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AN APPLICATION OF THE EXTENDED IVERSEN-TSUJI THEOREM

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ABSTRACT. An Application of the Extended Iversen-Tsuji Theorem As an immediate consequence of the extended Iversen-Tsuji Theorem. We have presented a result on the boundary behavior of analytic functions on a simply connected domain. It is shown that F. Bagemihl's result for the case of capacity 0 can be obtained as a special case of $\frac{1}{2}$-dimensional Hausdorff measure zero.

In [1] we have shown a curvilinear extension of the Iversen-Tsuji theorem for an arbitrary simply connected domain.

A subset of the boundary of a simply connected domain D with at least two boundary points will be called a D-conformal null set if it corresponds to a set of linear measure zero under a one-to-one conformal mapping onto the unit disc. The set of all prime ends of D will be denoted by $\tilde{D}$. If @ is an accessible boundary point of D, then @ determines a unique prime end $P(@)$. The complex coordinate of an accessible boundary point @ will be denoted by $z(@)$.

Now we are ready to state the curvilinear extension of the Iversen-Tsuji theorem given in [1].

Theorem 1. *Let D be a simply connected domain in the complex plane, which is not the whole plane, t_0 a boundary point of D, $\tilde{E}$ a conformal null set of prime ends of D. If $f(z)$ is meromorphic in D and bounded in the intersection of D with some neighborhood of t_0, then*

$$\lim_{z\to t_0}\sup_{z\in D}|f(z)| = \lim_{z(@)\to t_0}\sup_{P(@)\in\tilde{D}-\tilde{E}}(\inf_{A}(\lim_{z\to z(@)}\sup_{z\in A}|f(z)|)),$$

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where A *is an arc at an accessible boundary point* @ *with* $P(@) \in \widetilde{D} - \widetilde{E}$ *and the convergence is in the sense of the ordinary euclidean metric.*

As immediate consequences of the above thoerem we have stated the following corollaries.

Corollary 1. *Let* D *be a simply connected domain in the* z*-plane, which is not the whole plane, and* t_0 *a boundary point of* D, $\widetilde{E}$ *a conformal null set of prime ends of* D. *If* $f(z)$ *is meromorphic in* D *and bounded in the intersection of* D *with some neighborhood* $N(t_0)$ *of* t_0, *and at each accessible boundary point* @ *with* $P(@) \in \widetilde{D} - \widetilde{E}$, $z(@) \in \partial D \cap N(t_0)$, *there exists an arc* $A_@$ *at* $z(@)$ *on which* $\lim_{z \to z(@), z \in A_@} |f(z)| \leq m$, *then*

$$\lim_{z \to t_0} \sup_{z \in D} |f(z)| \leq m.$$

Corollary 2. *Let* D *be a simply connected domain in the* z*-plane, which is not the whole plane, and let* t_0 *be a boundary point of* D, E *a subset of* ∂D *such that the set* $\{P(@) : z(@) \in E, @$ *is an accessible boundary point of* $D\}$ *is a* D*-conformal null set. If* $u(z)$ *is harmonic in* D *and bounded above in the intersection of* D *with some neighborhood of* t_0, *then*

$$\lim_{z \to t_0} \sup_{z \in D} u(z) = \lim_{z(@) \to t_0} \sup_{P(@) \in \partial D - E} \left(\inf_{A_@} \left(\lim_{z \to z(@)} \sup_{z \in A_@} u(z) \right) \right).$$

where $A_@$ *is an arc at an accessible boundary point* @ *with* $z(@) \in \partial D - E$ *and the convergence is in the sense of the ordinary euclidean metric.*

In this note we state several corollaries as immediate consequences of the above theorem.

Corollary 3. *In the statement of Corollary 1 (respectively Corollary 2) let* ∂D *be a rectifiable Jordan curve. If the set* E *of exceptional points is of linear measure zero, then the same conclusion holds as in Corollary 1 (respectively Corollary 2).*

Proof. $E(= \widetilde{E})$ is a conformal null set (see [2]).

Corollary 4. *Under the same hypothesis as in Corollary 1 (respectively Corollary 2) if the set E of exceptional points is of capacity zero, the same conclusion holds as in Corollary 1 (respectively Corollary 2).*

Proof. It is well-known that E is a conformal null set.

Corollary 5. *Let D be a simply connected domain whose boundary is a Jordan curve J. Let E be a set of points on J, of linear measure zero if J is rectifiable, of $1/2$-dimemsional Hausdroff measure zero otherwise, and suppose that the function $f(z)$ is analytic in D.*

Assume that at every point $t \in J - E$ there is an arc A_t in D at t such that $lim_{z \to t} sup_{z \in A_t} |f(z)| \leq m$, such that $f(z)$ does not have the asymptotic value ∞ at any point of J. Then

$$\lim_{z \in D} |f(z)| \leq m.$$

Proof. The set E is a conformal null set in both cases. Thus these are special cases of Corollary 3.

Corollary 6 (Begemihl [3]). *In place of the assumption in Corollary 5 that E is of 1/2-dimensional Hausdorff measure zero, let E be of logarithmic capacity zero. Then we have the same conclusion.*

Proof. The Set E is a conformal null set.

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