{T}) = \emptyset$. By the above lemma, there is a continuous map $k : \mathrm{QF(Y)} \longrightarrow \mathrm{K}$ with $k \circ j = w$. Thus h is a homeomorphism.

In [7], it is shown that for any compact space X, $\Phi_X(\mathrm{Z(QF(X))}^\#) = \{\Phi_X(\mathrm{A}) : \mathrm{A} \in \mathrm{Z(QF(X))}^\#\} = \mathrm{Z(X)}^\#$. For any compactification Y of a space X, let $\Phi_{Y_X} : \Phi_Y^{-1}(\mathrm{X}) \longrightarrow \mathrm{X}$ be the restriction and corestriction of Φ_Y with respect to $\Phi_Y^{-1}(\mathrm{X})$ and X, respectively. For any separating nest generated intersection ring $\mathcal{F}$ on X, let $\mathcal{F}^\# = \{\mathrm{cl}_X(\mathrm{int}_X(\mathrm{A})) : \mathrm{A} \in \mathcal{F}\}$.

Corollary 3.4 Let X be a space and Y = $w(\mathrm{X}, \mathcal{F})$ for some a separating nest generated intersection ring $\mathcal{F}$ on X. Then there is a separating nest generated intersection ring $\mathcal{G}$ on $\Phi_Y^{-1}(\mathrm{X})$ such that $\Phi_{Y_X}(\mathcal{G}^\#) = \mathcal{F}^\#$.

Proof. Let $h = \Phi_{Y_X}$, $\mathrm{T} = \Phi_Y^{-1}(\mathrm{X})$ and $\mathcal{G} = \mathrm{Z(QF(Y))}_T$. By the above theorem, QF(Y) = $w(\mathrm{T}, \mathcal{G})$. Let K = QF(Y) and A $\in \mathcal{G}$, then there is a B $\in$ Z(K) with A = B $\cap$ T. Since T is dense in K and B is closed in K, by Lemma 2.2, $\mathrm{cl}_T(\mathrm{int}_T(\mathrm{A})) = \mathrm{cl}_K(\mathrm{int}_K(\mathrm{B})) \cap \mathrm{T}$. Since h is a covering map, $h(\mathrm{cl}_T(\mathrm{int}_T(\mathrm{A}))) = \Phi_Y(\mathrm{cl}_K(\mathrm{int}_K(\mathrm{B}))) \cap \mathrm{X}$ ([9]). Since $\Phi_Y(\mathrm{Z(K)}^\#) = \mathrm{Z(Y)}^\#$, $h(\mathrm{cl}_T(\mathrm{int}_T(\mathrm{A}))) \in \mathcal{F}^\#$. Hence $h(\mathcal{G}^\#) \subseteq \mathcal{F}^\#$. Let C $\in \mathcal{F}^\#$. Then $\mathrm{C} = \mathrm{cl}_X(\mathrm{int}_X(\mathrm{D}))$ for some D in $\mathcal{F}$ and hence there is E in Z(Y) with D = E $\cap$ X. By Lemma 2.2, $\mathrm{C} = \mathrm{cl}_Y(\mathrm{int}_Y(\mathrm{E})) \cap \mathrm{X}$. Since Φ_Y is a covering map, $\mathrm{cl}_K(\Phi_Y^{-1}(\mathrm{int}_Y(\mathrm{E}))) = \mathrm{cl}_K(\mathrm{int}_K(\Phi_Y^{-1}(\mathrm{E}))) \in \mathrm{Z(K)}^\#$ and so $\mathrm{cl}_K(\Phi_Y^{-1}(\mathrm{int}_Y(\mathrm{E}))) \cap \mathrm{T} \in \mathcal{G}^\#$. Thus $h(\mathrm{cl}_K(\Phi_Y^{-1}(\mathrm{int}_Y(\mathrm{E}))) \cap \mathrm{T}) = \Phi_Y(\mathrm{cl}_K(\Phi_Y^{-1}(\mathrm{int}_Y(\mathrm{E})))) \cap \mathrm{X} = \mathrm{cl}_Y(\mathrm{int}_Y(\mathrm{E})) \cap \mathrm{X} = \mathrm{C} \in h(\mathcal{G}^\#)$.

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